faster, notwithstanding the likelihood of nanosecond circuitry. This, too, can be designed to be generally recursive and nestable.[8,9]

3) The general problem is far bigger than generalized data searcher. Lee and Paull are near it in their mention that the cells themselves should guide the flow of information. This asks for generalization[9] in which we abolish finally the distinction between programs and data. In other words, we remove the "atom" from McCarthy's recursive logic,[8] or we remove the "individual" from Russell's logical theory of types.

Incidentally, input $\alpha_5\chi_3\beta$ to the memory string[10] would cause it to write out the string $\alpha_3\chi_1\beta\alpha_1\chi_2\beta\alpha_5\chi_3\beta\alpha_4\chi_4\beta$ and not the string stated.

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[8] J. McCarthy, "Recursive functions of symbolic expressions and their computation by machine," *Commun. ACM*, vol. 3, pp. 184–195; April, 1960.
[9] ——, "A basis for a mathematical theory of computation", in "Computer Programming and Formal Systems," P. Braffort and D. Hirschberg, Eds., North Holland Publishing Co., Amsterdam, Holland; 1963.
[10] See Lee and Paull, *op. cit.*, p. 929.

Relationships between Different Kinds of Network Parameters, Not Assuming Reciprocity or Equality of the Waveguide or Transmission Line Characteristic Impedances*

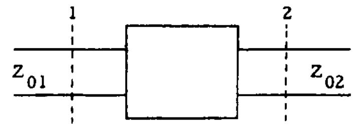
It is well known that linear networks can be characterized by different sets of parameters, and these different sets of parameters are each related, one to the other. Tables have been published[1,2] giving various relationships for two terminal-pair network parameters, or two ports, and they are useful in the analysis both of lumped element circuits and of waveguide junctions.

It is noted however that in these tabulations, the assumptions are often made that 1) nonreciprocal behavior is excluded, or ruled out at the beginning and 2) the characteristic impedances of the waveguide leads of transmission lines furnishing access to the network or junction are all equal to each other.

Nonreciprocal behavior is becoming commonplace and the choice of unequal characteristic impedances is frequently useful.

Therefore, the following more general table of relationships between different sets of parameters is felt to be timely and useful.

TABLE I



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{Y_{11}Y_{22} - Y_{12}Y_{21}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{Z_{11}Z_{22} - Z_{12}Z_{21}} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{C} \begin{bmatrix} A & AD - BC \\ 1 & D \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{B} \begin{bmatrix} D & -(AD - BC) \\ -1 & A \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{Z_{21}} \begin{bmatrix} Z_{11} & (Z_{11}Z_{22} - Z_{12}Z_{21}) \\ 1 & Z_{22} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{-Y_{21}} \begin{bmatrix} Y_{22} & 1 \\ (Y_{11}Y_{22} - Y_{12}Y_{21}) & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{r_{22}} \begin{bmatrix} r_{12} & (r_{11}r_{22} - r_{12}r_{21}) \\ 1 & -r_{21} \end{bmatrix}$$

$$\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} -(S_{11}S_{22} - S_{12}S_{21}) & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{\left(\frac{Z_{11}}{Z_{01}}+1\right)\left(\frac{Z_{22}}{Z_{02}}+1\right) - \frac{Z_{12}Z_{21}}{Z_{01}Z_{02}}} \begin{bmatrix} \left(\frac{Z_{11}}{Z_{01}}-1\right)\left(\frac{Z_{22}}{Z_{02}}+1\right) - \frac{Z_{12}Z_{21}}{Z_{01}Z_{02}} & 2\frac{Z_{12}}{Z_{02}} \\ 2\frac{Z_{21}}{Z_{01}} & \left(\frac{Z_{11}}{Z_{01}}+1\right)\left(\frac{Z_{22}}{Z_{02}}-1\right) - \frac{Z_{12}Z_{21}}{Z_{01}Z_{02}} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{\left(1+\frac{Y_{11}}{Y_{01}}\right)\left(1+\frac{Y_{22}}{Y_{02}}\right) - \frac{Y_{12}Y_{21}}{Y_{01}Y_{02}}} \begin{bmatrix} \left(1-\frac{Y_{11}}{Y_{01}}\right)\left(1+\frac{Y_{22}}{Y_{02}}\right) + \frac{Y_{12}Y_{21}}{Y_{01}Y_{02}} & -2\frac{Y_{12}}{Y_{01}} \\ -2\frac{Y_{21}}{Y_{02}} & \left(1+\frac{Y_{11}}{Y_{01}}\right)\left(1-\frac{Y_{22}}{Y_{02}}\right) + \frac{Y_{12}Y_{21}}{Y_{01}Y_{02}} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{(B+CZ_{01}Z_{02})(AZ_{02}+DZ_{01})} \begin{bmatrix} (B-CZ_{01}Z_{02}) + (AZ_{02}-DZ_{01}) & 2Z_{01}(AD-BC) \\ 2Z_{02} & (B-CZ_{01}Z_{02}) - (AZ_{02}-DZ_{01}) \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \begin{bmatrix} Z_{01}[(1+S_{11})(1-S_{22}) + S_{12}S_{21}] & 2Z_{01}S_{21} \\ 2Z_{02}S_{12} & Z_{02}[(1-S_{11})(1+S_{22}) + S_{12}S_{21}] \end{bmatrix}$$

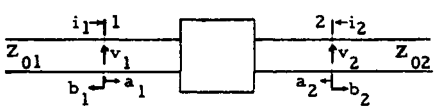
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{(1+S_{11})(1+S_{22}) - S_{12}S_{21}} \begin{bmatrix} Y_{01}[(1-S_{11})(1+S_{22}) + S_{12}S_{21}] & -2Y_{01}S_{12} \\ -2Y_{02}S_{21} & Y_{02}[(1+S_{11})(1-S_{22}) + S_{12}S_{21}] \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2S_{21}} \begin{bmatrix} [(1+S_{11})(1-S_{22}) + S_{12}S_{21}] & Z_{02}[(1+S_{11})(1+S_{22}) - S_{12}S_{21}] \\ \frac{1}{Z_{01}}[(1-S_{11})(1-S_{22}) - S_{12}S_{21}] & \frac{Z_{01}}{Z_{02}}[(1-S_{11})(1+S_{22}) + S_{12}S_{21}] \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2} \begin{bmatrix} [(r_{22}+r_{11}) + (r_{21}+r_{12})] & Z_{02}[(r_{22}-r_{11}) - (r_{21}-r_{12})] \\ \frac{1}{Z_{01}}[(r_{22}-r_{11}) + (r_{21}-r_{12})] & \frac{Z_{01}}{Z_{02}}[(r_{22}+r_{11}) - (r_{21}+r_{12})] \end{bmatrix}$$

In order to keep it of reasonable length the number of parameters was limited, but those chosen were selected because they are widely used.

The sets of parameters chosen were impedance Z, admittance Y, general circuit constants $ABCD$, scattering S and wave cascading coefficients r. They are defined in the following equations, which relate the terminal variables v and i and a and b in one waveguide to those in the other waveguide.



In matrix form,

$$v = Zi, \qquad i = Yv$$
$$b = Sa$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}.$$

Note that the terminal surfaces 1 and 2 may be replaced by terminal pairs when considering two terminal-pair networks.

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* Received June 24, 1962.
[1] E. A. Guillemin, "Communication Networks," John Wiley and Sons, Inc., New York, N. Y., vol. II, pp. 137–138; 1935.
[2] E. F. Bolinder, "Note on the matrix representation of linear two-port networks," IRE Trans. on Circuit Theory, (*Correspondence*); vol. CT-4, pp. 337–339; December, 1957.

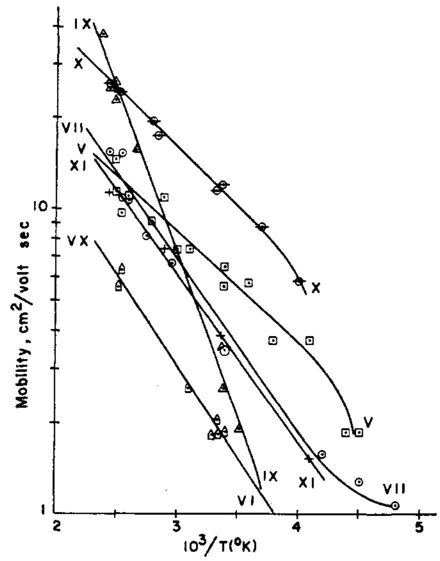

Fig. 1—Temperature dependence of Hall mobility μ_H for CdS films.

TABLE I

Sample	Resistivity (Ω cm) (300 K)	Hall Mobility (cm²/V sec) (300 K)	E_R (eV)	E_μ (eV)	E_ρ (eV)	Source T (°C)	Substrate T (°C)	Processing	Color
5	32	6	0.13	0.07	0.21	—	23	360°C H_2 bake	yellow
6	6	2	0.06	0.12	0.18	—	23	None	black
7	24	4.4	0.07	0.12	0.18	880	100	None	orange
8	10,000	—	—	—	0.32	820	200	None	yel-or.
9	270	3.2	0.05	0.20	0.25	750	200	None	yel-or.
10	1900	12	0.35	0.07	0.42	760	140	None	yel-or.
11	650	4	0.12	0.10	0.22	730	160	None	yel-or.

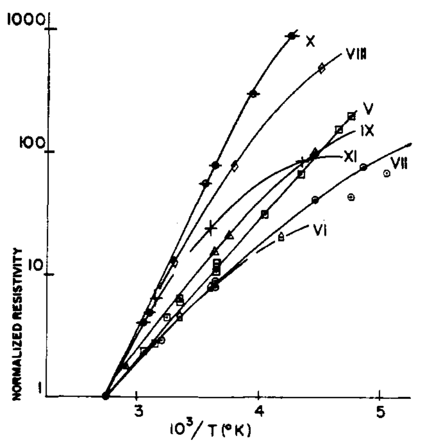

Fig. 2—Temperature dependence of the resistivity ρ of CdS films. The ordinate is normalized at 400°K.

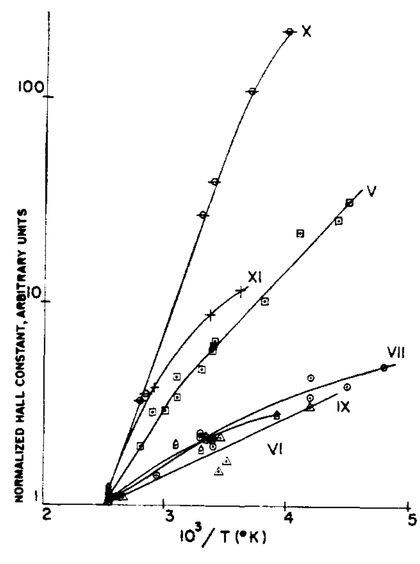

Fig. 3—Temperature dependence of the Hall constant R_H for deposited CdS films. The ordinate is normalized at 400°K.

of the reciprocal of the free-charge density, the sum of the activation energies for the mobility and the Hall coefficient should equal the activation energy of the resistivity, as is observed.

Discussion

The observation of an exponential dependence for Hall mobility on temperature in deposited CdS films was first reported by Berger.[3] Such a dependence has also been found in deposited films of PbS, and, following the analysis of Petritz,[4] it is often ascribed to scattering at the boundaries between the small crystallites which make up the film. There is reason to doubt this hypothesis chiefly because of the near independence of the observed Hall-mobility value on crystallite size. This view is corroborated by Berger.[3] Work is now going on in this laboratory to ascertain whether or not the observed mobility dependence is not resultant from the large deep-trap densities that are known to characterize these films. This information is being sought through photo-Hall effect measurements, and through thermally-stimulated trap emptying studies. Present technology in this effort permits the fabrication of films with significantly higher mobilities and resistivities than those discussed in this communication.

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[3] H. Berger, "Über das Ausheilen von Gitterfehlern frischaufgedampfter CdS-Schichten (I)," *Phys. Status Solidi*, vol. 1, pp. 739–757; July, 1961.
[4] R. L. Petritz, "Theory of photoconductivity in semiconductor films," *Phys. Rev.*, vol. 104, pp. 1508–1516; December, 1956.

Correction to "Relationships between Different Kinds of Network Parameters, Not Assuming Reciprocity or Equality of the Waveguide or Transmission Line Characteristics Impedances"[1]

The following has been called to the attention of the Editor. In the relationship having the S-matrix on the left and expressions involving A, B, C, D, Z_{01} and Z_{02} on the right, a plus sign should appear in the denominator between the terms $(B+CZ_{01}Z_{02})$ and $(AZ_{02}+DZ_{01})$.

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Manuscript received February 11, 1964.
[1] R. W. Beatty and D. M. Kearns, Proc. IEEE (*Correspondence*), vol. 52, p. 84; January, 1964.

Considerations Regarding the Use of Semiconductor Heterojunctions for Laser Operation

In a recent communication,[1] Kroemer has proposed a new injection scheme using heterojunctions for possible laser action, in which an indirect-gap semiconductor, say Ge, is sandwiched between two direct-gap semiconductors of opposite types, say n- and p-type GaAs. In our laboratory, we also have considered the feasibility of using heterojunctions for laser work based on a different scheme. Kroemer's proposal presupposes that 1) injected electrons and holes would be trapped in the center region by potential barriers at the two heterojunctions and 2) laser action would eventually occur at sufficiently high carrier injection levels. The argument presented in his communication, however, is rather vague and misleading. We would like to discuss theoretical considerations in using heterojunctions for laser operation and to present our scheme in view of these considerations.

Manuscript received January 31, 1964. The research reported herein is made possible through support received from the Departments of Army, Navy and Air Force under grant AF-AFOSR-139-63.
[1] H. Kroemer, "A proposed class of heterojunction injection lasers," Proc. IEEE (*Correspondence*), vol. 51, pp. 1782–1783; December, 1963.

Conversions Between S, Z, Y, h, $ABCD$, and T Parameters which are Valid for Complex Source and Load Impedances

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***Abstract*— This paper provides tables which contain the conversion between the various common two-port parameters, Z, Y, h, $ABCD$, S, and T. The conversion are valid for complex normalizing impedances. An example is provided which verifies the conversions to and from S parameters.**

I. Introduction

MOST microwave textbooks these days seem to provide a table of the conversion between the various 2-port parameters. These 2-port parameters often include Z (impedance), Y (admittance), h (hybrid), $ABCD$ (chain), S (scattering), and T (chain scattering or chain transfer). While the scattering parameters have been shown [1] to be valid for complex normalizing impedances (with positive real parts), the tables in [2]–[15] are not valid for complex source and load impedances. Often, the tables only provide conversions for the cases where port 1 and port 2 normalizing impedances are equal, i.e., $Z_{01} = Z_{02} = Z_0$. Some have results in which Z_{01} and Z_{02} are normalized to 1. Others provide equations for port 1 and port 2 impedances Z_{01} and Z_{02} to be unique. However, in all of these cases, the results are not valid when the impedances, Z_{01} and Z_{02}, or just Z_0, are complex.

Of the two-port parameters mentioned, only the S and T parameters are dependent upon the source and load impedances. In this paper, the derivations of the conversions from the S and T parameters to the other 2-port parameters includes complex source and load impedances. The equations developed in this work are valid with port 1 and port 2 normalizing impedances complex and unique. When the normalizing impedances are real, the results simplify to those shown in other references. To make the list complete, the conversions between the Z, Y, h, and $ABCD$ parameters as well as between S and T parameters are included.

II. Derivation

Two-port parameters are defined for a general 2-port network as shown in Fig. 1. Using the voltages and currents defined in this figure, the various 2-port parameters are written as

Manuscript received December 2, 1992; revised April 13, 1993.
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IEEE Log Number 9214525.

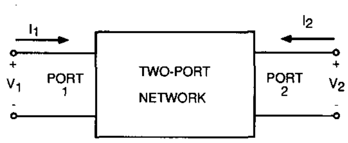

Fig. 1. A general two-port network with voltages and currents defined.

Z parameters

$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2 \quad (1a)$$

$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2, \quad (1b)$$

Y parameters

$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2 \quad (2a)$$

$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2, \quad (2b)$$

h parameters

$$V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2 \quad (3a)$$

$$I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2, \quad (3b)$$

$ABCD$ parameters

$$V_1 = A \cdot V_2 - B \cdot I_2 \quad (4a)$$

$$I_1 = C \cdot V_2 - D \cdot I_2, \quad (4b)$$

S parameters

$$b_1 = S_{11} \cdot a_1 + S_{12} \cdot a_2 \quad (5a)$$

$$b_2 = S_{21} \cdot a_1 + S_{22} \cdot a_2, \quad (5b)$$

TABLE I
EQUATIONS FOR THE CONVERSION BETWEEN S PARAMETERS AND Z, Y, h, AND $ABCD$ PARAMETERS WITH A SOURCE IMPEDANCE Z_{01} AND LOAD IMPEDANCE Z_{02}

$S_{11} = \frac{(Z_{11}-Z_{01}^*)(Z_{22}+Z_{02})-Z_{12}Z_{21}}{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}$	$Z_{11} = \frac{(Z_{01}^*+S_{11}Z_{01})(1-S_{22})+S_{12}S_{21}Z_{01}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$
$S_{12} = \frac{2Z_{12}(R_{01}R_{02})^{1/2}}{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}$	$Z_{12} = \frac{2S_{12}(R_{01}R_{02})^{1/2}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$
$S_{21} = \frac{2Z_{21}(R_{01}R_{02})^{1/2}}{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}$	$Z_{21} = \frac{2S_{21}(R_{01}R_{02})^{1/2}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$
$S_{22} = \frac{(Z_{11}+Z_{01})(Z_{22}-Z_{02}^*)-Z_{12}Z_{21}}{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}$	$Z_{22} = \frac{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$
$S_{11} = \frac{(1-Y_{11}Z_{01}^*)(1+Y_{22}Z_{02})+Y_{12}Y_{21}Z_{01}^*Z_{02}}{(1+Y_{11}Z_{01})(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}Z_{02}}$	$Y_{11} = \frac{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}$
$S_{12} = \frac{-2Y_{12}(R_{01}R_{02})^{1/2}}{(1+Y_{11}Z_{01})(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}Z_{02}}$	$Y_{12} = \frac{-2S_{12}(R_{01}R_{02})^{1/2}}{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}$
$S_{21} = \frac{-2Y_{21}(R_{01}R_{02})^{1/2}}{(1+Y_{11}Z_{01})(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}Z_{02}}$	$Y_{21} = \frac{-2S_{21}(R_{01}R_{02})^{1/2}}{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}$
$S_{22} = \frac{(1+Y_{11}Z_{01})(1-Y_{22}Z_{02}^*)+Y_{12}Y_{21}Z_{01}Z_{02}^*}{(1+Y_{11}Z_{01})(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}Z_{02}}$	$Y_{22} = \frac{(Z_{01}^*+S_{11}Z_{01})(1-S_{22})+S_{12}S_{21}Z_{01}}{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}$
$S_{11} = \frac{(h_{11}-Z_{01}^*)(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}{(Z_{01}+h_{11})(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}$	$h_{11} = \frac{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}$
$S_{12} = \frac{2h_{12}(R_{01}R_{02})^{1/2}}{(Z_{01}+h_{11})(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}$	$h_{12} = \frac{2S_{12}(R_{01}R_{02})^{1/2}}{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}$
$S_{21} = \frac{-2h_{21}(R_{01}R_{02})^{1/2}}{(Z_{01}+h_{11})(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}$	$h_{21} = \frac{-2S_{21}(R_{01}R_{02})^{1/2}}{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}$
$S_{22} = \frac{(Z_{01}+h_{11})(1-h_{22}Z_{02}^*)+h_{12}h_{21}Z_{02}^*}{(Z_{01}+h_{11})(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}$	$h_{22} = \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}$
$S_{11} = \frac{AZ_{02}+B-CZ_{01}^*Z_{02}-DZ_{01}^*}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$	$A = \frac{(Z_{01}^*+S_{11}Z_{01})(1-S_{22})+S_{12}S_{21}Z_{01}}{2S_{21}(R_{01}R_{02})^{1/2}}$
$S_{12} = \frac{2(AD-BC)(R_{01}R_{02})^{1/2}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$	$B = \frac{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}{2S_{21}(R_{01}R_{02})^{1/2}}$
$S_{21} = \frac{2(R_{01}R_{02})^{1/2}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$	$C = \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}(R_{01}R_{02})^{1/2}}$
$S_{22} = \frac{-AZ_{02}^*+B-CZ_{01}Z_{02}^*+DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$	$D = \frac{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}{2S_{21}(R_{01}R_{02})^{1/2}}$

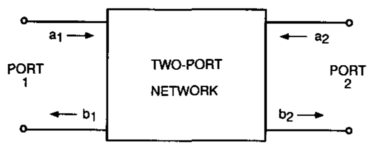

Fig. 2. A general two port network with a's and b's defined.

T parameters[1]

$$a_1 = T_{11} \cdot b_2 + T_{12} \cdot a_2 \quad (6a)$$

$$b_1 = T_{21} \cdot b_2 + T_{22} \cdot a_2 \quad (6b)$$

where the a's and b's are shown in Fig. 2 and defined below.

$$a_j = \left[\frac{Z_{0j} + Z_{0j}^*}{2}\right]^{1/2} \cdot I_{ji} \quad (7a)$$

$$b_j = \left[\frac{Z_{0j} + Z_{0j}^*}{2}\right]^{1/2} \cdot I_{jr} \quad (7b)$$

[1]Some authors, (e.g. Rizzi [16]) define the T parameters as $\mathrm{b}_1 = \mathrm{T}_{11} \cdot \mathrm{a}_2 + \mathrm{T}_{12} \cdot \mathrm{b}_2$, and $\mathrm{a}_1 = \mathrm{T}_{21} \cdot \mathrm{a}_2 + \mathrm{T}_{22}\mathrm{b}_2$. In this case, the parameters can just be switched from what is derived in this paper. T_{11} and T_{22} are switched, T_{12} and T_{21} are switched.

where * indicates complex conjugate and Z_{0j} is the normalizing impedance for the jth port. For two-port networks, Z_{01} and Z_{02} are the source and load impedances of the system in which the S parameters of the two-port are measured or calculated. I_{ji} and I_{jr} are the incident and reflected currents for the jth port. Knowing that,

$$I_j = I_{ji} - I_{jr} \quad (8)$$

we can solve (7a) and (7b) for I_{ji} and I_{jr} and substitute them into (8) to get,

$$I_j = \left[\frac{2}{Z_{0j} + Z_{0j}^*}\right]^{1/2} \cdot (a_j - b_j). \quad (9)$$

Knowing also that,

$$V_j = V_{ji} + V_{jr} \quad (10)$$

where V_{ji} and V_{jr} are the incident and reflected voltage at the jth port, we can substitute the expressions for I_{ji} and I_{jr} along with

$$V_{ji} = I_{ji} \cdot Z_{0j}^* \qquad V_{jr} = I_{jr} \cdot Z_{0j}$$

into (10) to get,

$$V_j = \left[\frac{2}{Z_{0j} + Z_{0j}^*}\right]^{1/2} \cdot (a_j \cdot Z_{0j}^* + b_j \cdot Z_{0j}). \quad (11)$$

TABLE II
EQUATIONS FOR THE CONVERSION BETWEEN T PARAMETERS AND Z, Y, h, AND $ABCD$ PARAMETERS WITH A SOURCE IMPEDANCE Z_{01} AND LOAD IMPEDANCE Z_{02}

$T_{11} = \frac{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}{2Z_{21}(R_{01}R_{02})^{1/2}}$	$Z_{11} = \frac{Z_{01}^*(T_{11}+T_{12})+Z_{01}(T_{21}+T_{22})}{T_{11}+T_{12}-T_{21}-T_{22}}$
$T_{12} = \frac{(Z_{11}+Z_{01})(Z_{02}^*-Z_{22})+Z_{12}Z_{21}}{2Z_{21}(R_{01}R_{02})^{1/2}}$	$Z_{12} = \frac{2(R_{01}R_{02})^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{T_{11}+T_{12}-T_{21}-T_{22}}$
$T_{21} = \frac{(Z_{11}-Z_{01}^*)(Z_{22}+Z_{02})-Z_{12}Z_{21}}{2Z_{21}(R_{01}R_{02})^{1/2}}$	$Z_{21} = \frac{2(R_{01}R_{02})^{1/2}}{T_{11}+T_{12}-T_{21}-T_{22}}$
$T_{22} = \frac{(Z_{01}^*-Z_{11})(Z_{22}-Z_{02}^*)+Z_{12}Z_{21}}{2Z_{21}(R_{01}R_{02})^{1/2}}$	$Z_{22} = \frac{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}-T_{22})}{T_{11}+T_{12}-T_{21}-T_{22}}$
$T_{11} = \frac{(-1-Y_{11}Z_{01})(1+Y_{22}Z_{02})+Y_{12}Y_{21}Z_{01}Z_{02}}{2Y_{21}(R_{01}R_{02})^{1/2}}$	$Y_{11} = \frac{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}-T_{22})}{T_{11}Z_{01}^*Z_{02}^*-T_{12}Z_{01}^*Z_{02}+T_{21}Z_{01}Z_{02}^*-T_{22}Z_{01}Z_{02}}$
$T_{12} = \frac{(1+Y_{11}Z_{01})(1-Y_{22}Z_{02}^*)+Y_{12}Y_{21}Z_{01}Z_{02}^*}{2Y_{21}(R_{01}R_{02})^{1/2}}$	$Y_{12} = \frac{-2(R_{01}R_{02})^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{T_{11}Z_{01}^*Z_{02}^*-T_{12}Z_{01}^*Z_{02}+T_{21}Z_{01}Z_{02}^*-T_{22}Z_{01}Z_{02}}$
$T_{21} = \frac{(Y_{11}Z_{01}^*-1)(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}^*Z_{02}}{2Y_{21}(R_{01}R_{02})^{1/2}}$	$Y_{21} = \frac{-2(R_{01}R_{02})^{1/2}}{T_{11}Z_{01}^*Z_{02}^*-T_{12}Z_{01}^*Z_{02}+T_{21}Z_{01}Z_{02}^*-T_{22}Z_{01}Z_{02}}$
$T_{22} = \frac{(1-Y_{11}Z_{01}^*)(1-Y_{22}Z_{02}^*)-Y_{12}Y_{21}Z_{01}^*Z_{02}^*}{2Y_{21}(R_{01}R_{02})^{1/2}}$	$Y_{22} = \frac{Z_{01}^*(T_{11}+T_{12})+Z_{01}(T_{21}+T_{22})}{T_{11}Z_{01}^*Z_{02}^*-T_{12}Z_{01}^*Z_{02}+T_{21}Z_{01}Z_{02}^*-T_{22}Z_{01}Z_{02}}$
$T_{11} = \frac{(-h_{11}-Z_{01})(1+h_{22}Z_{02})+h_{12}h_{21}Z_{02}}{2h_{21}(R_{01}R_{02})^{1/2}}$	$h_{11} = \frac{Z_{02}^*(T_{11}Z_{01}^*+T_{21}Z_{01})-Z_{02}(T_{12}Z_{01}^*+T_{22}Z_{01})}{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}+T_{22})}$
$T_{12} = \frac{(h_{11}+Z_{01})(1-h_{22}Z_{02}^*)+h_{12}h_{21}Z_{02}^*}{2h_{21}(R_{01}R_{02})^{1/2}}$	$h_{12} = \frac{2(R_{01}R_{02})^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}+T_{22})}$
$T_{21} = \frac{(Z_{01}^*-h_{11})(1+h_{22}Z_{02})+h_{12}h_{21}Z_{02}}{2h_{21}(R_{01}R_{02})^{1/2}}$	$h_{21} = \frac{-2(R_{01}R_{02})^{1/2}}{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}+T_{22})}$
$T_{22} = \frac{(h_{11}-Z_{01}^*)(1-h_{22}Z_{02}^*)+h_{12}h_{21}Z_{02}^*}{2h_{21}(R_{01}R_{02})^{1/2}}$	$h_{22} = \frac{T_{11}+T_{12}-T_{21}-T_{22}}{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}+T_{22})}$
$T_{11} = \frac{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}{2(R_{01}R_{02})^{1/2}}$	$A = \frac{Z_{01}^*(T_{11}+T_{12})+Z_{01}(T_{21}+T_{22})}{2(R_{01}R_{02})^{1/2}}$
$T_{12} = \frac{AZ_{02}^*-B+CZ_{01}Z_{02}^*-DZ_{01}}{2(R_{01}R_{02})^{1/2}}$	$B = \frac{Z_{02}^*(T_{11}Z_{01}^*+T_{21}Z_{01})-Z_{02}(T_{12}Z_{01}^*+T_{22}Z_{01})}{2(R_{01}R_{02})^{1/2}}$
$T_{21} = \frac{AZ_{02}+B-CZ_{01}^*Z_{02}-DZ_{01}^*}{2(R_{01}R_{02})^{1/2}}$	$C = \frac{T_{11}+T_{12}-T_{21}-T_{22}}{2(R_{01}R_{02})^{1/2}}$
$T_{22} = \frac{AZ_{02}^*-B-CZ_{01}^*Z_{02}^*+DZ_{01}^*}{2(R_{01}R_{02})^{1/2}}$	$D = \frac{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}-T_{22})}{2(R_{01}R_{02})^{1/2}}$

Solving (9) and (11) for a_j and b_j gives

$$a_j = \frac{V_j + Z_{0j}I_j}{[2(Z_{0j} + Z_{0j}^*)]^{1/2}} \tag{12}$$

$$b_j = \frac{V_j - Z_{0j}^*I_j}{[2(Z_{0j} + Z_{0j}^*)]^{1/2}}. \tag{13}$$

Equations (12) and (13) are (3) and (4) in [1] and served as the starting point.

The notation, $S \leftrightarrow Z$, indicates the conversion from S parameters to Z parameters and Z parameters to S parameters. Since S and T parameters are defined in terms of a's and b's, they will contain the source and load normalizing impedances Z_{01} and Z_{02}. The other 2-port parameters are defined independent of the source and load impedances.

To derive the conversions, $S \leftrightarrow Z$, $S \leftrightarrow Y$, $S \leftrightarrow h$, $S \leftrightarrow ABCD$, $T \leftrightarrow Z$, $T \leftrightarrow Y$, $T \leftrightarrow h$, and $T \leftrightarrow ABCD$, it is necessary to use (9), (11)–(13). For example, to derive the expressions for S parameters in terms of the Z parameters, first substitute (9) and (11) into (1a) and (1b) and solve for b_1 and b_2 to get in the form of (5a) and (5b). Likewise, to get the expressions for the Z parameters in terms of the S parameters, substitute (12) and (13) into (5a) and (5b) and solve for V_1 and V_2 to get in the form of (1a) and (1b).

Since Z, Y, h, and $ABCD$ parameters do not require normalizing impedances, the conversions, $Z \leftrightarrow Y$, $Z \leftrightarrow h$, $Z \leftrightarrow ABCD$, $Y \leftrightarrow h$, $Y \leftrightarrow ABCD$, and $h \leftrightarrow ABCD$, as well as $S \leftrightarrow T$, are straight forward. These conversions are accomplished by rearranging one set of equations into the form of the other. These conversions appear in many of the references cited and are included here for completeness.

III. RESULTS

The results are given in the following tables. In these tables, Z_{01} and Z_{02} are the source and load impedances of the system to which the S and T parameters pertain. Complex conjugate is indicated by *, and R_{01} and R_{02} are the real parts of Z_{01} and Z_{02}.

Table I gives the conversions between S parameters and Z, Y, h, and $ABCD$ parameters. Table II gives the conversions

TABLE III
EQUATIONS FOR THE CONVERSION BETWEEN S PARAMETERS AND NORMALIZED Z, Y, h, AND $ABCD$ PARAMETERS WITH A SOURCE IMPEDANCE Z_{01} AND LOAD IMPEDANCE Z_{02}

$$S_{11} = \frac{\left[Z_{11n} - \frac{Z_{01}^*}{Z_{01}}\right](Z_{22n}+1) - Z_{12n}Z_{21n}}{(Z_{11n}+1)(Z_{22n}+1) - Z_{12n}Z_{21n}}$$

$$S_{12} = \frac{2Z_{12n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(Z_{11n}+1)(Z_{22n}+1) - Z_{12n}Z_{21n}}$$

$$S_{21} = \frac{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(Z_{11n}+1)(Z_{22n}+1) - Z_{12n}Z_{21n}}$$

$$S_{22} = \frac{(Z_{11n}+1)\left[Z_{22n} - \frac{Z_{02}^*}{Z_{02}}\right] - Z_{12n}Z_{21n}}{(Z_{11n}+1)(Z_{22n}+1) - Z_{12n}Z_{21n}}$$

$$Z_{11n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right](1-S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}$$

$$Z_{12n} = \frac{2S_{12}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}$$

$$Z_{21n} = \frac{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}$$

$$Z_{22n} = \frac{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}$$

$$Z_{11n} = \frac{Z_{11}}{Z_{01}} \qquad Z_{12n} = \frac{Z_{12}}{(Z_{01}Z_{02})^{1/2}} \qquad Z_{21n} = \frac{Z_{21}}{(Z_{01}Z_{02})^{1/2}} \qquad Z_{22n} = \frac{Z_{22}}{Z_{02}}$$

$$S_{11} = \frac{\left[1 - Y_{11n}\left[\frac{Z_{01}^*}{Z_{01}}\right]\right](1+Y_{22n}) + Y_{12n}Y_{21n}\left[\frac{Z_{01}^*}{Z_{01}}\right]}{(1+Y_{11n})(1+Y_{22n}) - Y_{12n}Y_{21n}}$$

$$S_{12} = \frac{-2Y_{12n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1+Y_{11n})(1+Y_{22n}) - Y_{12n}Y_{21n}}$$

$$S_{21} = \frac{-2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1+Y_{11n})(1+Y_{22n}) - Y_{12n}Y_{21n}}$$

$$S_{22} = \frac{(1+Y_{11n})\left[1 - Y_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right] + Y_{12n}Y_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{(1+Y_{11n})(1+Y_{22n}) - Y_{12n}Y_{21n}}$$

$$Y_{11n} = \frac{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}$$

$$Y_{12n} = \frac{-2S_{12}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}$$

$$Y_{21n} = \frac{-2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}$$

$$Y_{22n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right](1-S_{22}) + S_{12}S_{21}}{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}$$

$$Y_{11n} = Y_{11}Z_{01} \qquad Y_{12n} = Y_{12}(Z_{01}Z_{02})^{1/2} \qquad Y_{21n} = Y_{21}(Z_{01}Z_{02})^{1/2} \qquad Y_{22n} = Y_{22}Z_{02}$$

$$S_{11} = \frac{\left[h_{11n} - \frac{Z_{01}^*}{Z_{01}}\right](1+h_{22n}) - h_{12n}h_{21n}}{(1+h_{11n})(1+h_{22n}) - h_{12n}h_{21n}}$$

$$S_{12} = \frac{2h_{12n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1+h_{11n})(1+h_{22n}) - h_{12n}h_{21n}}$$

$$S_{21} = \frac{-2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1+h_{11n})(1+h_{22n}) - h_{12n}h_{21n}}$$

$$S_{22} = \frac{(1+h_{11n})\left[1 - h_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right] + h_{12n}h_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{(1+h_{11n})(1+h_{22n}) - h_{12n}h_{21n}}$$

$$h_{11n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}$$

$$h_{12n} = \frac{2S_{12}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}$$

$$h_{21n} = \frac{-2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}$$

$$h_{22n} = \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}$$

$$h_{11n} = \frac{h_{11}}{Z_{01}} \qquad h_{12n} = h_{12}\left[\frac{Z_{02}}{Z_{01}}\right]^{1/2} \qquad h_{21n} = h_{21}\left[\frac{Z_{02}}{Z_{01}}\right]^{1/2} \qquad h_{22n} = h_{22}Z_{02}$$

$$S_{11} = \frac{A_n + B_n - C_n\left[\frac{Z_{01}^*}{Z_{01}}\right] - D_n\left[\frac{Z_{01}^*}{Z_{01}}\right]}{A_n + B_n + C_n + D_n}$$

$$S_{12} = \frac{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right](A_nD_n - B_nC_n)}{A_n + B_n + C_n + D_n} = \frac{2(AD-BC)}{A_n + B_n + C_n + D_n}$$

$$S_{21} = \frac{2}{A_n + B_n + C_n + D_n}$$

$$S_{22} = \frac{-A_n\left[\frac{Z_{02}^*}{Z_{02}}\right] + B_n - C_n\left[\frac{Z_{02}^*}{Z_{02}}\right] + D_n}{A_n + B_n + C_n + D_n}$$

$$A_n = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right](1-S_{22}) + S_{12}S_{21}}{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$B_n = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$C_n = \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$D_n = \frac{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$A_n = \frac{AZ_{02}}{(R_{01}R_{02})^{1/2}} \qquad B_n = \frac{B}{(R_{01}R_{02})^{1/2}} \qquad C_n = \frac{CZ_{01}Z_{02}}{(R_{01}R_{02})^{1/2}} \qquad D_n = \frac{DZ_{01}}{(R_{01}R_{02})^{1/2}}$$

TABLE IV
EQUATIONS FOR THE CONVERSION BETWEEN T PARAMETERS AND NORMALIZED Z, Y, h, AND $ABCD$ PARAMETERS WITH A SOURCE IMPEDANCE Z_{01} AND LOAD IMPEDANCE Z_{02}

$$T_{11} = \frac{(Z_{11n}+1)(Z_{22n}+1)-Z_{12n}Z_{21n}}{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{12} = \frac{(Z_{11n}+1)\left[\frac{Z_{02}^*}{Z_{02}}-Z_{22n}\right]+Z_{12n}Z_{21n}}{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{21} = \frac{\left[Z_{11n}-\frac{Z_{01}^*}{Z_{01}}\right](Z_{22n}+1)-Z_{12n}Z_{21n}}{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{22} = \frac{\left[\frac{Z_{01}^*}{Z_{01}}-Z_{11n}\right]\left[Z_{22n}-\frac{Z_{02}^*}{Z_{02}}\right]+Z_{12n}Z_{21n}}{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$Z_{11n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}}\right](T_{11}+T_{12})+(T_{21}+T_{22})}{T_{11}+T_{12}-T_{21}-T_{22}}$$

$$Z_{12n} = \frac{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{T_{11}+T_{12}-T_{21}-T_{22}}$$

$$Z_{21n} = \frac{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{T_{11}+T_{12}-T_{21}-T_{22}}$$

$$Z_{22n} = \frac{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}{T_{11}+T_{12}-T_{21}-T_{22}}$$

$$Z_{11n} = \frac{Z_{11}}{Z_{01}} \qquad Z_{12n} = \frac{Z_{12}}{(Z_{01}Z_{02})^{1/2}} \qquad Z_{21n} = \frac{Z_{21}}{(Z_{01}Z_{02})^{1/2}} \qquad Z_{22n} = \frac{Z_{22}}{Z_{02}}$$

$$T_{11} = \frac{(-1-Y_{11n})(1+Y_{22n})+Y_{12n}Y_{21n}}{2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{12} = \frac{(1+Y_{11n})\left[1-Y_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right]+Y_{12n}Y_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{21} = \frac{\left[Y_{11n}\left[\frac{Z_{01}^*}{Z_{01}}\right]-1\right](1+Y_{22n})-Y_{12n}Y_{21n}\left[\frac{Z_{01}^*}{Z_{01}}\right]}{2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{22} = \frac{\left[1-Y_{11n}\left[\frac{Z_{01}^*}{Z_{01}}\right]\right]\left[1-Y_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right]-Y_{12n}Y_{21n}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]}{2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$Y_{11n} = \frac{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}$$

$$Y_{12n} = \frac{-2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}$$

$$Y_{21n} = \frac{-2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}$$

$$Y_{22n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}}\right](T_{11}+T_{12})+(T_{21}+T_{22})}{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}$$

$$Y_{11n} = Y_{11}Z_{01} \qquad Y_{12n} = Y_{12}(Z_{01}Z_{02})^{1/2} \qquad Y_{21n} = Y_{21}(Z_{01}Z_{02})^{1/2} \qquad Y_{22n} = Y_{22}Z_{02}$$

$$T_{11} = \frac{(-h_{11n}-1)(1+h_{22n})+h_{12n}h_{21n}}{2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{12} = \frac{(h_{11n}+1)\left[1-h_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right]+h_{12n}h_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{21} = \frac{\left[\frac{Z_{01}^*}{Z_{01}}-h_{11n}\right](1+h_{22n})+h_{12n}h_{21n}}{2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$T_{22} = \frac{\left[h_{11n}-\frac{Z_{01}^*}{Z_{01}}\right]\left[1-h_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right]+h_{12n}h_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}$$

$$h_{11n} = \frac{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}$$

$$h_{12n} = \frac{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}$$

$$h_{21n} = \frac{-2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}$$

$$h_{22n} = \frac{T_{11}+T_{12}-T_{21}-T_{22}}{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}$$

$$h_{11n} = \frac{h_{11}}{Z_{01}} \qquad h_{12n} = h_{12}\left[\frac{Z_{02}}{Z_{01}}\right]^{1/2} \qquad h_{21n} = h_{21}\left[\frac{Z_{02}}{Z_{01}}\right]^{1/2} \qquad h_{22n} = h_{22}Z_{02}$$

$$T_{11} = \frac{A_n+B_n+C_n+D_n}{2}$$

$$T_{12} = \frac{A_n\left[\frac{Z_{02}^*}{Z_{02}}\right]-B_n+C_n\left[\frac{Z_{02}^*}{Z_{02}}\right]-D_n}{2}$$

$$T_{21} = \frac{A_n+B_n-C_n\left[\frac{Z_{01}^*}{Z_{01}}\right]-D_n\left[\frac{Z_{01}^*}{Z_{01}}\right]}{2}$$

$$T_{22} = \frac{A_n\left[\frac{Z_{02}^*}{Z_{02}}\right]-B_n-C_n\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]+D_n\left[\frac{Z_{01}^*}{Z_{01}}\right]}{2}$$

$$A_n = \frac{\left[\frac{Z_{01}^*}{Z_{01}}\right](T_{11}+T_{12})+(T_{21}+T_{22})}{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$B_n = \frac{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$C_n = \frac{T_{11}+T_{12}-T_{21}-T_{22}}{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$D_n = \frac{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$A_n = \frac{AZ_{02}}{(R_{01}R_{02})^{1/2}} \qquad B_n = \frac{B}{(R_{01}R_{02})^{1/2}} \qquad C_n = \frac{CZ_{01}Z_{02}}{(R_{01}R_{02})^{1/2}} \qquad D_n = \frac{DZ_{01}}{(R_{01}R_{02})^{1/2}}$$

TABLE V
EQUATIONS SHOWING THE CONVERSIONS BETWEEN Z, Y, h, AND $ABCD$ PARAMETERS

$Z_{11} = \frac{Y_{22}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	$Z_{11} = \frac{h_{11}h_{22}-h_{12}h_{21}}{h_{22}}$	$h_{11} = \frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{22}}$
$Z_{12} = \frac{-Y_{12}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$Y_{12} = \frac{-Z_{12}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	$Z_{12} = \frac{h_{12}}{h_{22}}$	$h_{12} = \frac{Z_{12}}{Z_{22}}$
$Z_{21} = \frac{-Y_{21}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$Y_{21} = \frac{-Z_{21}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	$Z_{21} = \frac{-h_{21}}{h_{22}}$	$h_{21} = \frac{-Z_{21}}{Z_{22}}$
$Z_{22} = \frac{Y_{11}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	$Z_{22} = \frac{1}{h_{22}}$	$h_{22} = \frac{1}{Z_{22}}$
$Z_{11} = \frac{A}{C}$	$A = \frac{Z_{11}}{Z_{21}}$	$Y_{11} = \frac{1}{h_{11}}$	$h_{11} = \frac{1}{Y_{11}}$
$Z_{12} = \frac{AD-BC}{C}$	$B = \frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{21}}$	$Y_{12} = \frac{-h_{12}}{h_{11}}$	$h_{12} = \frac{-Y_{12}}{Y_{11}}$
$Z_{21} = \frac{1}{C}$	$C = \frac{1}{Z_{21}}$	$Y_{21} = \frac{h_{21}}{h_{11}}$	$h_{21} = \frac{Y_{21}}{Y_{11}}$
$Z_{22} = \frac{D}{C}$	$D = \frac{Z_{22}}{Z_{21}}$	$Y_{22} = \frac{h_{11}h_{22}-h_{12}h_{21}}{h_{11}}$	$h_{22} = \frac{Y_{11}Y_{22}-Y_{12}Y_{21}}{Y_{11}}$
$Y_{11} = \frac{D}{B}$	$A = \frac{-Y_{22}}{Y_{21}}$	$h_{11} = \frac{B}{D}$	$A = \frac{h_{12}h_{21}-h_{11}h_{22}}{h_{21}}$
$Y_{12} = \frac{BC-AD}{B}$	$B = \frac{-1}{Y_{21}}$	$h_{12} = \frac{AD-BC}{D}$	$B = \frac{-h_{11}}{h_{21}}$
$Y_{21} = \frac{-1}{B}$	$C = \frac{Y_{12}Y_{21}-Y_{11}Y_{22}}{Y_{21}}$	$h_{21} = \frac{-1}{D}$	$C = \frac{-h_{22}}{h_{21}}$
$Y_{22} = \frac{A}{B}$	$D = \frac{-Y_{11}}{Y_{21}}$	$h_{22} = \frac{C}{D}$	$D = \frac{-1}{h_{21}}$

between T parameters and Z, Y, h, and $ABCD$ parameters. Tables III and IV provide the conversions from S and T parameters to the normalized Z, Y, h, and $ABCD$ parameters, respectively. From Tables III and IV, it is easy to see that if Z_{01} and Z_{02} are real, the conversions become those shown in many of the references cited, e.g., [2], [4], [7], [8], [11], [12], [14], [15]. Finally, Table V shows the conversions between Z, Y, h, and $ABCD$ parameters while Table VI shows the conversions between S and T parameters. These are included to make the table of conversions in this paper complete.

TABLE VI
EQUATIONS SHOWING THE CONVERSIONS BETWEEN S AND T PARAMETERS

$S_{11} = \frac{T_{21}}{T_{11}}$	$T_{11} = \frac{1}{S_{21}}$
$S_{12} = \frac{T_{11}T_{22}-T_{12}T_{21}}{T_{11}}$	$T_{12} = \frac{-S_{22}}{S_{21}}$
$S_{21} = \frac{1}{T_{11}}$	$T_{21} = \frac{S_{11}}{S_{21}}$
$S_{22} = \frac{-T_{12}}{T_{11}}$	$T_{22} = \frac{S_{12}S_{21}-S_{11}S_{22}}{S_{21}}$

IV. VERIFICATION

Using PSPICE, a SPICE based circuit analysis program, a lumped element model of an NE32000 HEMT was analyzed. The netlist was taken from the NEC databook and is shown below:

```
gl 5 6 3 4 0.045
lg 1 2 0.1nh
rg 2 3 2
cgs 3 4 0.2pf
cgd 3 5 0.016pf
cdg 5 4 6.7ff
ri 4 6 4
rs 6 7 3.5
ls 7 10 0.03nh
rds 5 6 200
cds 5 6 7.2ff
rd 5 8 4
ld 8 9 0.09nh.
```

By properly configuring a source at first port 1 then port 2, and opening and shorting out the other port, PSPICE will provide the complex voltages and currents required to calculate the Z, Y, h, and $ABCD$ parameters. Tables VII and VIII show the voltages and currents from PSPICE under the conditions listed in those tables. The Z, Y, h, and $ABCD$ parameters are calculated from these using (1)–(4) and are shown in Table IX.

The NE32000 lumped element model was also analyzed using Super Compact. For no particular reason, I chose to calculate the S parameters for the NE32000 in a system with a source impedance, Z_{01}, equal to $70+j\,30$ and load impedance, Z_{02}, equal to $25-j\,35$ at the single frequency of 10 GHz. The results of the Super Compact analysis are shown in Table X.

If a person uses the Z, Y, h, or $ABCD$ parameters of Table IX, in the equations of Table I, with $Z_{01} = 70+j\,30$ and $Z_{02} = 25-j\,35$, they will find that the calculated S parameters agree with those from Super Compact. In a like fashion, using the S parameters of Super Compact in the other equations in Table I will result in Z, Y, h, and $ABCD$ parameters shown in Table IX.

V. CONCLUSION

This paper developed the equations for converting between the various common 2-port parameters, Z, Y, h, $ABCD$, S, and T. The equations are derived from the definitions of the various 2-port parameters, the definition of a_j and b_j, and basic transmission line theory. As a result, the equations are completely general and are valid for complex and unique source and load impedances.

The validity of these results is shown by first calculating S parameters from Z, Y, h, and $ABCD$ parameters for an NE32000 HEMT in a system with $Z_S = 70 + j\,30$ and $Z_L = 25 - j\,35$. These results agreed with the S parameters produced by Super Compact. Also, beginning with the S parameters from Super Compact, the Z, Y, h, and $ABCD$ parameters are calculated using the equations developed. The results are the same as those calculated from the voltages and currents produced by PSPICE.

TABLE VII
VOLTAGES AND CURRENTS FOR THE NE32000 HEMT AT 10 GHz WITH THE SOURCE AT PORT 1. THE VOLTAGES AND CURRENTS ARE DEFINED IN FIG. 1

$V_1 = 1 + j\,0$			
$I_2 = 0$ (Port 2 Open Circuited)		$V_2 = 0$ (Port 2 Short Circuited)	
I_1	V_2	I_1	I_2
$8.844\text{E}-03 + j\,2.371\text{E}-02$	$-8.181\text{E}+00 + j\,5.615\text{E}+00$	$2.010\text{E}-03 + j\,1.292\text{E}-02$	$4.018\text{E}-02 - j\,1.071\text{E}-02$

TABLE VIII
VOLTAGES AND CURRENTS FOR THE NE32000 HEMT AT 10 GHz WITH THE SOURCE AT PORT 2. THE VOLTAGES AND CURRENTS ARE DEFINED IN FIG. 1

$V_2 = 1 + j\,0$			
$I_1 = 0$ (Port 1 Open Circuited)		$V_1 = 0$ (Port 1 Short Circuited)	
I_2	V_1	I_2	I_1
$8.032\text{E}-03 + j\,1.119\text{E}-03$	$9.661\text{E}-02 + j\,1.869\text{E}-02$	$3.949\text{E}-03 + j\,1.402\text{E}-03$	$4.741\text{E}-05 - j\,1.286\text{E}-03$

TABLE IX
Z, Y, h, AND $ABCD$ PARAMETERS FOR THE NE3200 HEMT AT 10 GHz. THESE PARAMETERS WERE CALCULATED FROM THE VOLTAGES AND CURRENTS IN TABLES VII AND VIII USING (1)–(4)

	11	12	21	22
Z	$1.380\text{E}+01 - j\,3.702\text{E}+01$	$1.212\text{E}+01 + j\,6.395\text{E}-01$	$9.518\text{E}+01 + j\,3.803\text{E}+02$	$1.221\text{E}+02 - j\,1.701\text{E}+01$
Y	$2.010\text{E}-03 + j\,1.292\text{E}-02$	$4.741\text{E}-05 - j\,1.286\text{E}-03$	$4.018\text{E}-02 - j\,1.071\text{E}-02$	$3.949\text{E}-03 + j\,1.402\text{E}-03$
h	$1.176\text{E}+01 - j\,7.557\text{E}+01$	$9.661\text{E}-02 + j\,1.869\text{E}-02$	$-3.370\text{E}-01 - j\,3.162\text{E}+00$	$8.032\text{E}-03 + j\,1.119\text{E}-03$
	A	B	C	D
$ABCD$	$-8.309\text{E}-02 - j\,5.703\text{E}-02$	$-2.324\text{E}+01 - j\,6.194\text{E}+00$	$6.173\text{E}-04 - j\,2.474\text{E}-03$	$3.332\text{E}-02 - j\,3.127\text{E}-01$

TABLE X
SUPER COMPACT RESULTS FOR THE NE32000 HEMT

		$Z_S = 70 + j\,30$					$Z_L = 25 - j\,35$		
		MICROWAVE HARMONICA PC V1.06 File: ne320-l.ckt 25-FEB-92 21:42:46							
Freq GHz	MS11 mag NE320L	PS11 deg NE320L	MS21 mag NE320L	PS21 deg NE320L	MS12 mag NE320L	PS12 deg NE320L	MS22 mag NE320L	PS22 deg NE320L	MS21 dB NE320L
10.000	0.665	−121.4	2.194	118.3	0.068	45.3	0.796	−12.4	6.82

ACKNOWLEDGMENT

The author would like to thank Dr. Ulrich Rohde and Dr. Ray Pengelly of Compact Software for the use of Microwave Harmonica. He also wishes to acknowledge as his Lord and Savior Jesus Christ, and thank Him for His love and guidance.

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[14] G. Gonzalez, *Microwave Transistor Amplifiers: Analysis and Design.* Englewood Cliffs, NJ: Prentice-Hall, 1984, pp. 24–25.
[15] R. Soares, Ed., *GaAs MESFET Circuit Design.* Butler, WI: Artech 1988, pp. 92–93.
[16] P. A. Rizzi, Microwave Engineering: Passive Circuits. Englewood Cliffs, NJ: Prentice-Hall, 1988, p. 538.

Dean A. Frickey (S'76–M'82) was born in Sheridan, WY on March 10, 1958. He received the B.S. and M.S. degrees in electrical engineering from the South Dakota School of Mines and Technology, Rapid City in 1980 and 1981, respectively. He is pursuing, part-time, the Ph.D. degree in electrical engineering through the University of Idaho, Moscow.

He spent one year with the Boeing Military Airplane Company, Seattle, WA, in a systems analysis group before becoming involved in microwave technology at Raytheon Missile Systems Division, Tewksbury, MA, where he spent 6 years mostly involved in the analysis and design of hybrid microwave integrated circuits, but with some time spent working with W-Band IMPATT diodes. In November 1989, he joined EG&G Idaho, which operates the Idaho National Engineering Laboratory. His research interests are in microwave theory and applications.

Network Theory

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He is expertise in Design and Development of cooling systems for large size electrical generators, and the C&I of process industries. He has been in academia for about twelve years. Presently, he is with VSS University of Technology, Burla, India at the capacity of Head and Associate Professor, EEE from Dec. 2016. He has more than 65 publications in various Journals and Conferences of Internationally repute to his credit. He also holds a patent as well, and filed one more. He also adapted one international edition book published by Pearson India. He received research grants of US$90,000 (INR 50 lakhs). He has been supervised 09 Masters' theses, and registered 04 PhD theses. He has also been recognized with many national and international awards by elite bodies. He has been awarded with CICS award under the head of Indian National Science Academy for travel support to USA, MHRD Fellowship by Govt. of India, and Gopabandhu Das Scholarship in his career. His major areas of interests are Power System Instrumentation, Industrial Automation, Robust and Intelligent Control, the Smart Sensors, IoT enabled Smart Sensors, the Smart Grid, Fuel Cell lead Sustainable Sources of Energy, and System Reliability.

Dr. Biswal is a Fellow IE (India), Senior Member of IEEE, USA, and Life Member of ISTE, India. He is actively involved in review panels of different societies of international repute viz. IEEE, IFAC, and the ISA. Currently, he is also actively involved as a Member of IEEE–SA (Standards Association) working groups; IEEE P1876 WG, IEEE P21451-001 WG, and IEEE P1415. He has also been invited for delivering guest lectures at World Congress on Sustainable Technologies (WCST) Conf. 2012, London, UK, INDICON 2015, New Delhi, India, National Power Training Institute (NPTI), Nangal, India, and G.B. Pant Engineering College, Pauri, Gharwal, India, Surendra Sai University of Technology (formerly UCE), Burla, and as a guest expert in 2016 IEEE PES General Meeting Boston, MA, USA.

Syllabus

Network Theory

MODULE-I (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Analysis of Coupled Circuits: Self-inductance and Mutual inductance, Coefficient of coupling, Series connection of coupled circuits, Dot convention, Ideal Transformer, Analysis of multi-winding coupled circuits, Analysis of single tuned and double tuned coupled circuits.

Transient Response: Transient study in series RL, RC, and RLC networks by time domain and Laplace transform method with DC and AC excitation. Response to step, impulse and ramp inputs of series RL, RC and RLC circuit.

MODULE-II (7 HOURS) [Online mode: 5 HOURS + 1 Test]

Two Port networks: Types of port Network, short circuit admittance parameter, open circuit impedance parameters, Transmission parameters, Condition of Reciprocity and Symmetry in two port network, Inter-relationship between parameters, Input and Output Impedances in terms of two port parameters, Image impedances in terms of ABCD parameters, Ideal two port devices, ideal transformer. Tee and Pie circuit representation, Cascade and Parallel Connections.

MODULE-III (8 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Functions & Responses: Concept of complex frequency, driving point and transfer functions for one port and two port network, poles & zeros of network functions, Restriction on Pole and Zero locations of network function, Time domain behavior and stability from pole-zero plot, Time domain response from pole zero plot.

Three Phase Circuits: Analysis of unbalanced loads, Neutral shift, Symmetrical components, Analysis of unbalanced system, power in terms of symmetrical components.

MODULE-IV (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Synthesis: Realizability concept, Hurwitz property, positive realness, properties of positive real functions, Synthesis of R-L, R-C and L-C driving point functions, Foster and Cauer forms.

MODULE-V (6 HOURS) [Online mode: 5 HOURS + 1 Test]

Graph theory: Introduction, Linear graph of a network, Tie-set and cut-set schedule, incidence matrix, Analysis of resistive network using cut-set and tie-set, Dual of a network.

Filters: Classification of filters, Characteristics of ideal filters.

Text and Reference Books

Recommended Text Books:

1. "Introductory Circuit Analysis", Robert L. Boylestad, Pearson, 12^{th} ed., 2012.

2. "Network Analysis", M. E. Van Valkenburg, Pearson, 3^{rd} ed., 2006.

3. "Engineering Circuit Analysis", W. Hayt, TMH, 2006.

4. "Network Analysis & Synthesis", Franklin Fa-Kun. Kuo, John Wiley & Sons.

Reference Books:

* "Basic Circuit Theory, Huelsman, PHI, 3^{rd} ed.,

* "HUGHES Electrical and Electronic Technology", Revised by J. Hiley, K. Brown, and I. M. Smith, Pearson, 10^{th} ed., 2011.

* "Circuits and Networks", Sukhija and Nagsarkar, Oxford Univ. Press, 2012.

* "Fundamentals of Electric Circuits", C. K. Alexander and M. N. O. Sadiku, McGraw-Hill Higher Education, 3^{rd} ed., 2005.

* "Fundamentals of Electrical Engineering", L. S. Bobrow, Oxford University Press, 2^{nd} ed., 2011.

* "Circuit Theory (Analysis and Synthesis)", A. Chakrabarti, Dhanpat Rai pub.

Other Important References

Reference Sites:	
1.	NPTEL, The National Programme on Technology Enhanced Learning (NPTEL): https://nptel.ac.in/
2.	MIT OpenCourseWare : https://ocw.mit.edu/index.htm

Course Outcomes

Upon successful completion of this course, you (students) will be able to

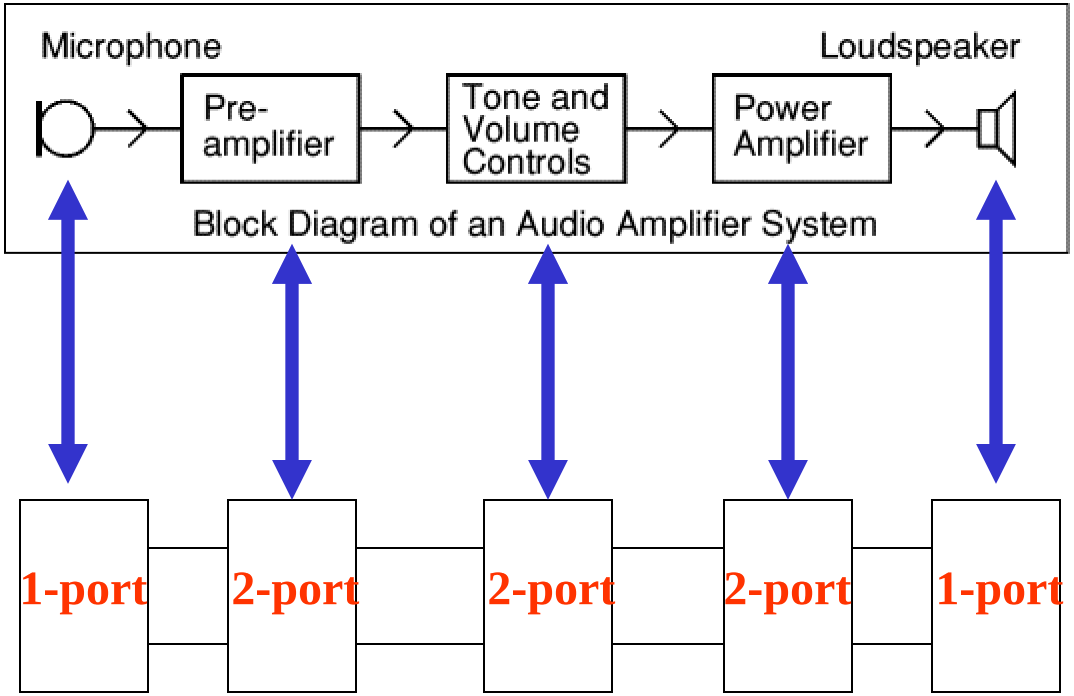
CO1	Analyze coupled circuits and understand the difference between the steady state and transient response of 1st and 2nd order circuit and understand the concept of time constant.
CO2	Learn the different parameters of two port network.
CO3	Concept of network function and three phases circuit and know the difference of balanced and unbalanced system and importance of complex power and its components.
CO4	Synthesis the electrical network.
CO5	Analyse the network using graph theory and understand the importance of filters in electrical system.

Two-Port Analysis

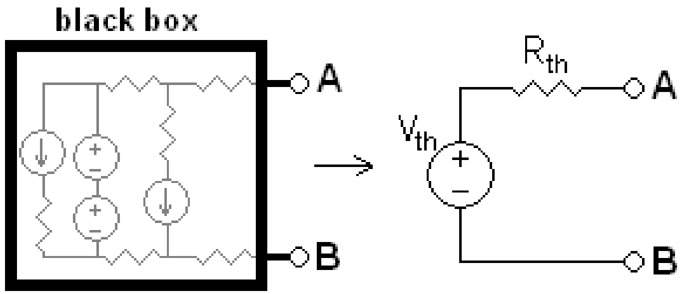
- **Special thanks to Oxford University Press, India and**
- **The University of Tennessee, USA**

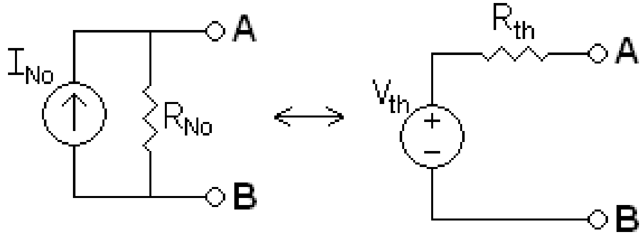
- Review of one ports
- Various two-port descriptions
- Terminated nonlinear two-ports
- Impedance and admittance matrices of two-ports
- Other two-port parameter matrices
- The hybrid matrices
- The transmission matrices
- Interconnection of networks

Review of one ports



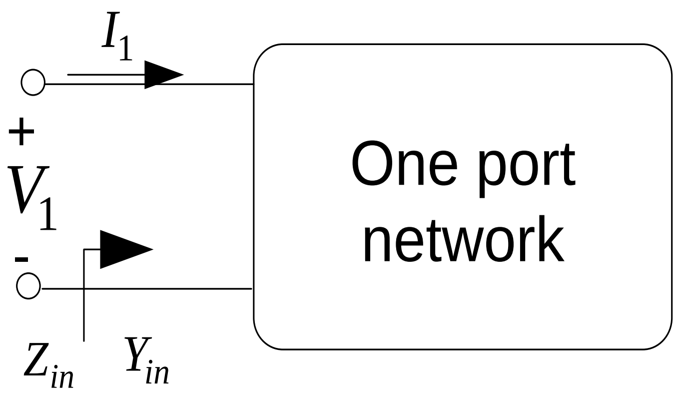
Thevenin's Equivalent Circuit





Norton's Equivalent Circuit

LTI one ports



Input impedance

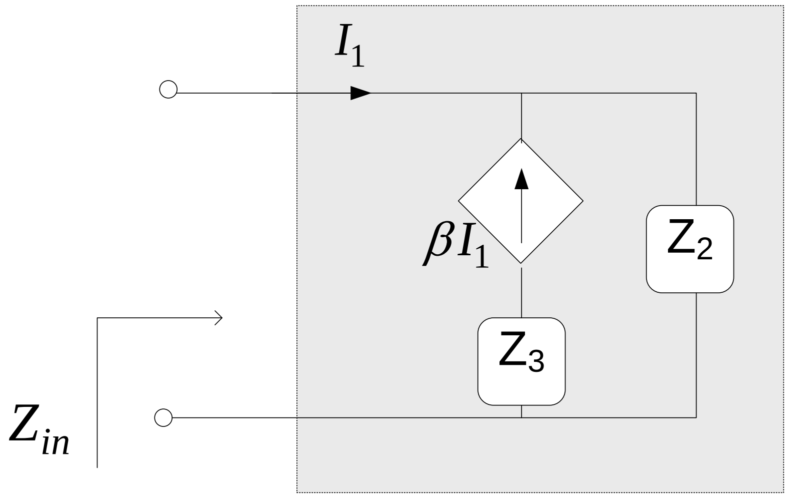
Input admittance

$$Z_{in} = \frac{V_1}{I_1}$$

$$Y_{in} = \frac{I_1}{V_1}$$

Example 1

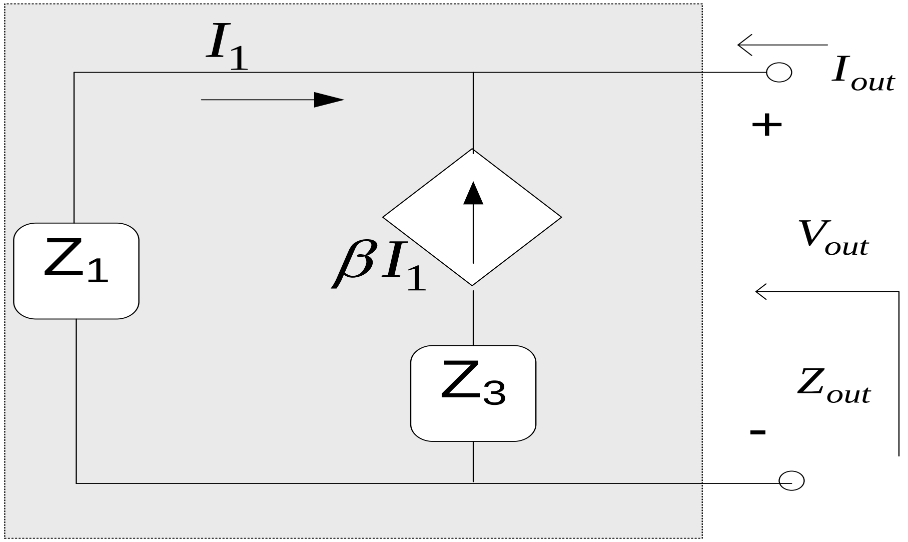
Determine the input impedance of the circuit in Fig.



$$I_{in} = I_1 = -\beta I_1 + \frac{V_{in}}{Z_2} \qquad V_{in} = (1+\beta)Z_2 I_{in} \qquad Z_{in} = (1+\beta)Z_2$$

Example 2

Determine the output impedance of the circuit in Fig.

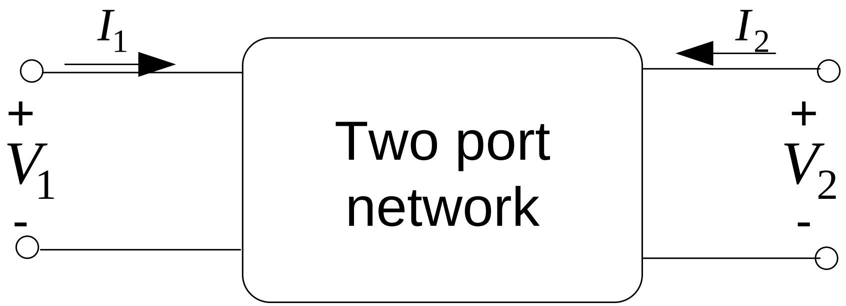


$$I_{out} = -I_1 - \beta I_1 = (1+\beta)\frac{V_{out}}{Z_1}$$

$$Z_{out} = \frac{V_{out}}{I_{out}} = \frac{Z_1}{1+\beta}$$

A two-port network

- Circuits can be considered by theirs terminal variables
- Voltages and currents are terminal's variables
- Complex circuit can be analyzed more easily.
- There are many kinds of two port parameters.



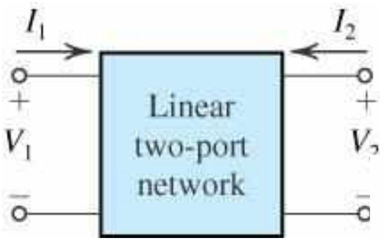
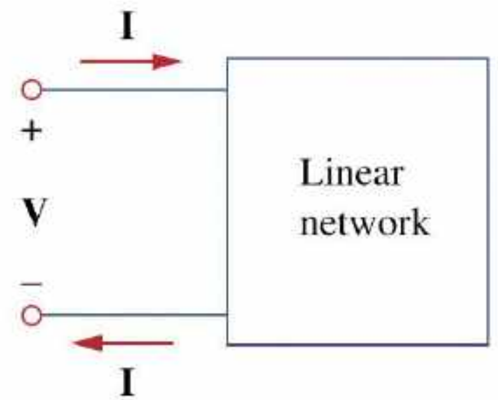
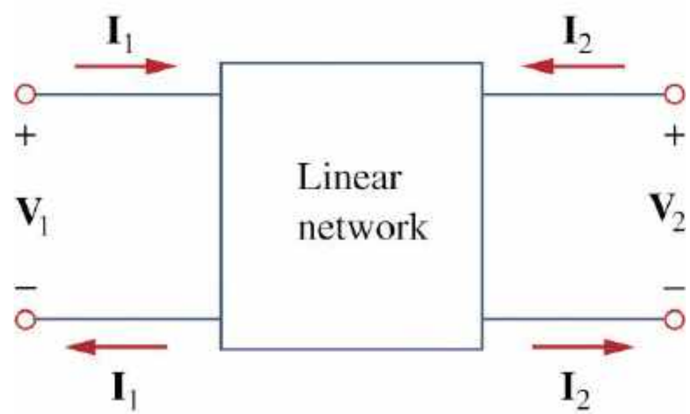


Figure: The reference directions of the four port variables in a linear two-port network.



(a)



(b)

- A **two-port Network** is an electrical network with two separate ports for input and output.

Various two-port descriptions

$$\mathbf{i} = g(\mathbf{v})$$ or $$i_1 = g_1(v_1, v_2)$$ $$i_2 = g_2(v_1, v_2)$$

Port current

Port voltage

$$\mathbf{v} = r(\mathbf{i})$$ or $$v_1 = r_1(i_1, i_2)$$ $$v_2 = r_2(i_1, i_2)$$

Or hybrid

$$v_1 = h_1(i_1, v_2)$$
$$i_2 = h_2(i_1, v_2)$$

Impedance Parameters

- Impedance parameters are very useful in designing impedance matching and power distribution system. Two port network can either be voltage or current driven. The input and output terminal voltage can be presented as follows:

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

where impedance parameters of the system is $z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$

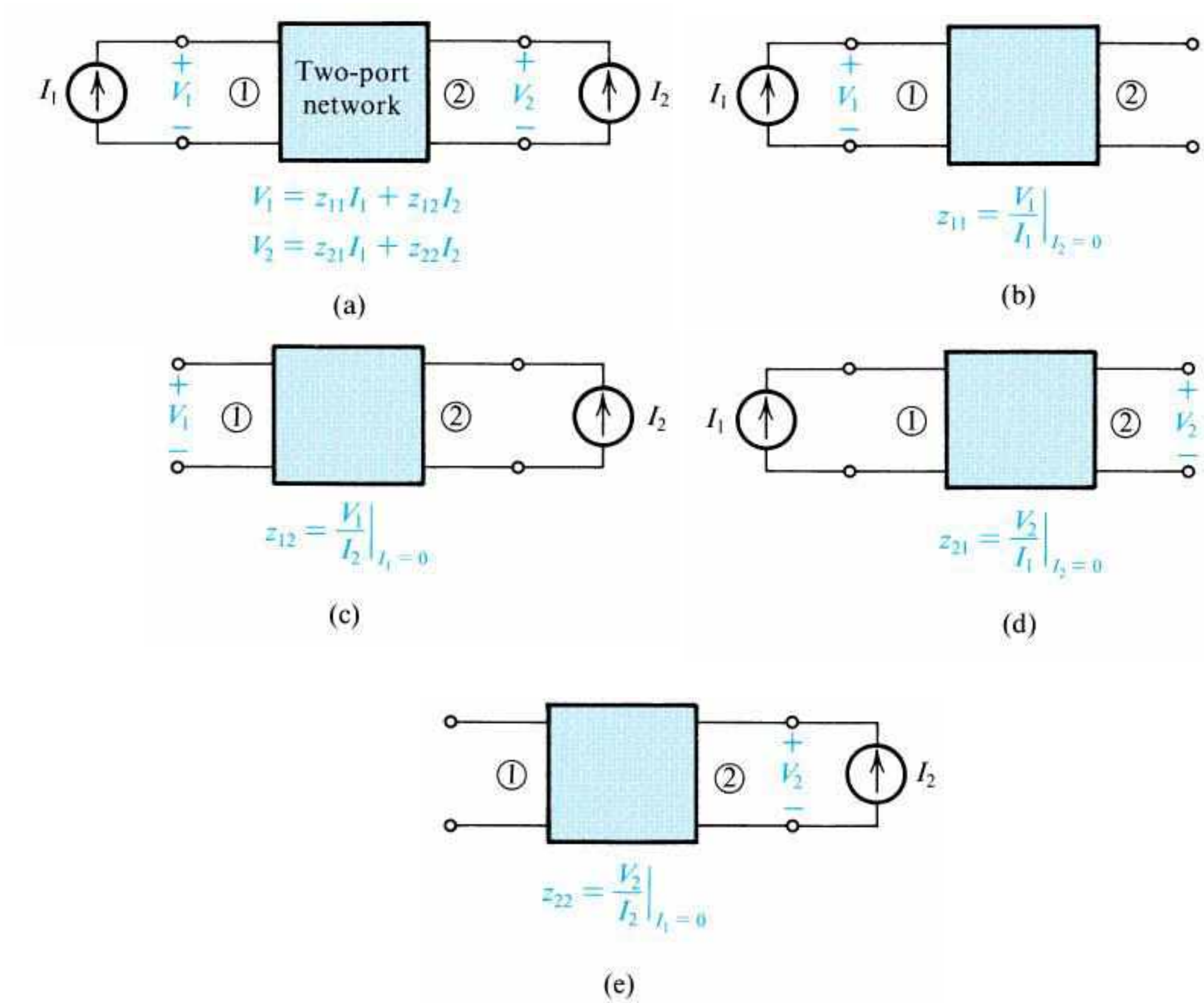
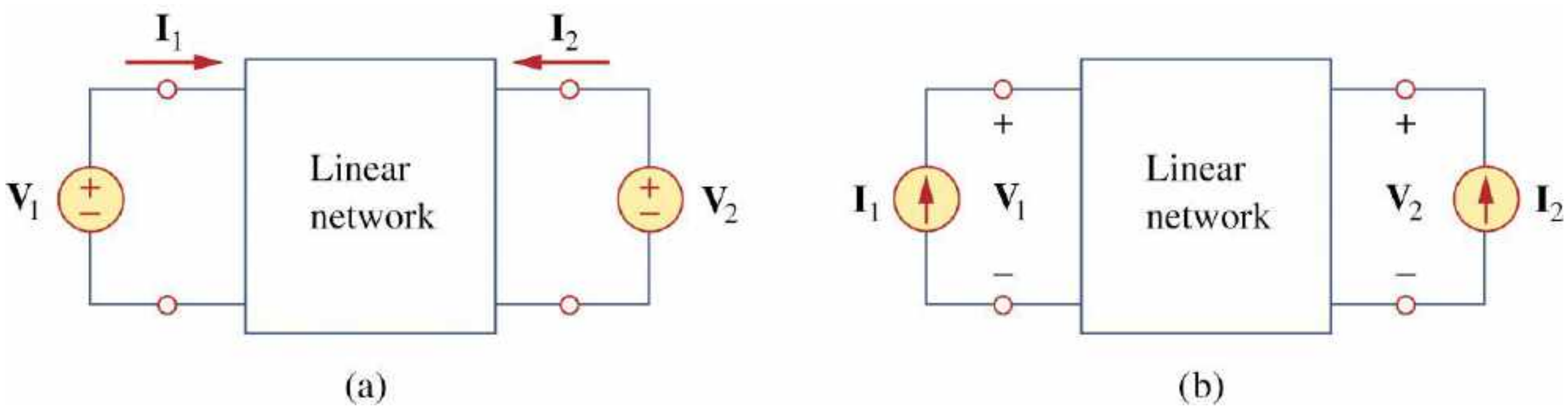


Figure: Definition and conceptual measurement circuits for *z* parameters.

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{z}_{11} & \mathbf{z}_{12}\\ \mathbf{z}_{21} & \mathbf{z}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = [\mathbf{z}]\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$



$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

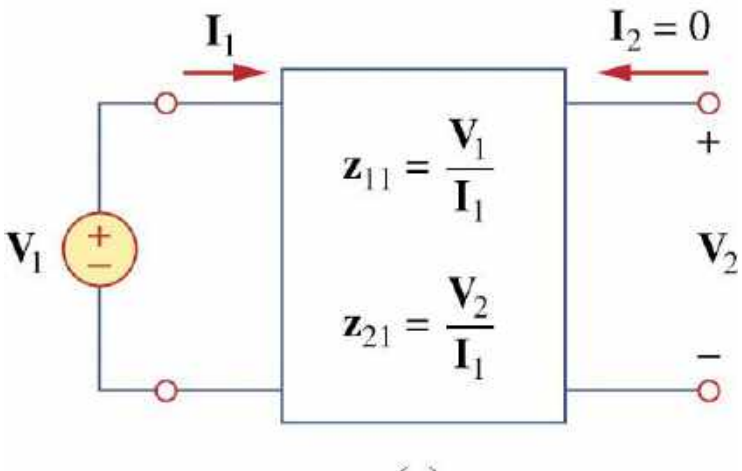
$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$\mathbf{z}_{11}$ = Open-circuit input impedance

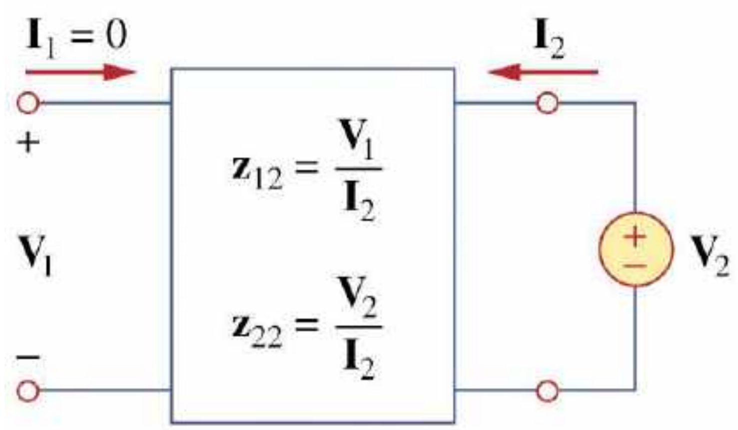
$\mathbf{z}_{12}$ = Open-circuit transfer impedance from port 1 to port 2

$\mathbf{z}_{21}$ = Open-circuit transfer impedance from port 2 to port 1

$\mathbf{z}_{22}$ = Open-circuit output impedance



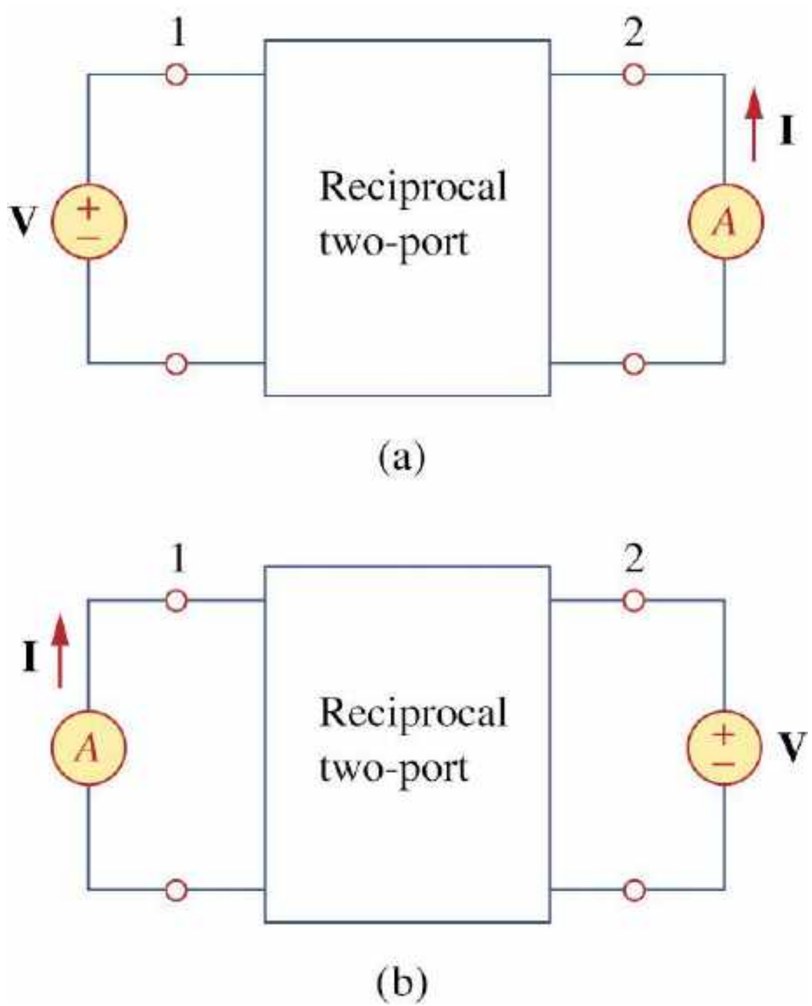
(a)



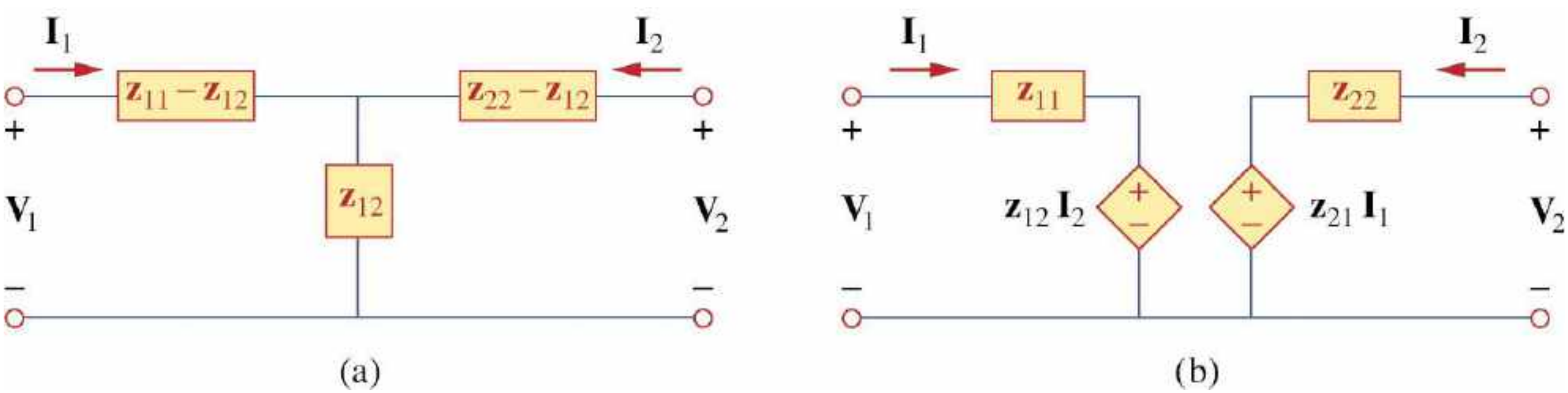
(b)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \quad \mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$$

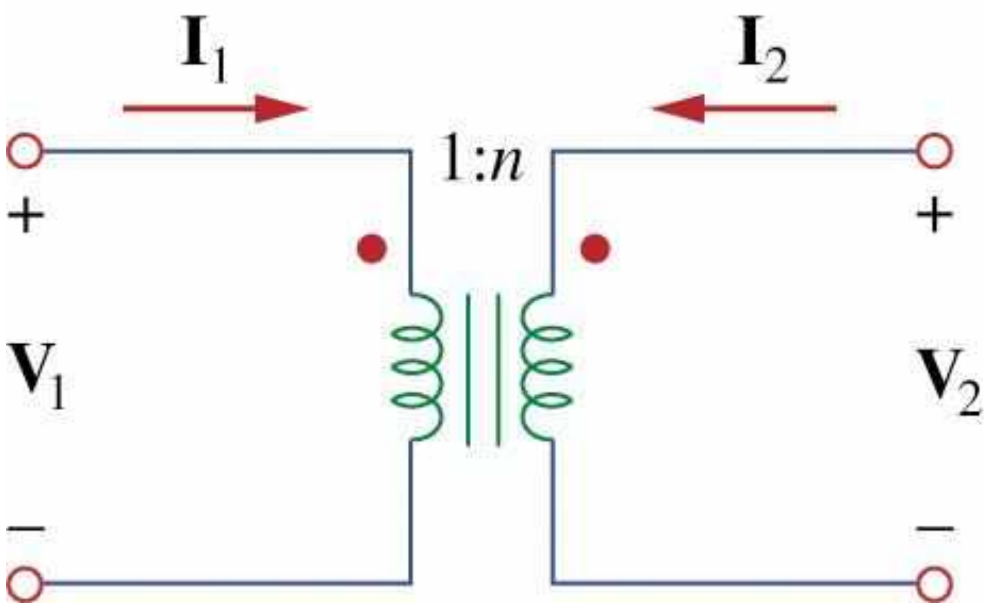
$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}, \quad \mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$$



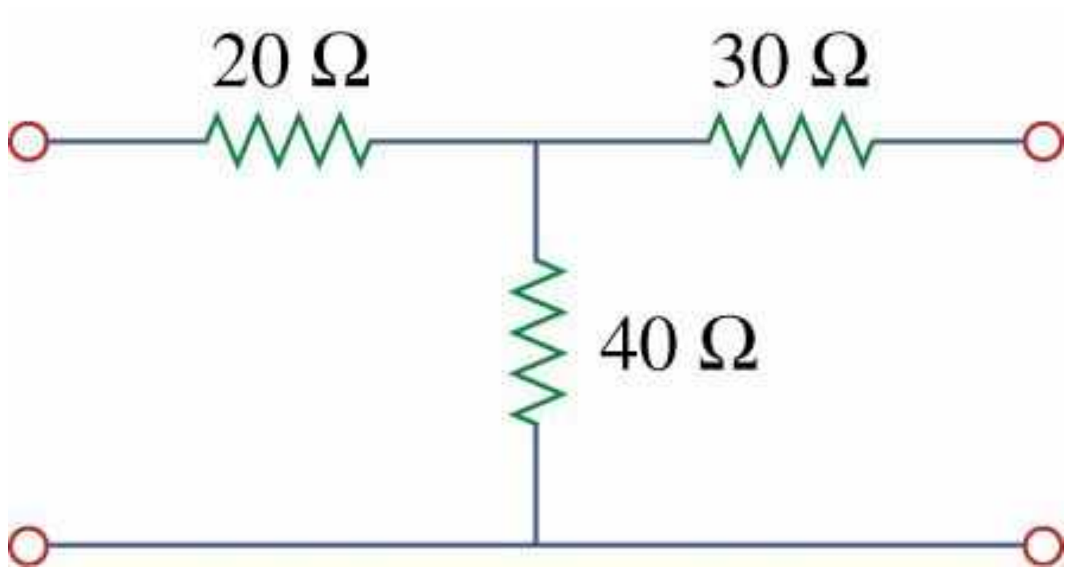
$$\mathbf{z}_{21} = \mathbf{z}_{12}$$


I1
I2
z11 − z12
z22 − z12
z12
+
V1
−
+
V2
−
(a)
I1
I2
z11
z22
z12 I2
z21 I1
+
V1
−
+
V2
−
(b)

$$\mathbf{V}_1 = \frac{1}{n}\mathbf{V}_2, \quad \mathbf{I}_1 = -n\mathbf{I}_2$$



- Determine the *z* parameters for the circuit:

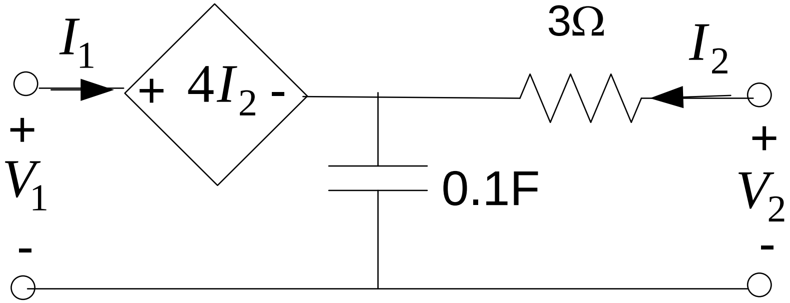


z_{11}	3.000E+01
z_{21}	2.000E+01

z_{12}	2.000E+01
z_{22}	5.000E+01

Example

Determine the impedance parameters from the circuit in Fig.



In frequency domain

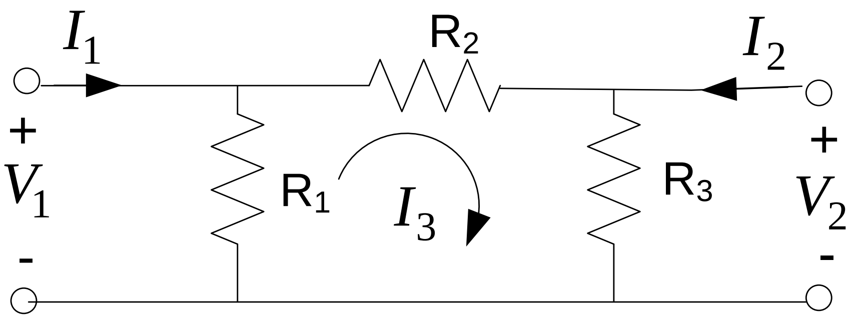
$$V_1 = 4I_2 + \frac{10}{s}(I_1 + I_2) = \frac{10}{s}I_1 + (4 + \frac{10}{s})I_2$$

$$V_2 = 3I_2 + \frac{10}{s}(I_1 + I_2) = \frac{10}{s}I_1 + (3 + \frac{10}{s})I_2$$

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{10}{s} & \frac{4s+10}{s} \\ \frac{10}{s} & \frac{3s+10}{s} \end{bmatrix}$$

Example

Compute the z-parameter of the circuit in Fig.



Or the simplest approach is apply Delta-Star conversion, and then solve through Z-parameter

$$V_1 = R_1 I_1 - R_1 I_3$$

$$V_2 = R_3 I_2 + R_3 I_3$$

$$0 = -R_1 I_1 + R_3 I_2 + (R_1 + R_2 + R_3) I_3$$

$$I_3 = \frac{R_1}{R_1 + R_2 + R_3} I_1 - \frac{R_3}{R_1 + R_2 + R_3} I_2$$

$$V_1 = (R_1 - \frac{R_1^2}{R_1 + R_2 + R_3})I_1 + \frac{R_1 R_3}{R_1 + R_2 + R_3} I_2$$

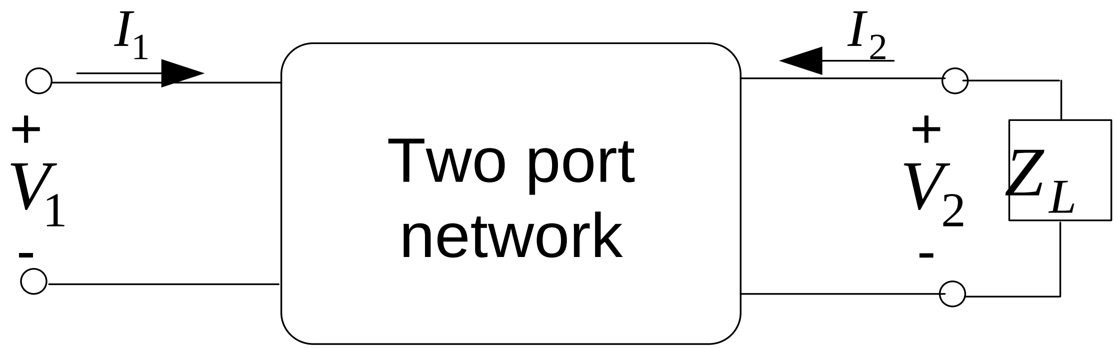
$$= \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} I_1 + \frac{R_1 R_3}{R_1 + R_2 + R_3} I_2$$

$$V_2 = \frac{R_1 R_3}{R_1 + R_2 + R_3} I_1 + (R_3 - \frac{R_3^2}{R_1 + R_2 + R_3})I_2$$

$$= \frac{R_1 R_3}{R_1 + R_2 + R_3} I_1 + \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} I_2$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{21} \end{bmatrix} = \begin{bmatrix} \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} & \frac{R_1 R_3)}{R_1 + R_2 + R_3} \\ \frac{R_1 R_3)}{R_1 + R_2 + R_3} & \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \end{bmatrix}$$

Z parameter analysis of terminated two-port



Z-parameter equations

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad V_2 = -Z_L I_2$$

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} + Z_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

From Crammer’s rules

$$I_1 = \frac{\begin{vmatrix} V_1 & z_{12} \\ 0 & z_{22}+Z_L \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22}+Z_L \end{vmatrix}} = \frac{(z_{22}+Z_L)V_1}{z_{11}(z_{22}+Z_L)-z_{12}z_{21}}$$

The input impedance Z_{in}

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22}+Z_L}$$

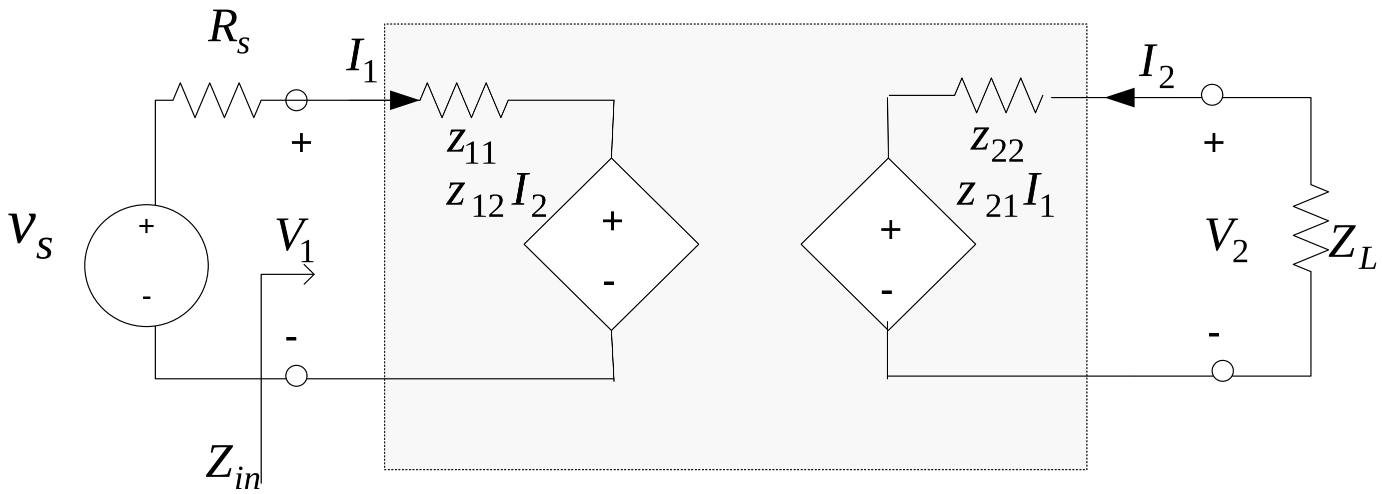
and

$$z_{21}I_1 = -(z_{22}+Z_L)I_2$$

$$I_2 = -\frac{z_{21}}{z_{22}+Z_L}I_1$$

$$V_1 = z_{11}I_1 + z_{12}I_2 = \left(z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L} \right) I_1$$

Gain: $$\frac{V_2}{V_s} = \frac{V_1}{V_s} \bullet \frac{V_2}{V_1} = \frac{Z_{in}}{Z_{in} + Z_s} \bullet \frac{Z_L}{z_{22} + Z_L} \frac{z_{21}}{Z_{in}} = \frac{Z_L}{z_{22} + Z_L} \bullet \frac{z_{21}}{Z_{in} + Z_s}$$

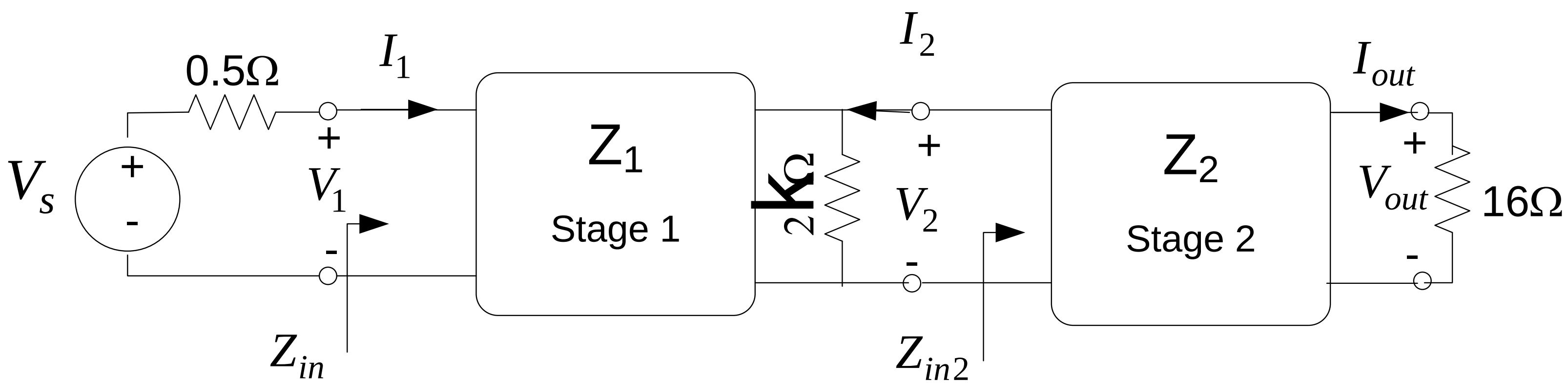


Terminated two-port Z-parameter model

Example:

The circuit in Figure is a two-stage transistor amplifier. The Z-parameters for each stage are

$$Z_1 = \begin{bmatrix} 350 & 2.667 \\ -10^6 & 6,667 \end{bmatrix} \qquad Z_2 = \begin{bmatrix} 1.0262\times10^6 & 6,790.8 \\ 1.0258\times10^6 & 6,793.5 \end{bmatrix}$$



Determine a) The input impedance Z_{in}

b) The overall voltage gain

c) Check the matching of the load and output impedance

Solution

$$Z_{in2} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_L}$$

$$= 1.0262 \cdot 10^6 - \frac{6790.8 \times 1.0258 \cdot 10^6}{6793.5 + 16}$$

$$= 3{,}159 \quad \Omega$$

$$\frac{V_{out}}{V_2} = \frac{Z_L}{z_{22} + Z_L} \frac{z_{21}}{Z_{in2}}$$

$$= \frac{16(1.0258 \cdot 10^6)}{(16 + 6793.5)3{,}159}$$

$$= 0.7629$$

$$Z_{L1} = 2k \,//\, Z_{in2} = 2000 \,//\, 3159 = 1224.7\Omega$$

$$Z_{in} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_{L1}}$$

$$= 350 + \frac{2.667 \times 10^6}{6667 + 1224.7}$$

$$= 687.9 \quad \Omega$$

$$\frac{V_2}{V_s} = \frac{Z_{L1}}{Z_{L1} + z_{22}} \frac{z_{21}}{Z_s + Z_{in}}$$

$$= \left(\frac{1224.7}{1224.7 + 6667} \right) \left(\frac{-10^6}{75 + 687.9} \right)$$

$$= -203.4$$

0.902 225.6

$$\frac{V_2}{V_s} = \frac{V_1}{V_s} \cdot \frac{V_2}{V_1} = \frac{Z_{in}}{Z_{in} + Z_s} \cdot \frac{Z_L}{z_{22} + Z_L} \frac{z_{21}}{Z_{in}} = \frac{Z_L}{z_{22} + Z_L} \cdot \frac{z_{21}}{Z_{in} + Z_s}$$

The overall voltage gain

$$A_{VS} = \frac{V_{out}}{V_s} = \frac{V_{out}}{V_2}\frac{V_2}{V_s}$$

$$= 0.7629 \times (-203.4)$$

$$= -155.2\ V/V$$

Out put impedance

$$Z_{out} = \left.\frac{V_2}{I_2}\right|_{V_s=0}$$

Job for you… To show that?

$$Z_{out} = z_{22} - \frac{z_{12}z_{21}}{R_s + z_{11}}$$

$$Z_{out1} = z_{22} - \frac{z_{12} z_{21}}{R_s + z_{11}}$$

$$= 6667 + \frac{2.667 \times 10^6}{0.5 + 350}$$

$$= 14.276 \, \text{k}\,\Omega$$

$$R_{s2} = Z_{out1} \,//\, 2k = 1.7542 \, \text{k}\,\Omega$$

$$Z_{out} = 6793.5 - \frac{6790.8 \cdot 1.0258 \times 10^6}{1754.24 + 1.0262 \times 10^6}$$

$$= 16.93 \Omega$$

Thus, the load is closely matched to the output impedance.

Admittance Parameters

- Admittance parameters are very useful for describing the network when impedance parameters may not be existed. This is solved by finding the second set of parameters by expressing the terminal current in term of the voltage. The input and output terminal current can be presented as follows:

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

where admittance parameters of the system is y = $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

These parameters are call short-circuited admittance parameters.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

where y_{11} short-circuit input impedance

y_{12} short-circuit transfer impedance from port 2 to 1

y_{21} short-circuit transfer impedance from port 1 to 2

y_{22} short-circuit output impedance

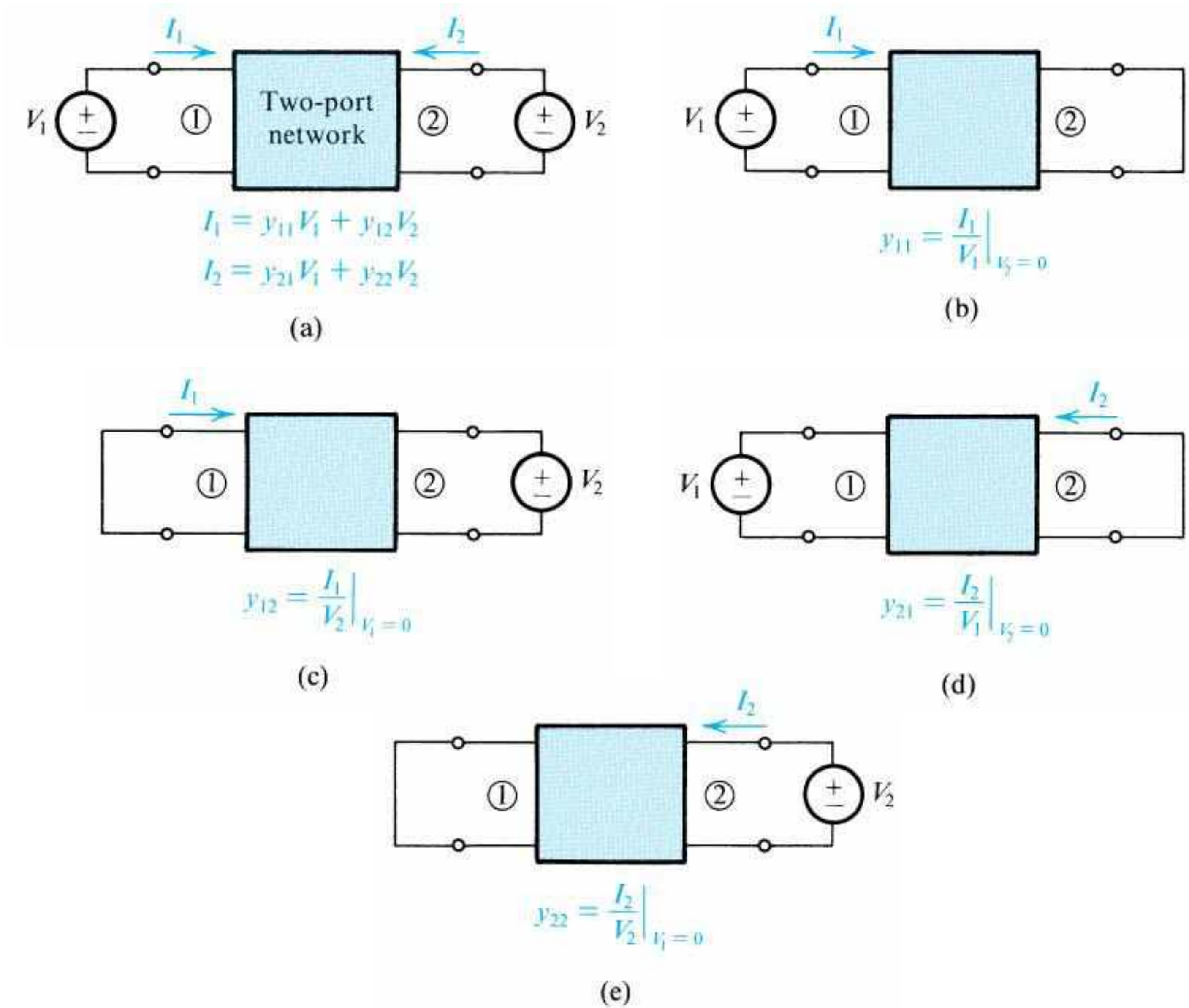
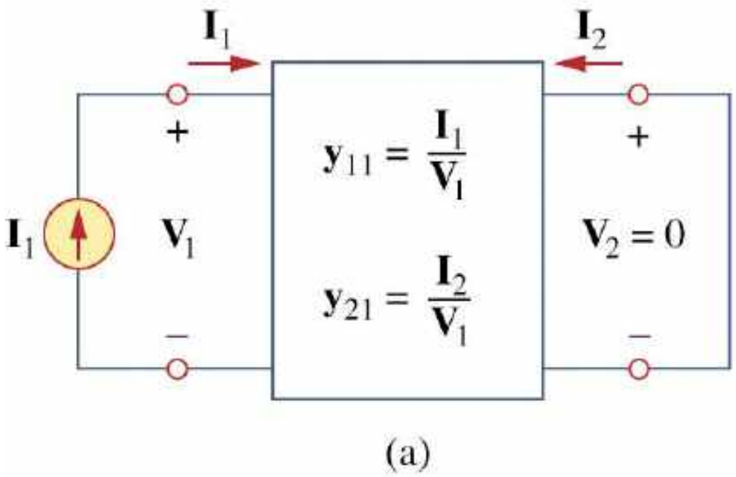
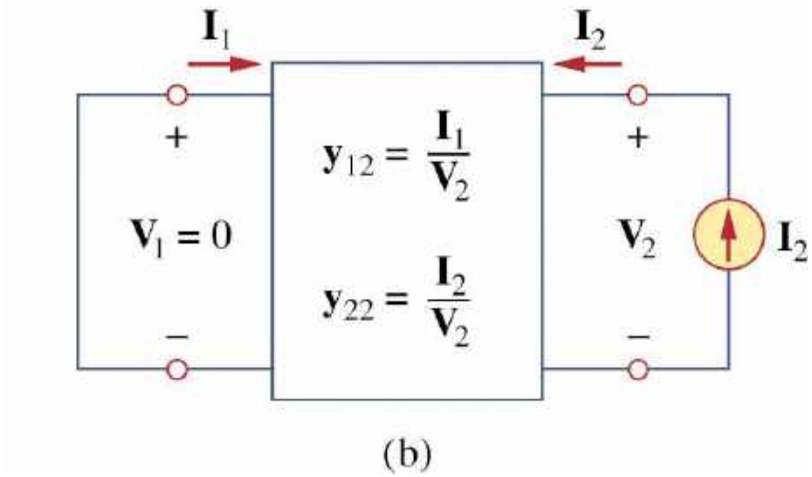


Figure: Definition and conceptual measurement circuits for y parameters.





$$
\boxed{\begin{aligned} \mathbf{I}_1 &= \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{aligned}}
$$

$$
\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}
$$

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$\mathbf{y}_{11}$ = Short-circuit input admittance

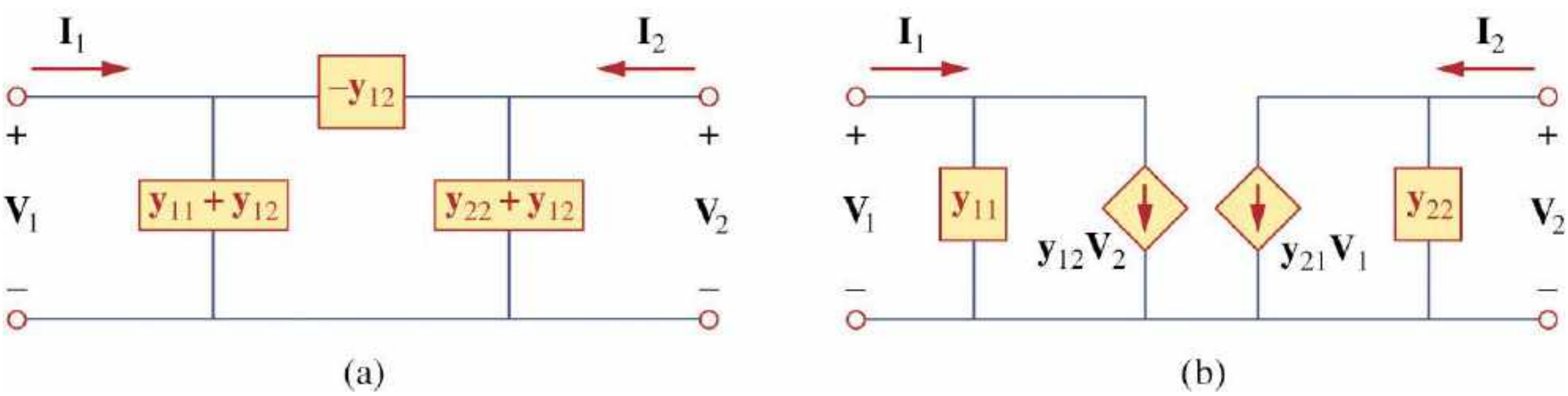
$\mathbf{y}_{12}$ = Short-circuit transfer admittance from port 1 to port 2

$\mathbf{y}_{21}$ = Short-circuit transfer admittance from port 2 to port 1

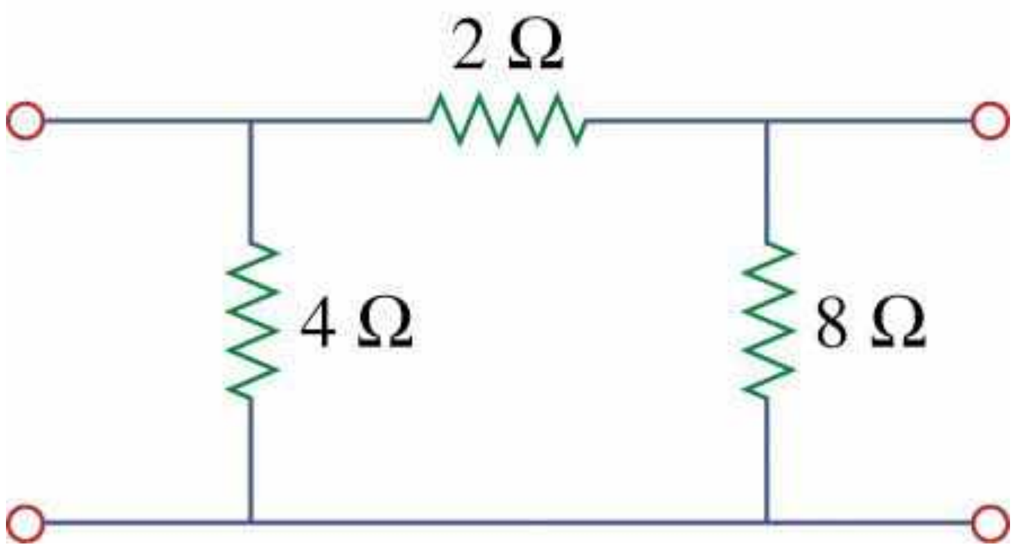
$\mathbf{y}_{22}$ = Short-circuit output admittance

$$\mathbf{y}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \quad \mathbf{y}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$$

$$\mathbf{y}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}, \quad \mathbf{y}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$$

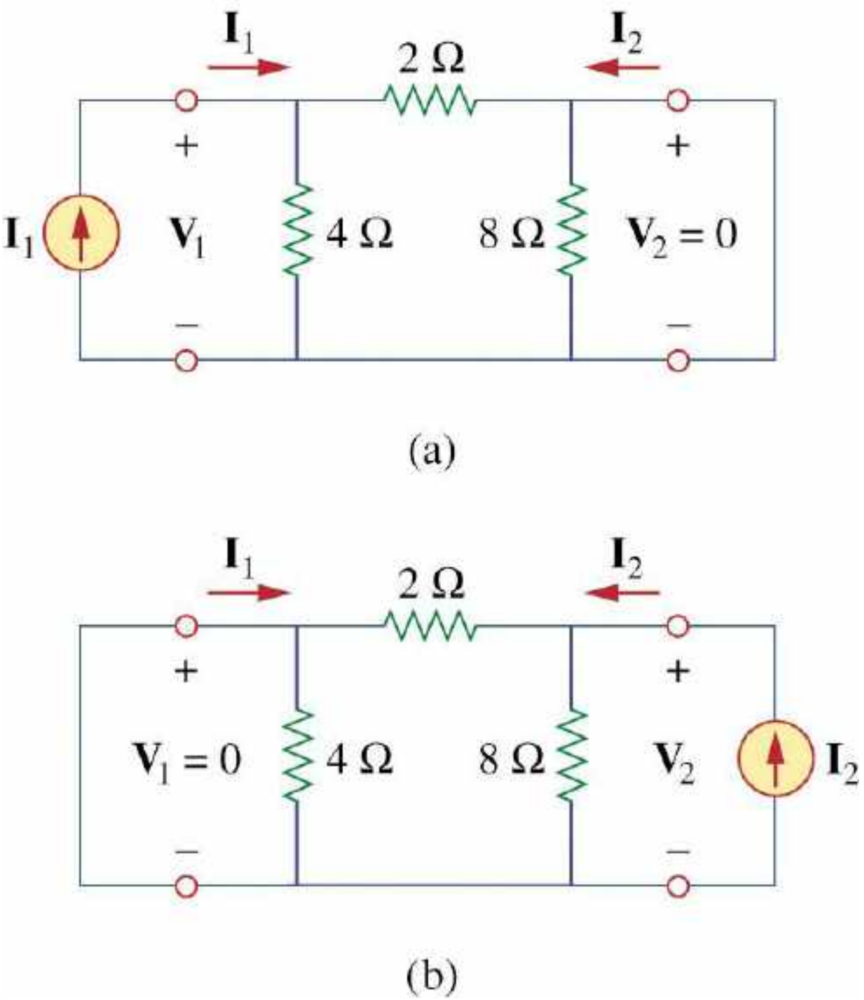


- Obtain the *y* parameters for the $\prod$ network shown:



y_{11}	7.500E-01
y_{21}	-5.000E-01

y_{12}	-5.000E-01
y_{22}	6.250E-01

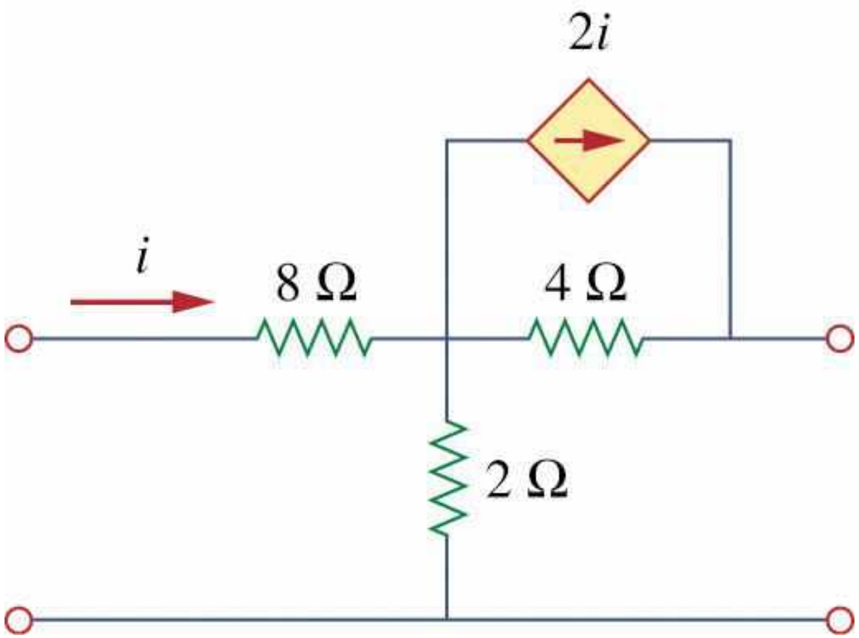


$$\mathbf{y}_{12} = -\frac{1}{2}\mathrm{S} = \mathbf{y}_{21}$$

$$\mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{4} \Rightarrow \mathbf{y}_{11} = \frac{1}{4} - \mathbf{y}_{12} = 0.75\,\mathrm{S}$$

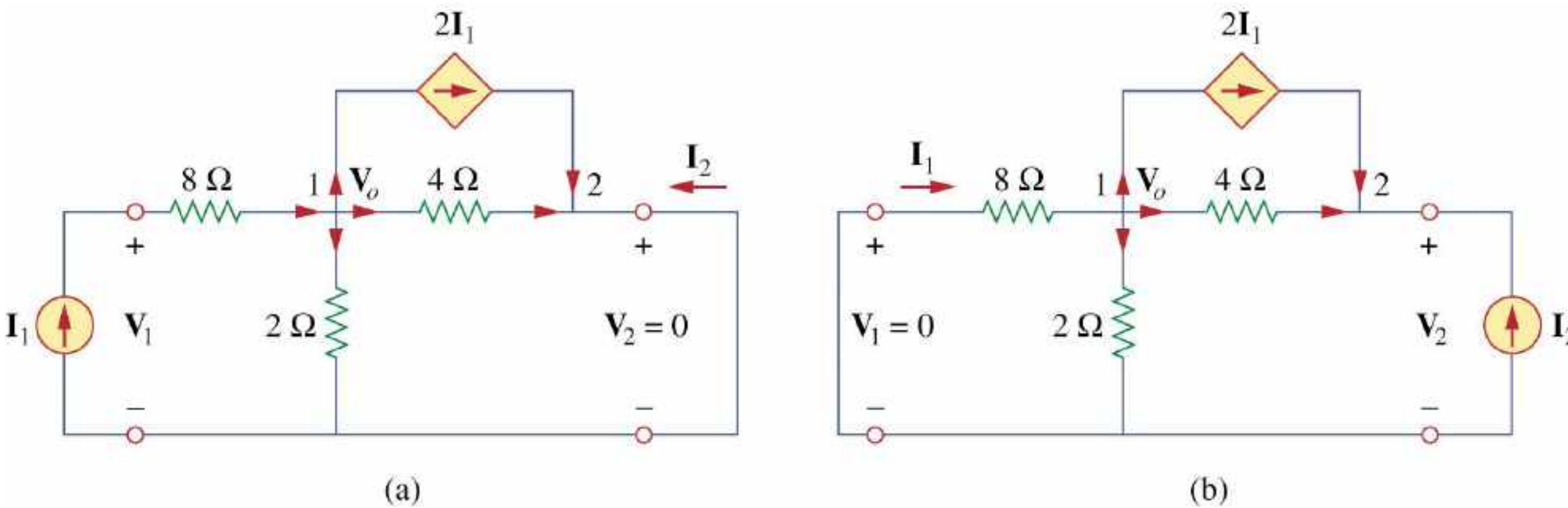
$$\mathbf{y}_{22} + \mathbf{y}_{12} = \frac{1}{8} \Rightarrow \mathbf{y}_{22} = \frac{1}{8} - \mathbf{y}_{12} = 0.625\,\mathrm{S}$$

- Determine the *y* parameters for the *T* network shown:



y_{11}	1.500E-01
y_{21}	-2.500E-01

y_{12}	-5.000E-02
y_{22}	2.500E-01


2I1
8 Ω
1
Vo
4 Ω
2
I2
+
I1
V1
2 Ω
V2 = 0
–
(a)
2I1
I1
8 Ω
1
Vo
4 Ω
2
+
V1 = 0
2 Ω
V2
I2
–
(b)

$$\text{At node 1, } \frac{\mathbf{V}_1 - \mathbf{V}_o}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - 0}{4}$$

$$\text{But } \mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{8}, \text{ therefore, } 0 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{8} + \frac{3\mathbf{V}_o}{4}$$

$$0 = \mathbf{V}_1 - \mathbf{V}_o + 6\mathbf{V}_o \Rightarrow \mathbf{V}_1 = -5\mathbf{V}_o$$

$$\text{Hence, } \mathbf{I}_1 = \frac{-5\mathbf{V}_o - \mathbf{V}_o}{8} = -0.75\,\mathbf{V}_o$$

$$\text{and } \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{-0.75\mathbf{V}_o}{-5\mathbf{V}_o} = 0.15\,\text{S}$$

At node 2, $\dfrac{\mathbf{V}_o - 0}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$

or $-\mathbf{I}_2 = 0.25\mathbf{V}_o - 1.5\mathbf{V}_o = -1.25\mathbf{V}_o$

Hence, $\mathbf{y}_{21} = \dfrac{\mathbf{I}_2}{\mathbf{V}_1} = \dfrac{1.25\mathbf{V}_o}{-5\mathbf{V}_o} = -0.25 \text{ S}$

Similarly, we get $\mathbf{y}_{12}$ and $\mathbf{y}_{21}$ using Fig. (b). At node 1,

$$\frac{0 - \mathbf{V}_o}{8} = 2\mathbf{I_1} + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{4}$$

But $\mathbf{I}_1 = \dfrac{0 - \mathbf{V}_o}{8}$, therefore, $0 = -\dfrac{\mathbf{V}_o}{8} + \dfrac{\mathbf{V}_o}{2} + \dfrac{\mathbf{V}_o - \mathbf{V}_2}{4}$

or $0 = -\mathbf{V}_o + 4\mathbf{V}_o + 2\mathbf{V}_o - 2\mathbf{V}_2 \Rightarrow \mathbf{V}_2 = 2.5\mathbf{V}_o$

$$\text{Hence, } \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\mathbf{V}_o/8}{2.5\mathbf{V}_o} = -0.05\,\text{S}$$

$$\text{At node 2, } \frac{\mathbf{V}_o - \mathbf{V}_2}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$$

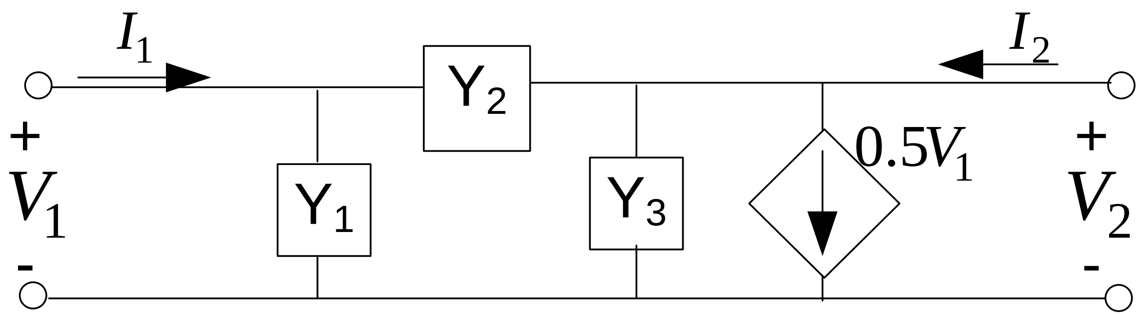
$$\text{or } -\mathbf{I}_2 = 0.25\mathbf{V}_o - \frac{1}{4}(2.5)\mathbf{V}_o - \frac{2\mathbf{V}_o}{8} = -0.625\mathbf{V}_o$$

$$\text{Thus, } \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{0.625\mathbf{V}_o}{2.5\mathbf{V}_o} = 0.25\,\text{S}$$

Notice that $\mathbf{y}_{12} \neq \mathbf{y}_{21}$ in this case, since the network is not reciprocal.

Example

Determine the admittance parameters from the circuit in Fig.



$$I_1 = Y_1V_1 + Y_2(V_1 - V_2) = (Y_1 + Y_2)V_1 - Y_2V_2$$

$$I_2 = 0.5V_1 + Y_3V_2 + Y_2(V_2 - V_1) = (0.5 - Y_2)V_1 + (Y_2 + Y_3)V_2$$

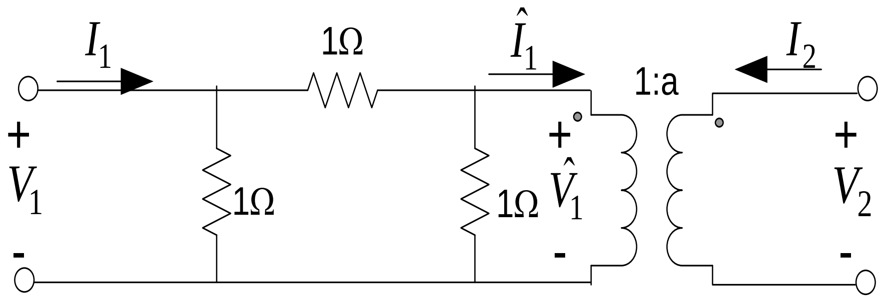
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ 0.5 - Y_2 & Y_2 + Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = Y_1 + Y_2 \;,\; y_{12} = -Y_2$$

$$y_{21} = 0.5 - Y_2 \;,\; y_{22} = Y_2 + Y_3$$

Example

Compute the y-parameter of the circuit in Fig.



$$I_1 = V_1 + (V_1 - \hat{V}_1) = 2V_1 - \hat{V}_1 = 2V_1 - \frac{1}{a}V_2$$

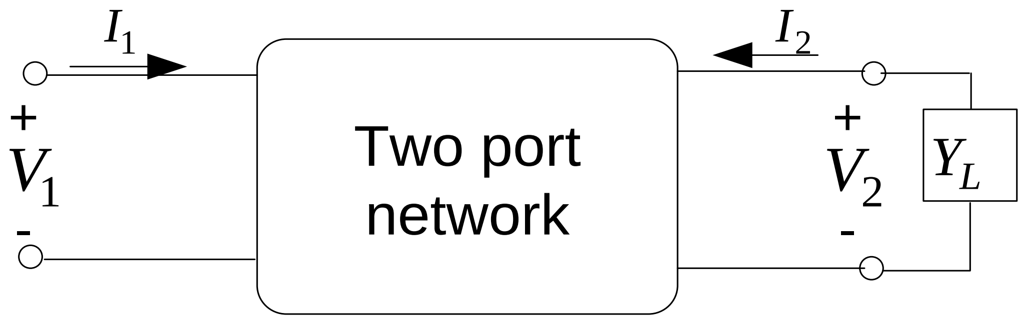
$$I_2 = -\frac{1}{a}\hat{I}_1 = -\frac{1}{a}\left[-\hat{V}_1 + (V_1 - \hat{V}_1)\right] = -\frac{1}{a}V_1 + \frac{2}{a^2}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{1}{a} \\ -\frac{1}{a} & \frac{2}{a^2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = 2, \quad y_{12} = -1/a$$

$$y_{21} = -1/a, \quad y_{22} = 2/a^2$$

Y parameter analysis of terminated two-port



Y-parameter equations

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad I_2 = -Y_L V_2$$

$$\begin{bmatrix} I_1 \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} + Y_L \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

From Crammer’s rules

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ 0 & y_{22} + Y_L \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} + Y_L \end{vmatrix}} = \frac{(y_{22} + Y_L)I_1}{y_{11}(y_{22} + Y_L) - y_{12}y_{21}}$$

The input admittance Y_{in}

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{11}(y_{22} + Y_L)}$$

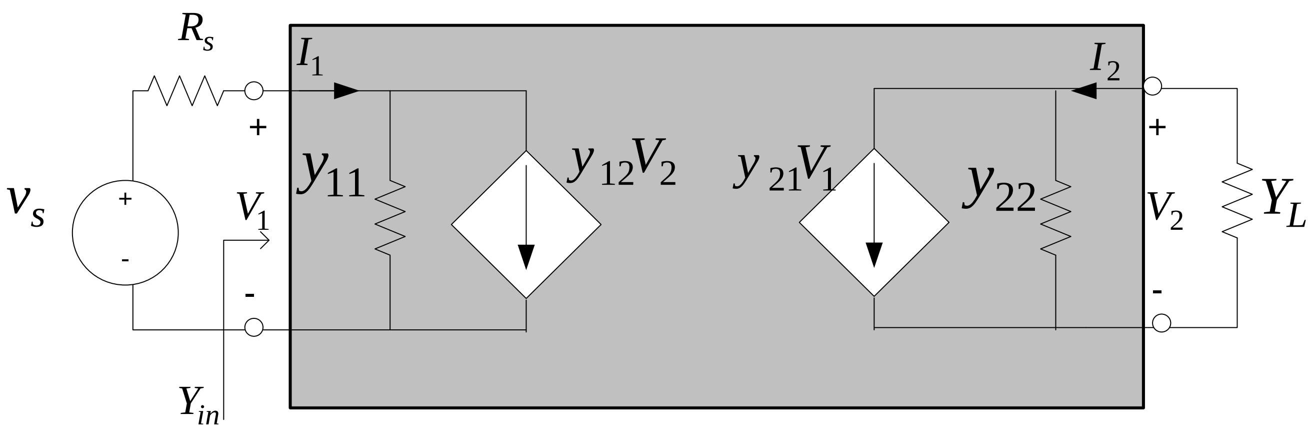
and

$$y_{21}V_1 = -(y_{22} + Y_L)V_2$$

$$V_2 = -\frac{y_{21}}{y_{22} + Y_L}V_1$$

$$I_1 = y_{11}V_1 + y_{12}V_2 = \left(y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} \right) V_1$$

Gain: $$\frac{V_2}{V_1} = -\frac{y_{21}}{y_{22} + Y_L}$$



Terminated two-port Y-parameter model

Two-port Devices and the Hybrid Model

❖H-parameter is the combination of Z and Y parameter defined by

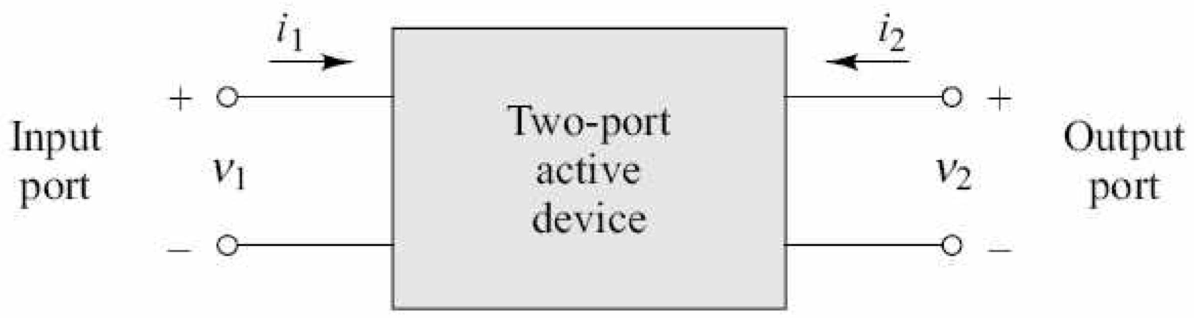


Fig. 8.4 *A two-port network.*

- If the current i_1 and the voltage v_2 are independent and if the two-port is linear.

$$v_1 = h_{11} i_1 + h_{12} v_2$$

$$i_2 = h_{21} i_1 + h_{22} v_2$$

- The quantities h_{11}, h_{12}, h_{21}, and h_{22} are called the *h, or hybrid, parameters.*

- H-parameter is commonly used in transistor modeling.

$$h_{11} \equiv \left.\frac{v_1}{i_1}\right|_{v_2=0}$$ = input resistance with output short-circuit (ohms).

$$h_{12} \equiv \left.\frac{v_1}{v_2}\right|_{i_1=0}$$ = fraction of output voltage at input with input open-circuited, or more simply, reverse-open-circuit voltage amplification (dimensionless).

$$h_{21} \equiv \left.\frac{i_2}{i_1}\right|_{v_2=0}$$ = negative of current transfer ratio (or current gain) with output short-circuited. (Note that the current into a load across the output port would be the negative of i_2.) This parameter is usually referred to, simply, as the *short-circuit current gain* (dimensionless).

$$h_{22} \equiv \left.\frac{i_2}{v_2}\right|_{i_1=0}$$ = output conductance with input open-circuited (mhos).

$i = 11 =$ input $\qquad o = 22 =$ output

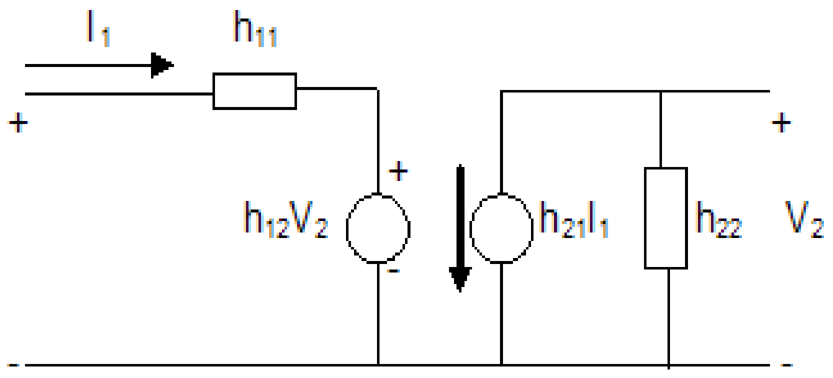
$f = 21 =$ forward transfer $\qquad r = 12 =$ reverse transfer

In the case of transistors, another subscript (b, e, or c) is added to designate the type of configuration. For example,

$h_{ib} = h_{11b} =$ input resistance in common-base configuration

$h_{fe} = h_{21c} =$ short-circuit forward current gain in common-emitter circuit

- The h-parameters are named specifically as follows:
  - h_{11} = short circuit input impedance
  - h_{12} = open circuit reverse voltage gain
  - h_{21} = short circuit forward current gain
  - h_{22} = open circuit output admittance



$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

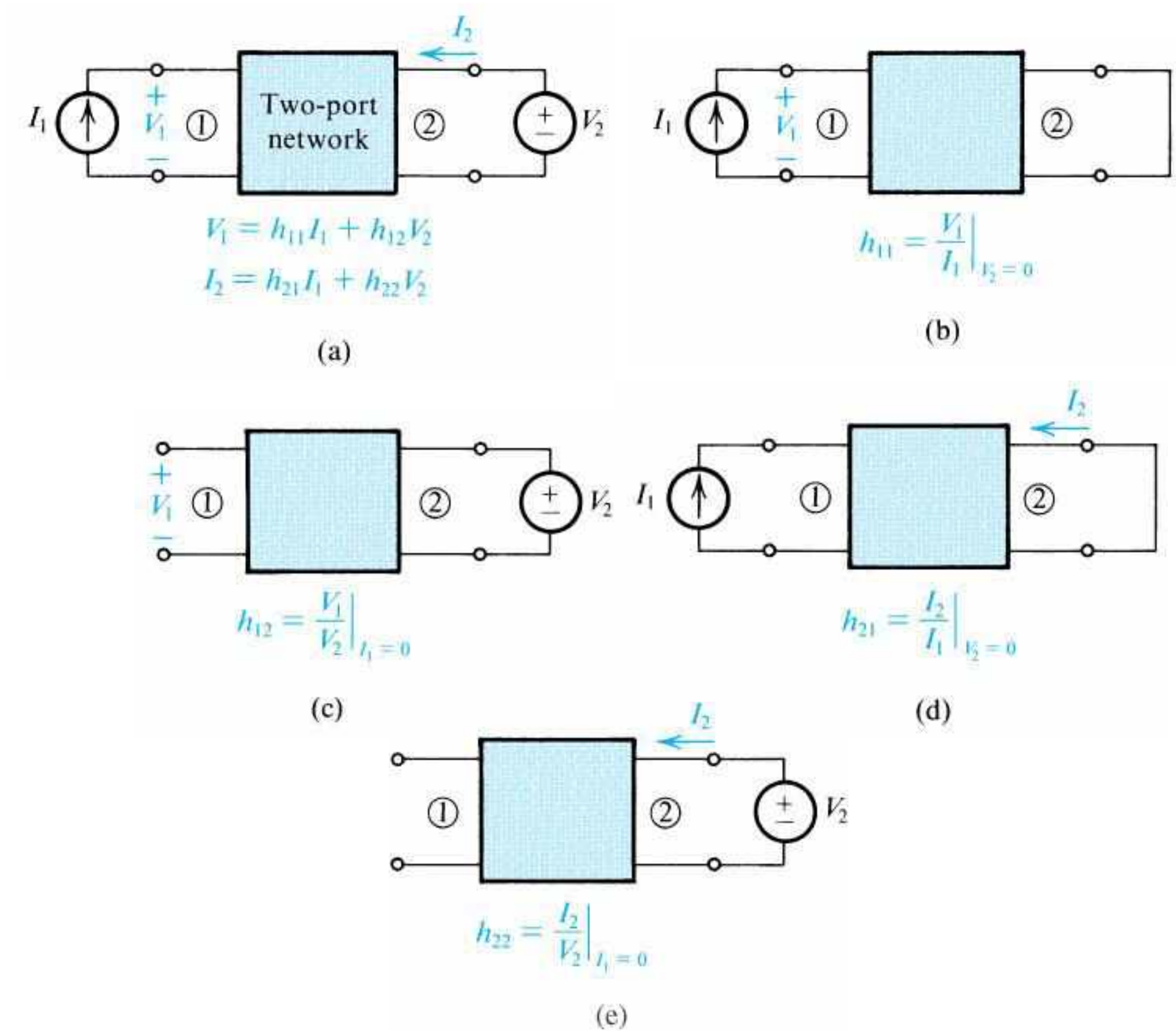


Figure : Definition and conceptual measurement circuits for *h* parameters.

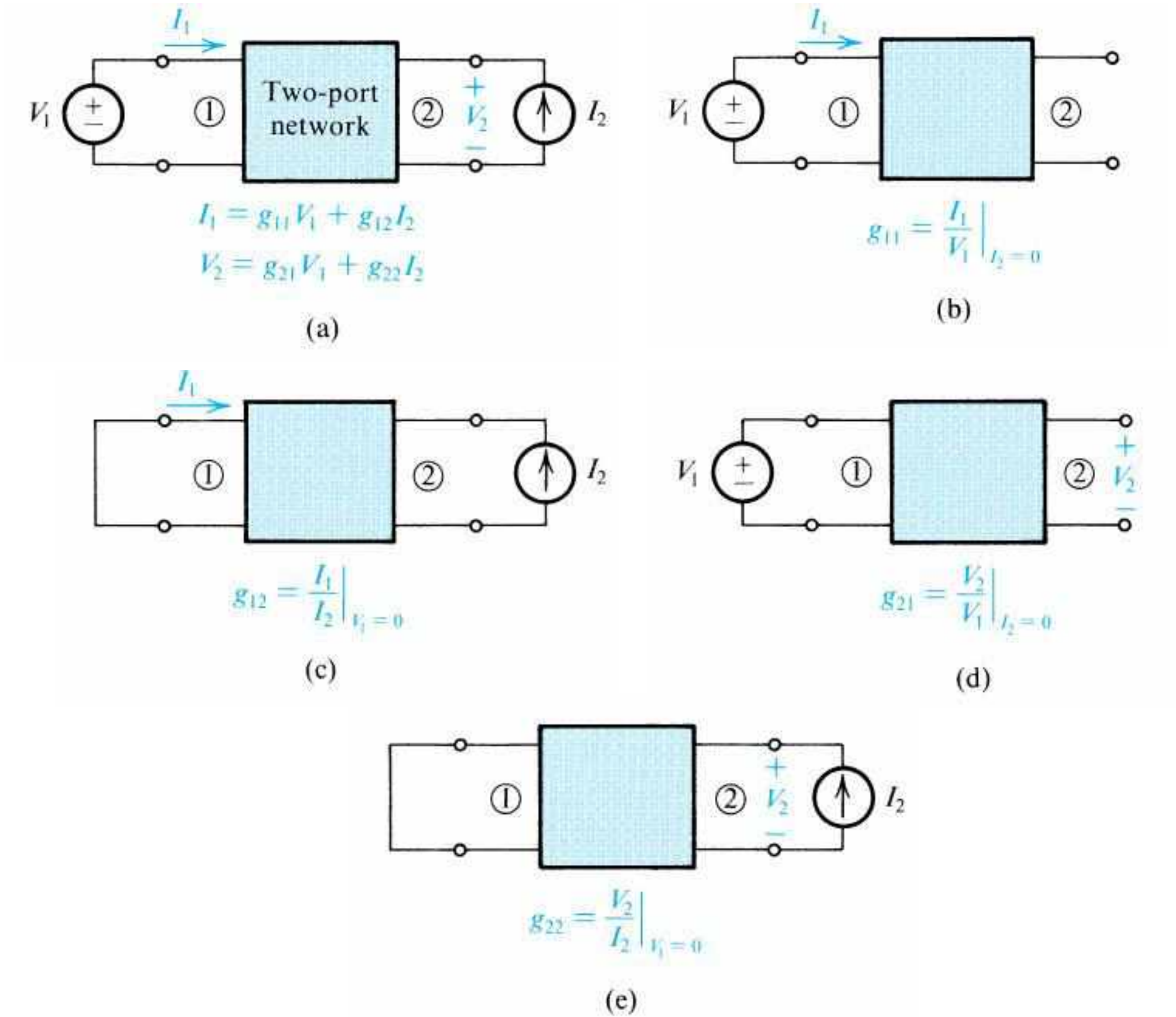
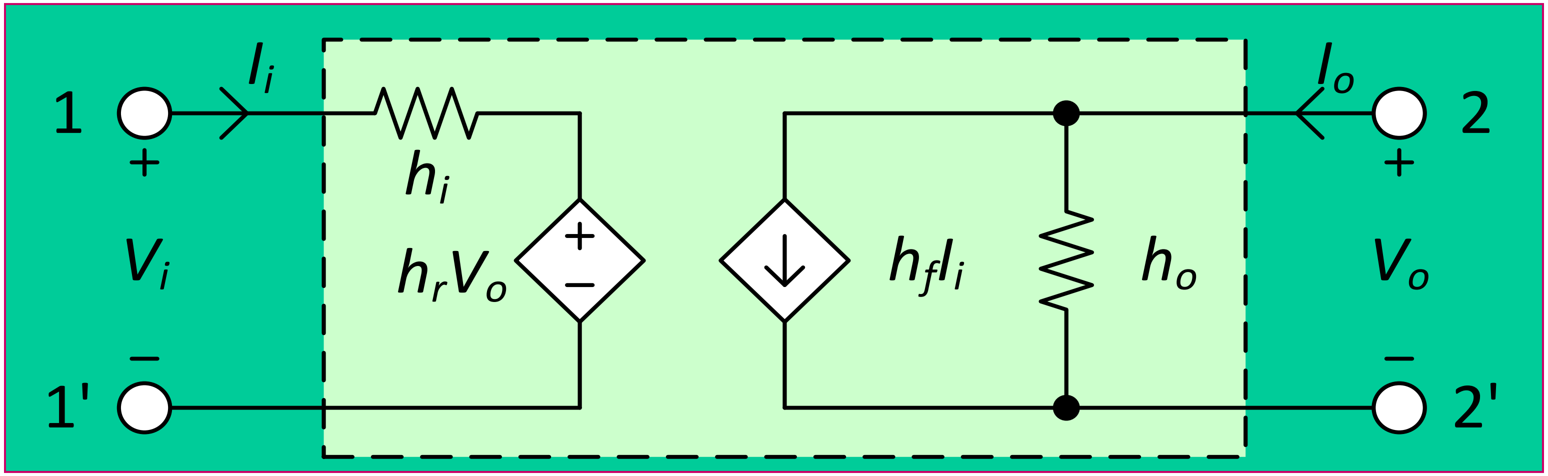


Figure: Definition and conceptual measurement circuits for *g* parameters.

Hybrid Parameter Model



$$V_i = h_{11}I_i + h_{12}V_o = h_iI_i + h_rV_o$$
$$I_o = h_{21}I_i + h_{22}V_o = h_fI_i + h_oV_o$$

h-Parameters

$$h_{11} = \frac{V_i}{I_i}\bigg|_{V_o = 0} \qquad h_{12} = \frac{V_i}{V_o}\bigg|_{I_i = 0}$$

$$h_{21} = \frac{I_o}{I_i}\bigg|_{V_o = 0} \qquad h_{22} = \frac{I_o}{V_o}\bigg|_{I_i = 0}$$

$h_{11} = h_i$ = Input Resistance

$h_{12} = h_r$ = Reverse Transfer Voltage Ratio

$h_{21} = h_f$ = Forward Transfer Current Ratio

$h_{22} = h_o$ = Output Admittance

The Model

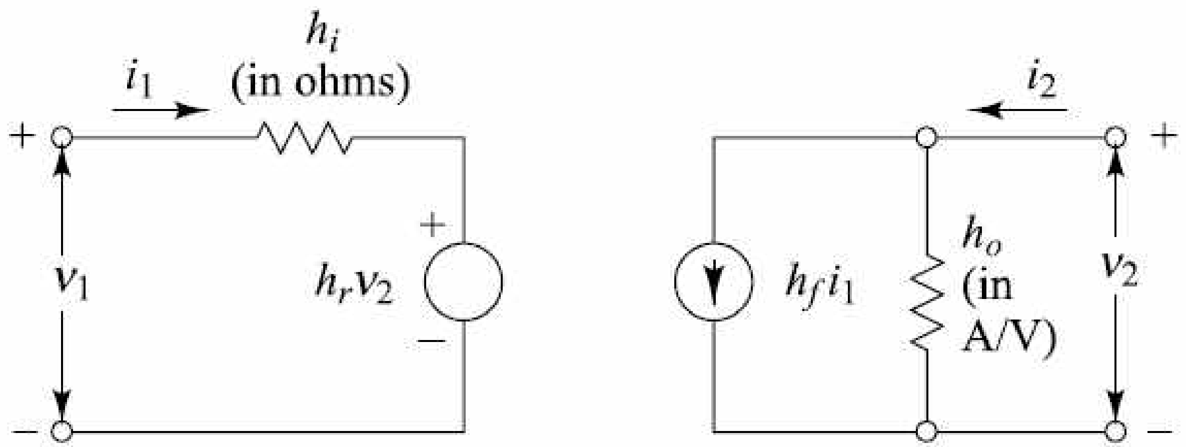


Fig. 8.5 *The hybrid model for the two-port network of Fig. 8.4. The parameters h_r and h_f are dimensionless.*

Transistor Hybrid Model

- *CE- configuration*

$$v_B = f_1(i_B, v_C)$$
$$i_C = f_2(i_B, v_C)$$

- Using Taylor's series expansion

$$\Delta v_B = \left.\frac{\partial f_1}{\partial i_B}\right|_{V_C} \Delta i_B + \left.\frac{\partial f_1}{\partial v_C}\right|_{I_B} \Delta v_C$$
$$\Delta i_C = \left.\frac{\partial f_2}{\partial i_B}\right|_{V_C} \Delta i_B + \left.\frac{\partial f_2}{\partial v_C}\right|_{I_B} \Delta v_C$$

- For small signal (incremental analysis)

$$v_b = h_{ie} i_b + h_{re} v_c$$
$$i_c = h_{fe} i_b + h_{oe} v_c$$

Where

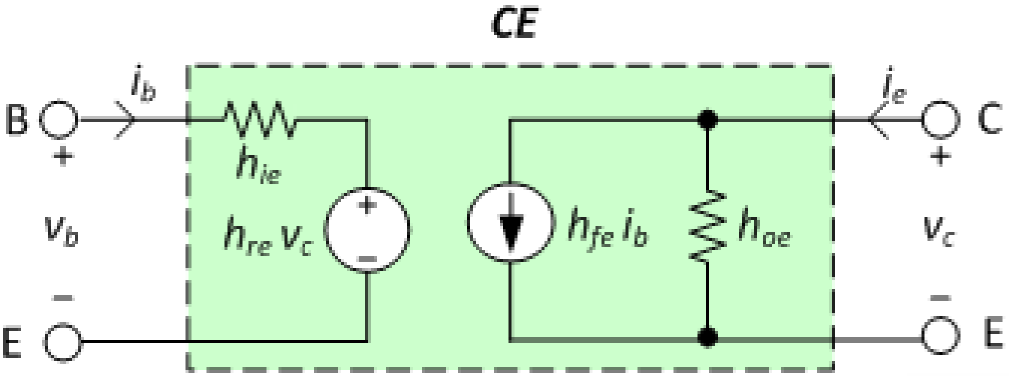
$$h_{ie} = \frac{\partial f_1}{\partial i_B} = \left.\frac{\partial v_B}{\partial i_B}\right|_{V_C} \qquad h_{re} = \frac{\partial f_1}{\partial v_C} = \left.\frac{\partial v_B}{\partial v_C}\right|_{I_B}$$

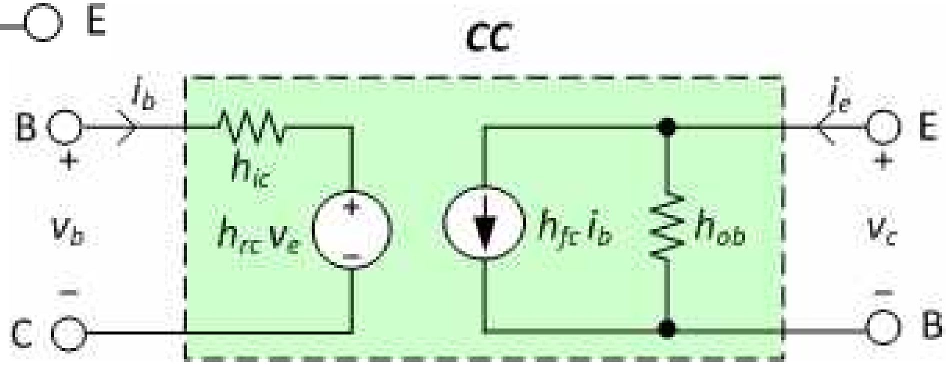
$$h_{fe} = \frac{\partial f_2}{\partial i_B} = \left.\frac{\partial i_C}{\partial i_B}\right|_{V_C} \qquad h_{oe} = \frac{\partial f_2}{\partial v_C} = \left.\frac{\partial i_C}{\partial v_C}\right|_{I_B}$$

- If a parameter is constant, its incremental change is zero.

For example: if V_C is constant, then it is equivalent to v_c=0.
if I_B is constant, then it is equivalent to i_b=0.

$$h_{re} = \left.\frac{\partial v_B}{\partial v_C}\right|_{I_B} = \left.\frac{v_b}{v_c}\right|_{i_b=0} \quad or \quad h_{re} = \left.\frac{V_b}{V_c}\right|_{I_b=0}$$





Graphical analysis of CE configuration

▪ The large-signal response of transistors are obtained graphically. For small signals the transistor operates with reasonable linearity, and we inquire into small-signal linear models which represent the operation of the transistor in the active region.

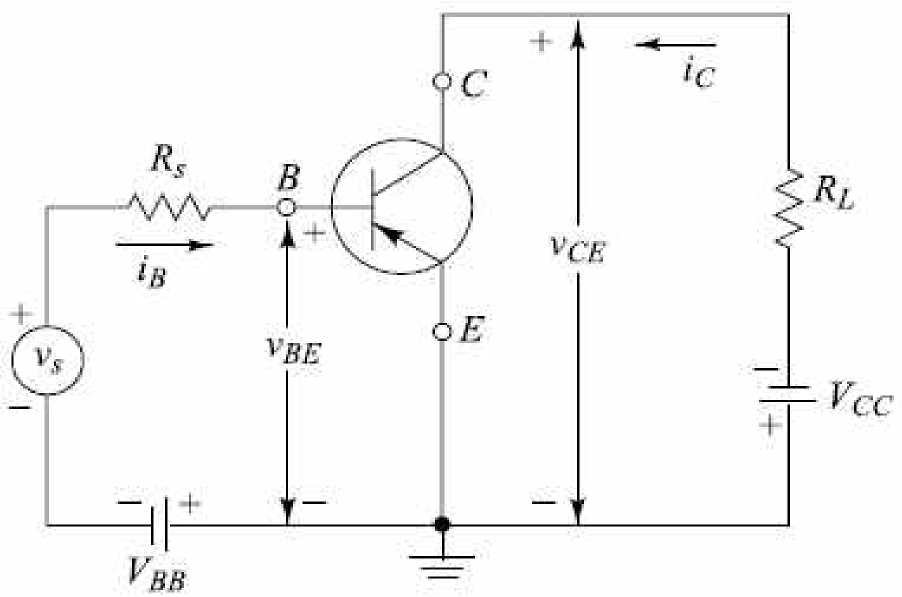


Fig. 8.1 *The CE transistor configuration.*

Graphical Analysis of the CE Configuration

- **Notation** instantaneous values are represented by lowercase letters (*i* for current, *v* for voltage, and *p* for power). Maximum, average (dc), and effective, or root-mean-square (rms), values are represented by the uppercase letter (dc) values instantaneous total values are indicated by the uppercase subscript of the proper electrode varying components from some quiescent value are indicated by the lowercase subscript.

$$I_C = f\left(I_B, V_{CE}\right)$$

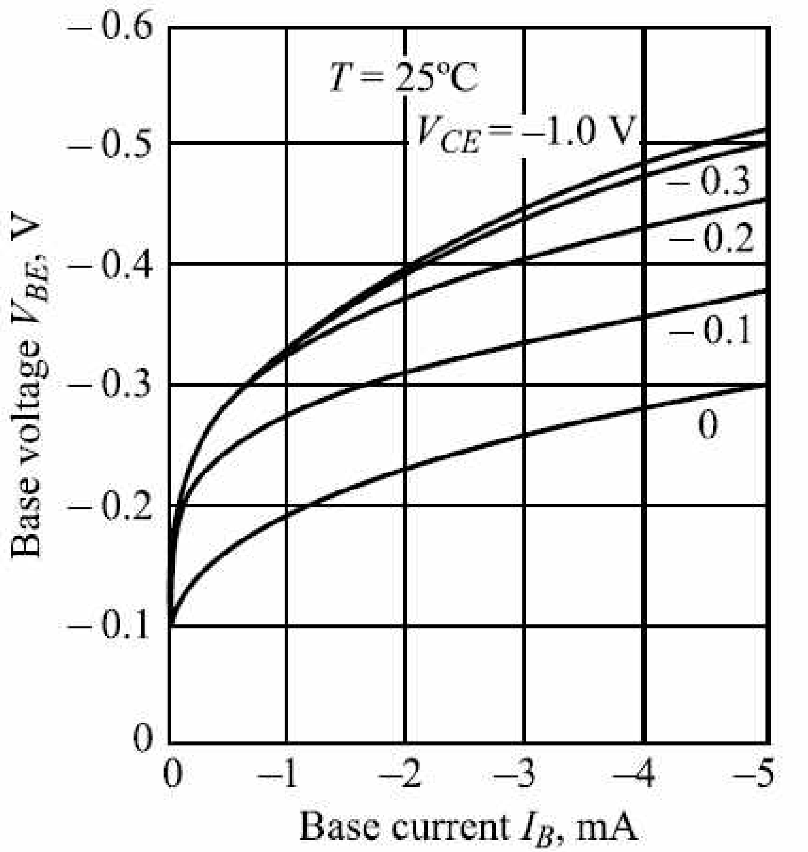
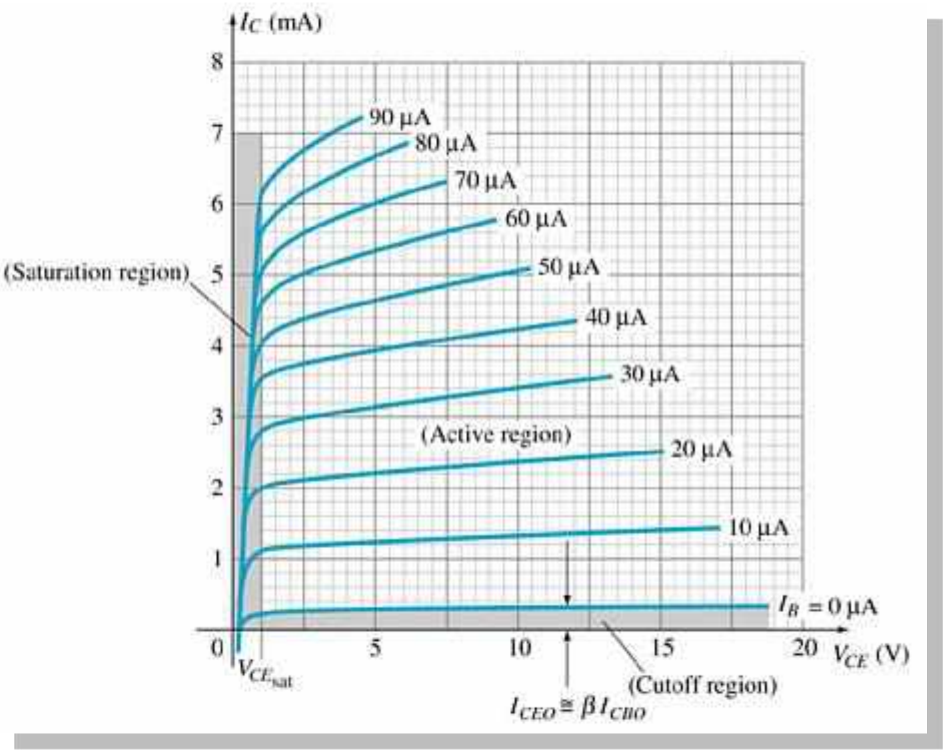
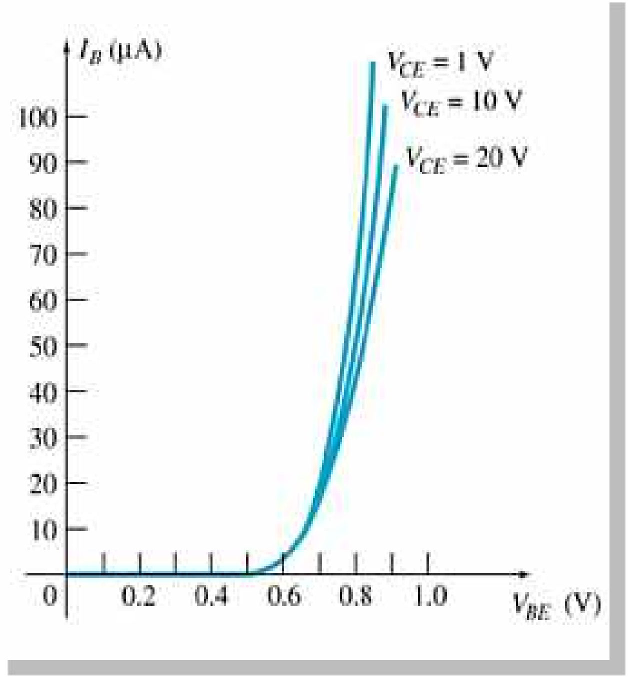


Fig. 5.11 *Typical common-emitter input characteristics of the p-n-p germanium junction transistor of Fig. 5.10.*

I/P and O/P Characteristics



O/P or Collector Characteristics (*n-p-n*)

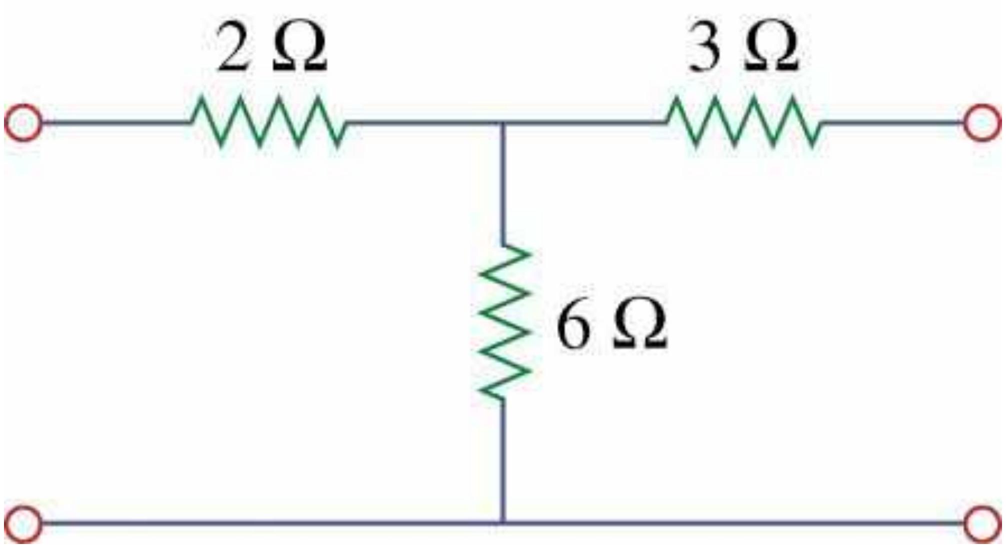


I/P or Base Characteristics (*n-p-n*)

$$I_C = f\left(I_B, V_{CE}\right)$$

$$V_{BE} = f\left(I_B, V_{CE}\right)$$

- Find the hybrid parameters for the two-port network:



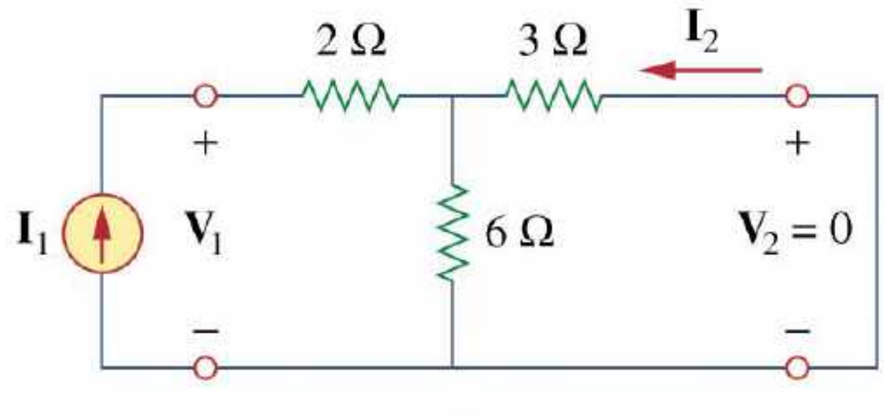
From Fig. (a),

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3 \| 6) = 4\mathbf{I}_1$$

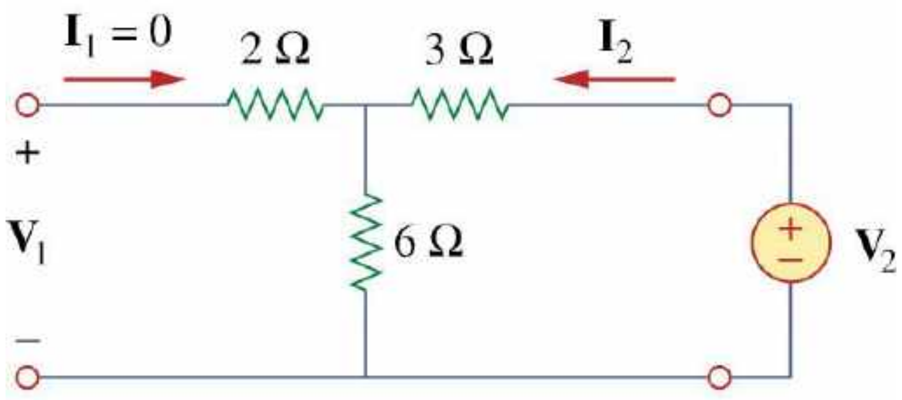
Hence, $\mathbf{h}_{11} = \dfrac{\mathbf{V}_1}{\mathbf{I}_1} = 4\ \Omega$

$$-\mathbf{I}_2 = \frac{6}{6+3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Hence, $h_{21} = \dfrac{\mathbf{I}_2}{\mathbf{I}_1} = -\dfrac{2}{3}$



(a)



(b)

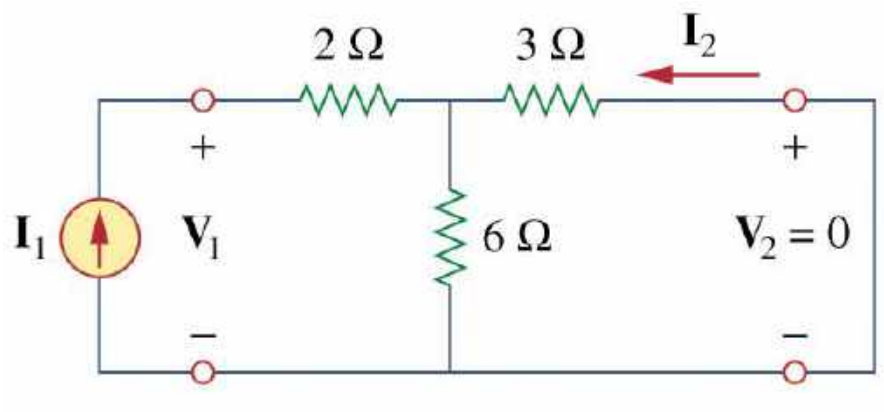
From Fig.(b),

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2$$

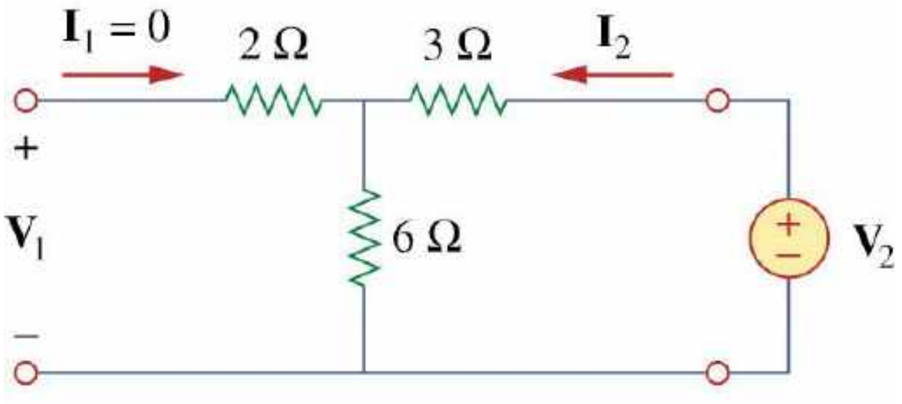
Hence, $\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$

Also, $\mathbf{V}_2 = (3+6)\mathbf{I}_2 = 9\mathbf{I}_2$

Thus, $\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{9}$ S



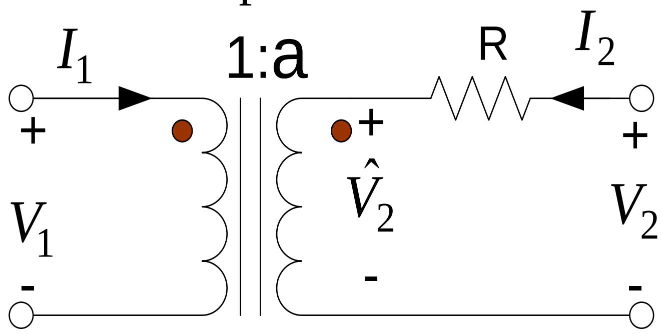
(a)



(b)

Example

Determine the h-parameter of the two-port circuit shown in Fig.



$$V_1 = \frac{1}{a}\hat{V}_2 \qquad I_1 = -aI_2$$

$$\hat{V}_2 = V_2 - RI_2 = \frac{R}{a}I_1 + V_2$$

$$V_1 = \frac{R}{a^2}I_1 + \frac{1}{a}V_2$$

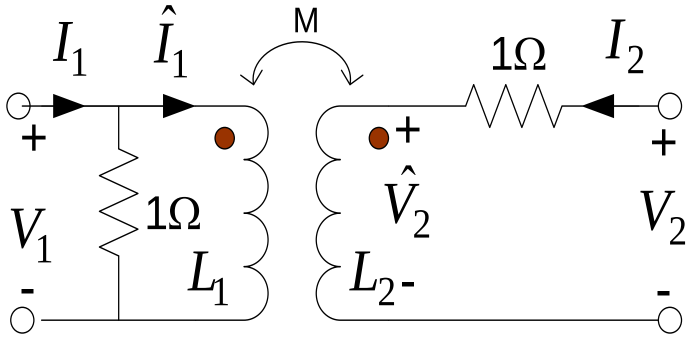
$$I_2 = \frac{V_2 - \hat{V}_2}{R} = -\frac{\hat{V}_2}{R} + \frac{V_2}{R}$$

$$= -\frac{1}{a}I_1 + 0V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{R}{a^2} & \frac{1}{a} \\ -\frac{1}{a} & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Example

Find the h-parameter of the circuit in Fig. assuming L_1=L_2=M=1H



In frequency domain

$$V_1 = sL_1\hat{I}_1 + sMI_2$$

$$\hat{I}_1 = I_1 - V_1$$

$$(1+sL_1)V_1 - sMI_2 = sL_1I_1$$

$$V_2 = \hat{V}_2 + I_2$$

$$\hat{V}_2 = sL_2 I_2 + sM\hat{I}_1 = sL_2 I_2 + sM(I_1 - V_1)$$

$$V_2 = (1 + sL_2)I_2 + sM(I_1 - V_1)$$

$$sMV_1 - (1 + sL_2)I_2 = sMI_1 - V_2$$

In matrix form

$$\begin{bmatrix} 1 + sL_1 & -sM \\ sM & -(1 + sL_2) \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} sL_1 & 0 \\ sM & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 + sL_1 & -sM \\ sM & -(1 + sL_2) \end{bmatrix}^{-1} \begin{bmatrix} sL_1 & 0 \\ sM & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

With $L_1 = L_2 = M = 1$ H

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1+s & -s \\ s & -(1+s) \end{bmatrix}^{-1} \begin{bmatrix} s & 0 \\ s & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$= \frac{1}{2s+1} \begin{bmatrix} s & s \\ -s & s+1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Transmission parameter

The T-parameter or transmission parameters are used in power system and it is called **ABCD parameter**. The transmission parameter is defined by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

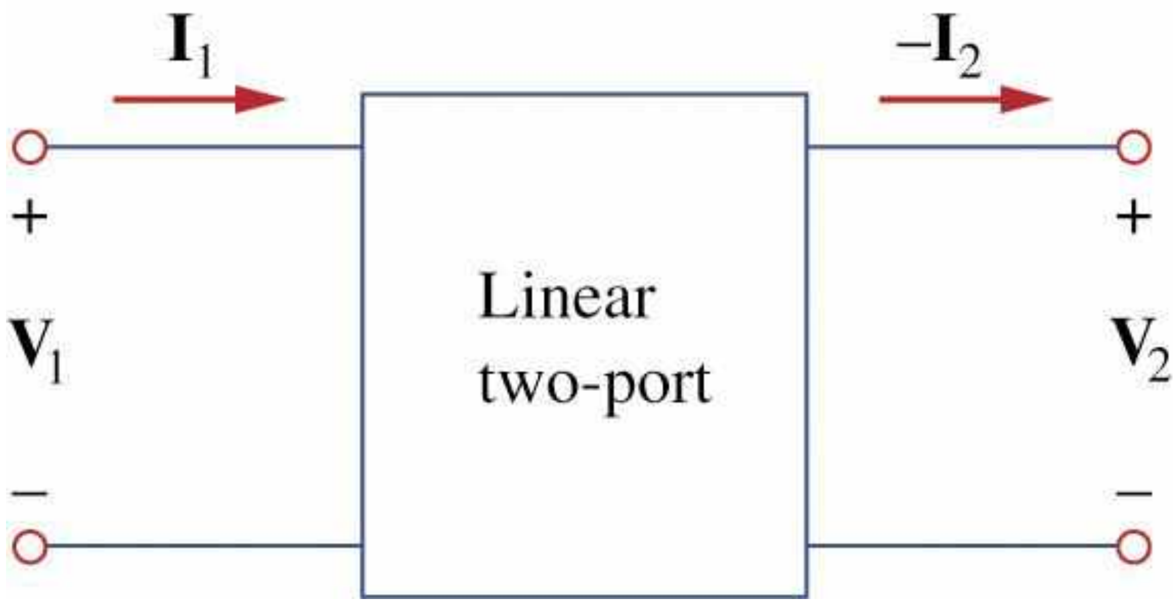
This means that the power flows into the input port and flow out to the load from the output port.

T-parameter can be calculated from

$$t_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad t_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$t_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad t_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Open or short circuit at the output port


$\mathbf{I}_1$
$-\mathbf{I}_2$
+
+
$\mathbf{V}_1$
Linear
two-port
$\mathbf{V}_2$
−
−

a = Open-circuit voltage gain

b = Negative short-circuit transfer impedance

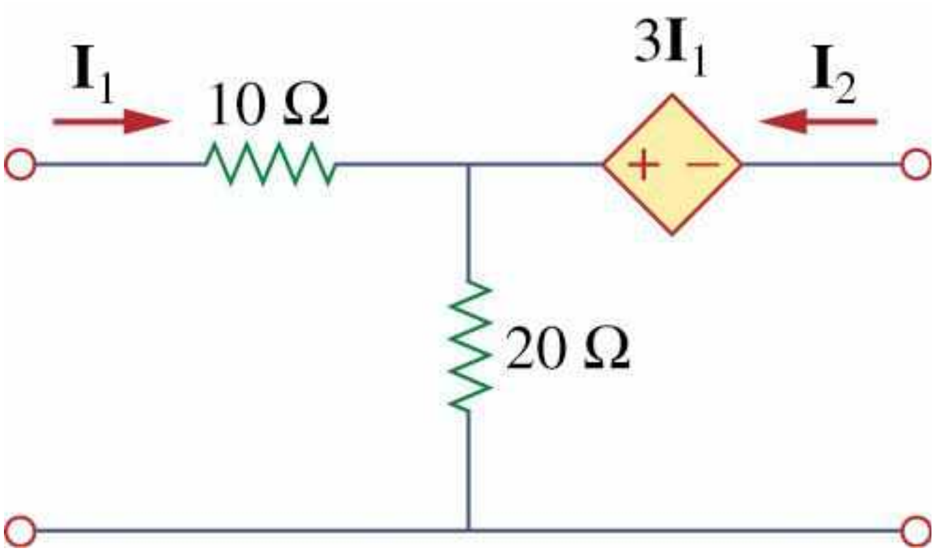
c = Open-circuit transfer admittance

d = Negative short-circuit current gain

$$\mathbf{AD} - \mathbf{BC} = 1, \quad \mathbf{ad} - \mathbf{bc} = 1$$

$$\mathbf{A} = \mathbf{D}$$

- Find the transmission parameters for the two-port network:

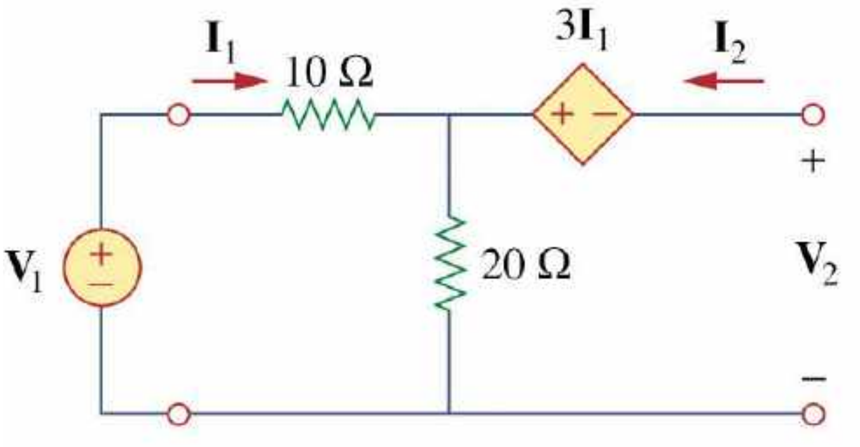


From Fig.(a),

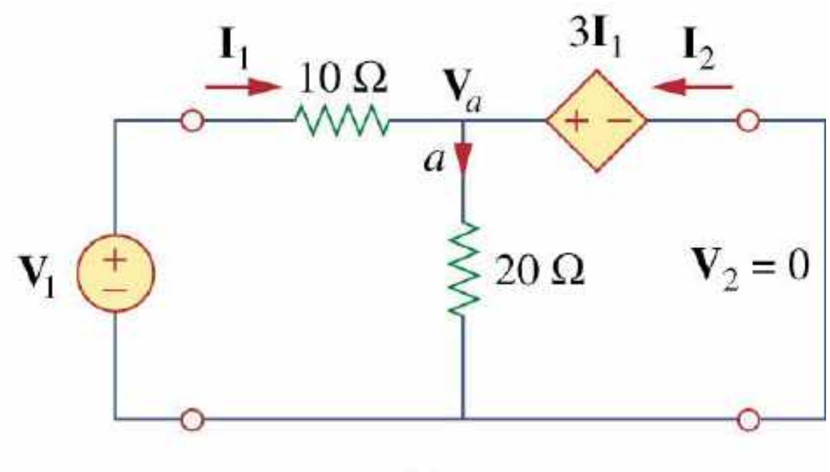
$$\mathbf{V}_1 = (10+20)\mathbf{I}_1 = 30\mathbf{I}_1 \text{ and } \mathbf{V}_2 = 20\mathbf{I}_1 - 3\mathbf{I}_1 = 17\mathbf{I}_1$$

Thus

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{30\mathbf{I}_1}{17\mathbf{I}_1} = 1.765, \quad \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_1}{17\mathbf{I}_1} = 0.0588 \text{ S}$$



(a)



(b)

From Fig.(b),

$$\frac{\mathbf{V}_1 - \mathbf{V}_a}{10} - \frac{\mathbf{V}_a}{20} + \mathbf{I}_2 = 0$$

But $\mathbf{V}_a = 3\mathbf{I}_1$ and $\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_a)/10,$

$$\Rightarrow \mathbf{V}_a = 3\mathbf{I}_1, \quad \mathbf{V}_1 = 13\mathbf{I}_1$$

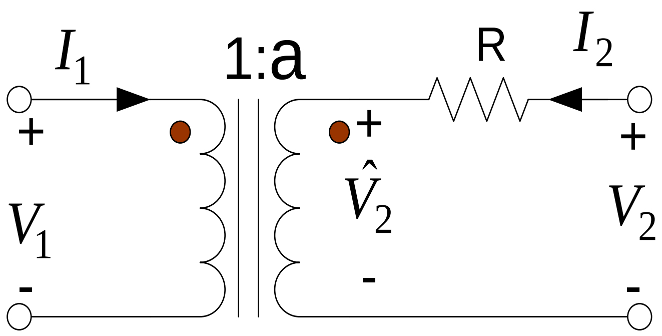
$$\Rightarrow \mathbf{I}_1 - \frac{3\mathbf{I}_1}{20} + \mathbf{I}_2 = 0 \Rightarrow \frac{17}{20}\mathbf{I}_1 = -\mathbf{I}_2$$

Thus,

$$\mathbf{D} = -\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{20}{17} = 1.176, \quad \mathbf{B} = -\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{-13\mathbf{I}_1}{(17/20)\mathbf{I}_1} = 15.29\ \Omega$$

Example

Determine the t-parameter of the circuit shown in Fig .



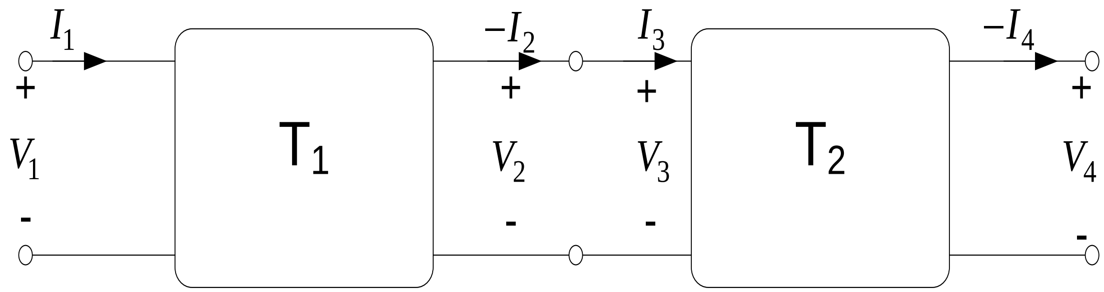
$$V_1 = \frac{1}{a}\hat{V}_2 = \frac{1}{a}(V_2 - RI_2)$$

$$I_1 = -aI_2$$

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & \frac{R}{a} \\ 0 & a \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Example

One of the most importance characteristics of the two-port circuit with T-parameter is to determine the overall cascade parameter.



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_1 \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = T_2 \begin{bmatrix} V_4 \\ -I_4 \end{bmatrix} \qquad V_2 = V_3\,, \quad -I_2 = I_3$$

Thus

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_1 T_2 \begin{bmatrix} V_4 \\ -I_4 \end{bmatrix}$$

Inverse Transmission parameter

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0} \qquad B' = -\left.\frac{V_2}{I_1}\right|_{V_1=0}$$

$$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0} \qquad D' = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

TABLE 19.1 Conversion of two-port parameters.

	z		y		h		g		T		t	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$	$\frac{d}{c}$	$\frac{1}{c}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta_t}{c}$	$\frac{a}{c}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$	$\frac{a}{b}$	$-\frac{1}{b}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$-\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta_t}{b}$	$\frac{d}{b}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$-\frac{g_{12}}{\Delta_g}$	$\frac{B}{D}$	$\frac{\Delta_T}{D}$	$\frac{b}{a}$	$\frac{1}{a}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$	$-\frac{1}{D}$	$\frac{C}{D}$	$\frac{\Delta_t}{a}$	$\frac{c}{a}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	g_{11}	g_{12}	$\frac{C}{A}$	$-\frac{\Delta_T}{A}$	$\frac{c}{d}$	$-\frac{1}{d}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}	$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta_t}{d}$	$-\frac{b}{d}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	A	B	$\frac{d}{\Delta_t}$	$\frac{b}{\Delta_t}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta_g}{g_{21}}$	C	D	$\frac{c}{\Delta_t}$	$\frac{a}{\Delta_t}$
t	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta_z}{z_{12}}$	$-\frac{y_{11}}{y_{12}}$	$-\frac{1}{y_{12}}$	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta_g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$	$\frac{D}{\Delta_T}$	$\frac{B}{\Delta_T}$	a	b
	$\frac{1}{z_{12}}$	$\frac{z_{11}}{z_{12}}$	$-\frac{\Delta_y}{y_{12}}$	$-\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta_h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$	$\frac{C}{\Delta_T}$	$\frac{A}{\Delta_T}$	c	d

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta_T = AD - BC$
$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta_g = g_{11}g_{22} - g_{12}g_{21}$, $\Delta_t = ad - bc$

Some Extra Problems

- Find [**z**] and [**g**] of a two-port network if

$$[\mathbf{T}] = \begin{bmatrix} 10 & 1.5\,\Omega \\ 2\,\text{S} & 4 \end{bmatrix}$$

- Solution:

If $\mathbf{A} = 10$, $\mathbf{B} = 1.5$, $\mathbf{C} = 2$, $\mathbf{D} = 4$, the determinant of the matrix is

$$\Delta_T = \mathbf{AD} - \mathbf{BC} = 40 - 3 = 37$$

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} = \frac{10}{2} = 5, \quad \mathbf{z}_{12} = \frac{\Delta_T}{\mathbf{C}} = \frac{37}{2} = 18.5$$

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}} = \frac{1}{2} = 0.5, \quad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}} = \frac{4}{2} = 2$$

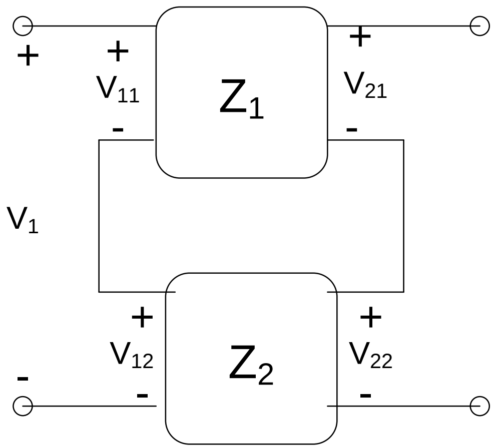
$$\mathbf{g}_{11} = \frac{\mathbf{C}}{\mathbf{A}} = \frac{2}{10} = 0.2, \quad \mathbf{g}_{12} = -\frac{\Delta_T}{\mathbf{A}} = -\frac{37}{10} = -3.7$$

$$\mathbf{g}_{21} = \frac{1}{\mathbf{A}} = \frac{1}{10} = 0.1, \quad \mathbf{g}_{22} = \frac{\mathbf{B}}{\mathbf{A}} = \frac{1.5}{10} = 0.15$$

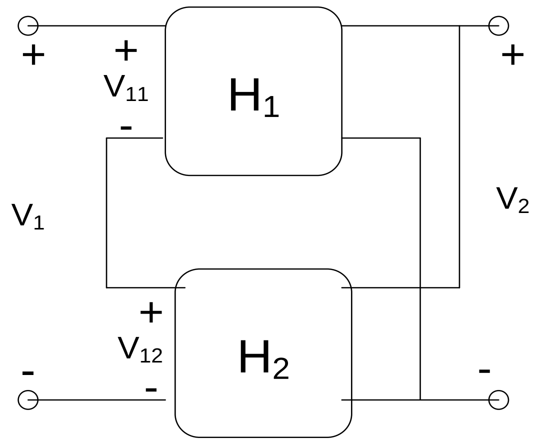
$$\text{Thus, } [\mathbf{z}] = \begin{bmatrix} 5 & 18.5 \\ 0.5 & 2 \end{bmatrix}\Omega, \qquad [\mathbf{g}] = \begin{bmatrix} 0.2\,\text{S} & -3.7 \\ 0.1 & 0.15\,\Omega \end{bmatrix}$$

Interconnection of two-port network

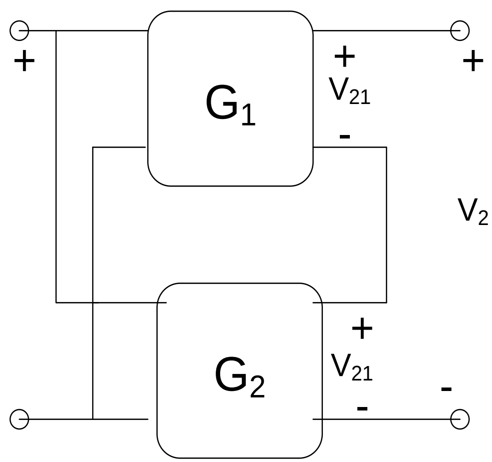
- Two port networks can be connected in series parallel or cascaded
- Series and parallel of two-port have 4 configurations
  - Series input-series output (Z-parameter)
  - Series input-parallel output (h-parameter)
  - Parallel in put-series output (g or h^{-1}-parameter)
  - Parallel input-parallel output (Y-parameter)
- With proper choice of parameters the combined parameters can be added together.



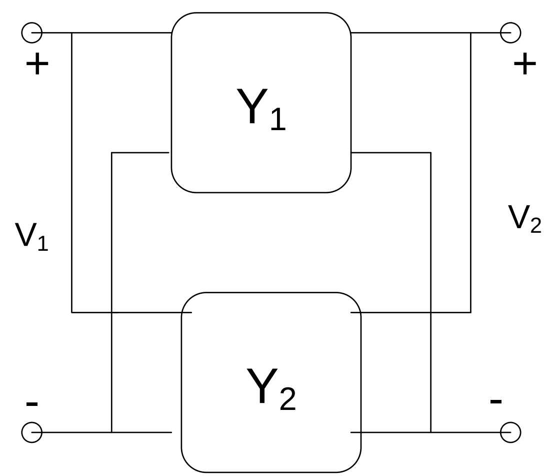
$Z=Z_1+Z_2$



$H=H_1+H_2$



$G=G_1+G_2$

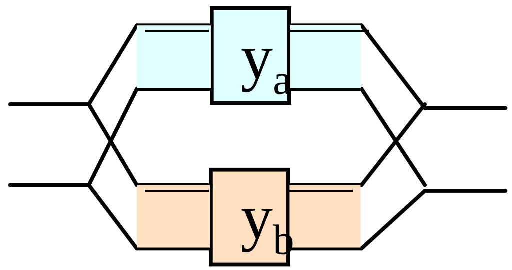


$Y=Y_1+Y_2$

Interconnection of Two- Port Networks

Three ways that two ports are interconnected:

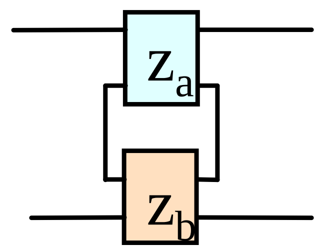
* Parallel



Y parameters

$$[y]=[y_a]+[y_b]$$

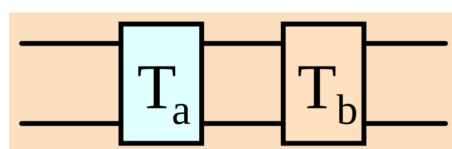
* Series



Z parameters

$$[z]=[z_a]+[z_b]$$

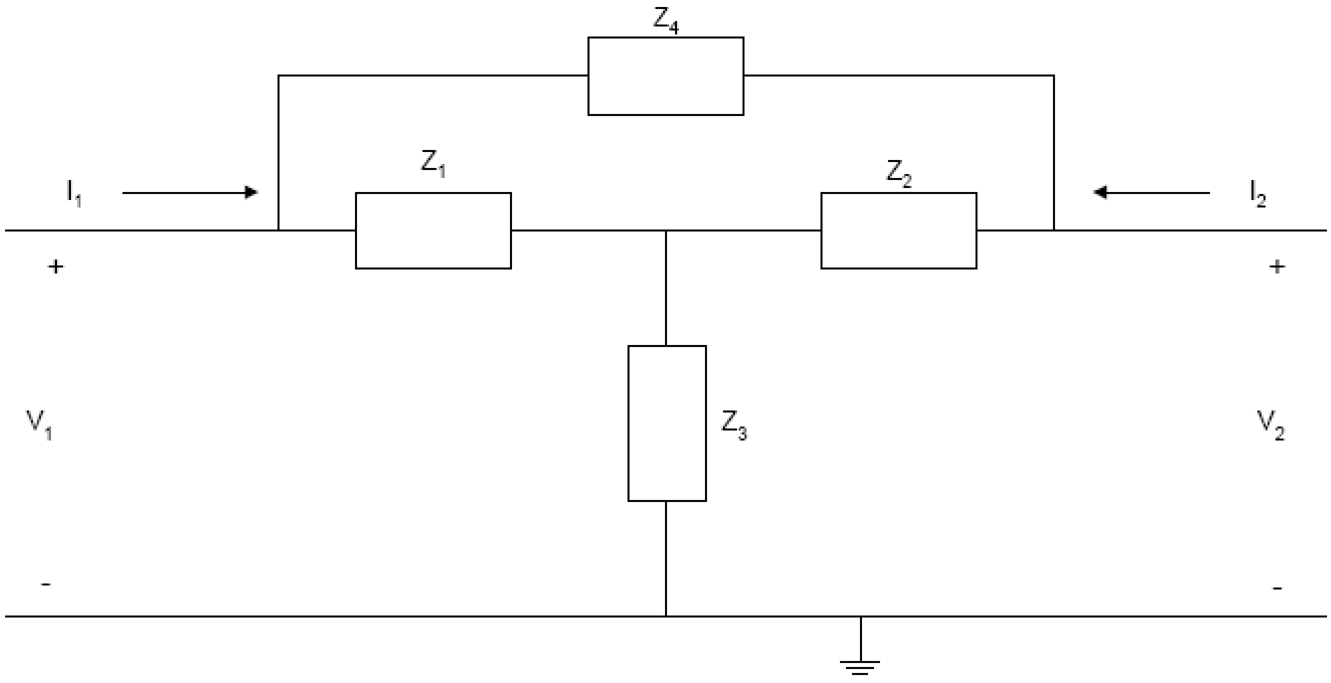
* Cascade

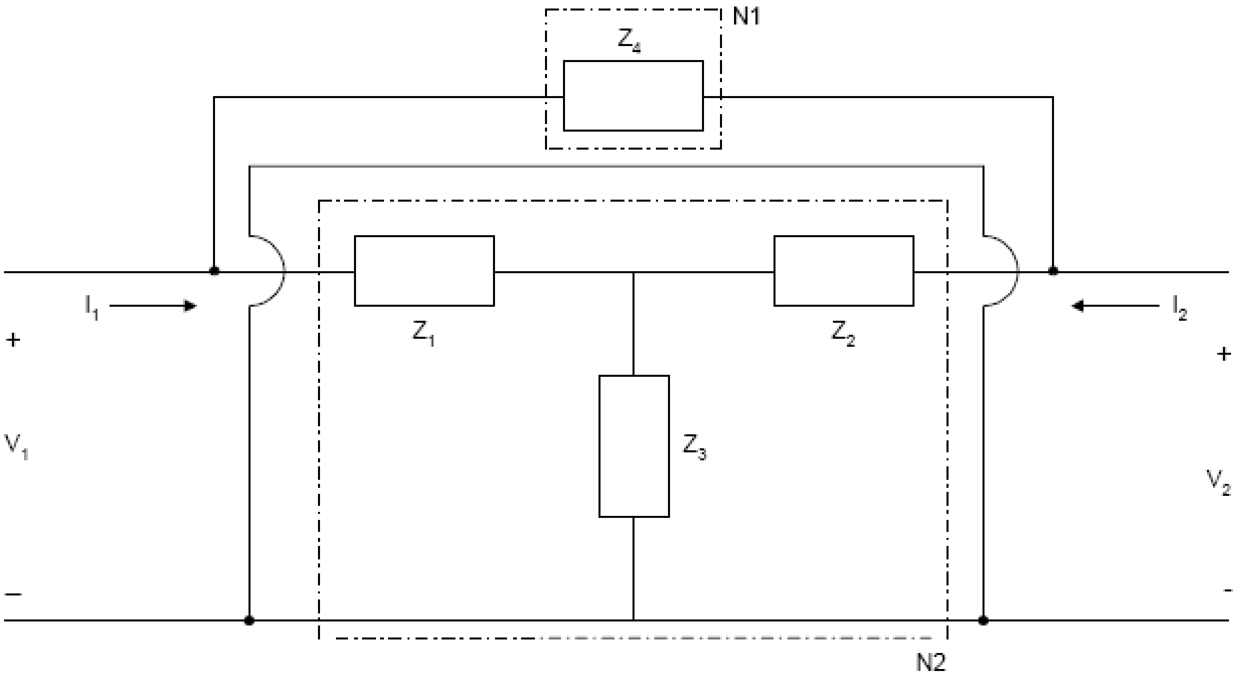


ABCD parameters

$$[T]=[T_a]\,[T_b]$$

Example: Bridge-T network





N1 // N2

For network N2

$$[Z] = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \longrightarrow$$

$$y_{11} = \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$y_{12} = \frac{-Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$y_{21} = \frac{-Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$y_{22} = -\frac{Z_1 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

For network N1

$$[T] = \begin{bmatrix} 1 & Z_4 \\ 0 & 1 \end{bmatrix} \longrightarrow$$

$$y_{11} = \frac{1}{Z_4}$$

$$y_{12} = -\frac{1}{Z_4}$$

$$y_{21} = -\frac{1}{Z_4}$$

$$y_{22} = \frac{1}{Z_4}$$

Y-parameters of the bridge-T network are

$$y_{11eq} = \frac{1}{Z_4} + \frac{Z_2 + Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

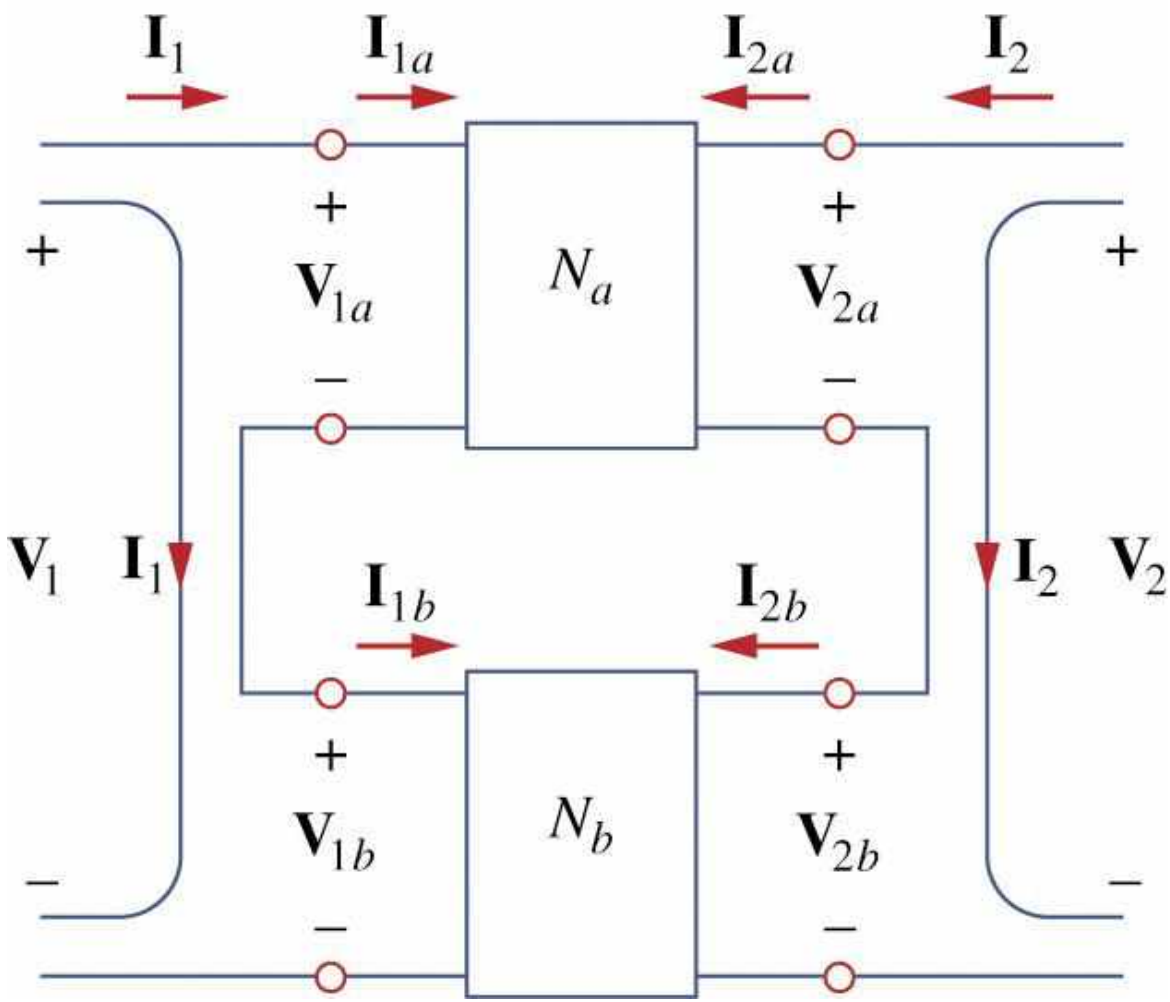
$$y_{12eq} = -\frac{1}{Z_4} - \frac{Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$y_{21eq} = -\frac{1}{Z_4} - \frac{Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

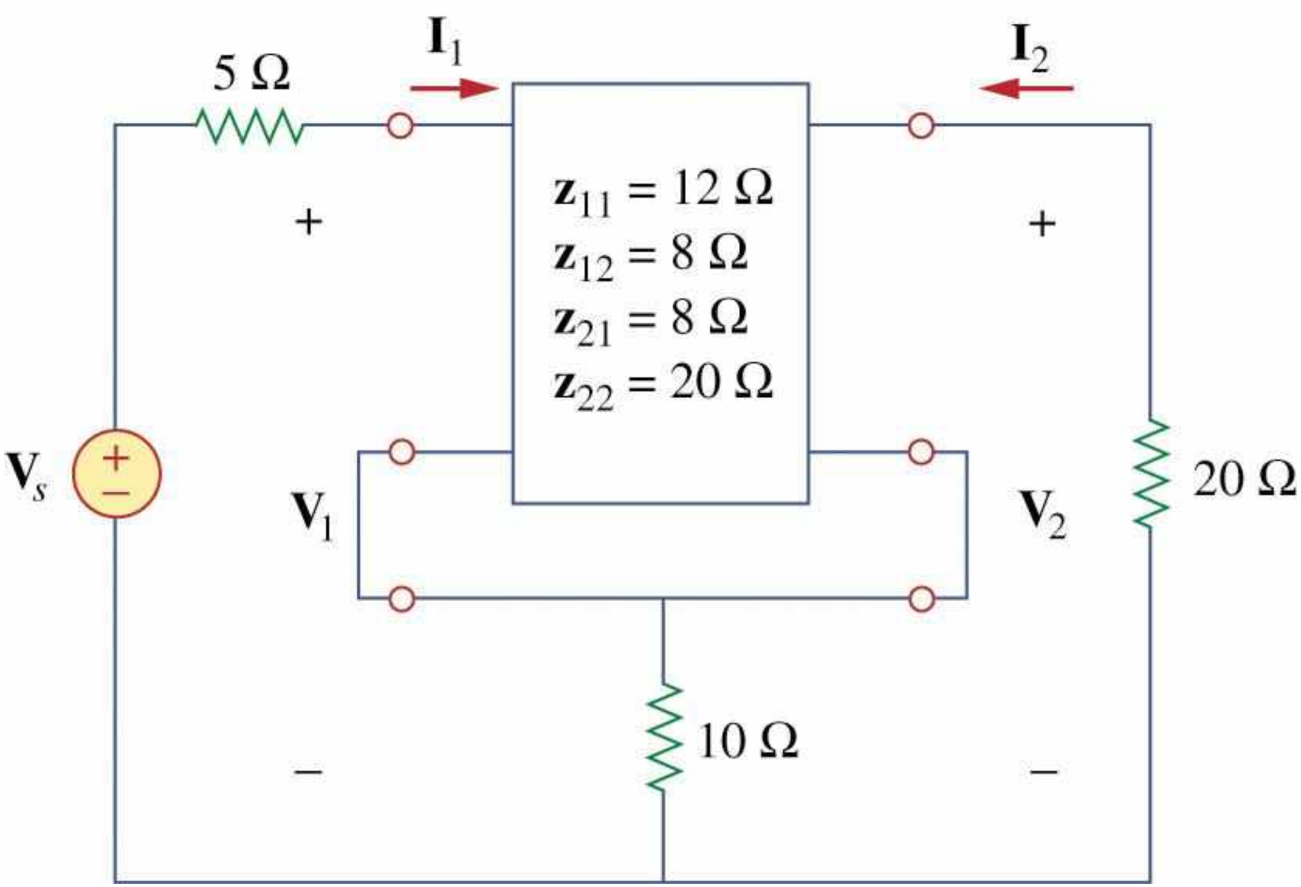
$$y_{22eq} = \frac{1}{Z_4} + \frac{Z_1 + Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

Interconnection of Networks

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]$$



- Evaluate $\mathbf{V}_2/\mathbf{V}_1$ in the circuit in Fig.:



This may be regarded as two - ports in series.

For N_b,

$$\mathbf{z}_{12b} = \mathbf{z}_{21b} = 10 = \mathbf{z}_{11b} = \mathbf{z}_{22b}$$

Thus,

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

But

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 = 22\mathbf{I}_1 + 18\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{32}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 = 18\mathbf{I}_1 + 30\mathbf{I}_2$$

Also, at the input port $\mathbf{V}_1 = \mathbf{V}_s - 5\mathbf{I}_1$

and at the output port $\mathbf{V}_2 = -20\mathbf{I}_2 \Rightarrow \mathbf{I}_2 = -\frac{\mathbf{V}_2}{20}$

$$\Rightarrow \mathbf{V}_s - 5\mathbf{I}_1 = 22\mathbf{I}_1 - \frac{18}{20}\mathbf{V}_2 \Rightarrow \mathbf{V}_s = 27\mathbf{I}_1 - 0.9\mathbf{V}_2$$

$$\Rightarrow \mathbf{V}_2 = 18\mathbf{I}_1 - \frac{30}{20}\mathbf{V}_2 \Rightarrow \mathbf{I}_1 = \frac{2.5}{18}\mathbf{V}_2$$

$$\Rightarrow \mathbf{V}_s = 27 \times \frac{2.5}{18}\mathbf{V}_2 - 0.9\mathbf{V}_2 = 2.85\mathbf{V}_2$$

And also, $\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{1}{2.85} = 0.3509$

Thank you

International Journal of Electronics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tetn20

Relations between two-port parameters

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Published online: 06 Apr 2007.

INT. J. ELECTRONICS, 1991, VOL. 71, NO. 1, 107–116

Relations between two-port parameters

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In the course of our research activities we have had frequent need for a set of relations between the parameters of two-port networks. Whilst some of these are scattered throughout the literature, it is not possible to find a comprehensive, error-free collection; with this in mind, we have compiled such a set.

Notation

The following notation is used below:

$$M=\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

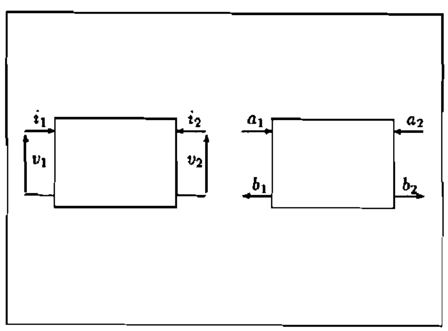
where M is any of the matrices $Z, Y, H, G, A, B, S, T, U$. These in turn are defined in terms of the quantities indicated in the Figure.

Throughout, Z represents the impedance matrix and Y the admittance matrix:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

The h-parameter matrix is H and G is its inverse:

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$



Received 3 December 1990; accepted 3 December 1990.

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0020-7217/91 $3.00

The elements of A are known as the general circuit constants and conventionally designated A, B, C, D; we depart from this convention by using the more logical notation $A = a_{11}$, $B = a_{12}$, $C = a_{21}$ and $D = a_{22}$. Our matrix B is defined as the inverse of A, so that

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix}$$

In order to make our results of as general application as possible, we have allowed for unequal characteristic impedances by using the following definitions of a_1, a_2, b_1, b_2 in terms of v_1, v_2, i_1, i_2:

$$a_1 \triangleq \frac{v_1 + Z_{01} i_1}{2\sqrt{Z_{01}}}, \quad b_1 \triangleq \frac{v_1 - Z_{01} i_1}{2\sqrt{Z_{01}}}$$

$$a_2 \triangleq \frac{v_2 + Z_{02} i_2}{2\sqrt{Z_{02}}}, \quad b_2 \triangleq \frac{v_2 - Z_{02} i_2}{2\sqrt{Z_{02}}}$$

The scattering matrix is denoted by S and is defined as follows:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

We denote the scattering transfer matrix by T and the inverse of T by U, i.e.

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} b_1 \\ a_1 \end{bmatrix}$$

Throughout the following it has been convenient to use

$$\Delta_M \triangleq m_{11} m_{22} - m_{12} m_{21}$$

for the determinant of any of the above matrices.

We present below all the relationships between the various two-port parameters mentioned above. The two-port parameters themselves fall naturally into two classes:

(*a*) the wave parameters S, T, U which comprise one class and are so called because they relate the forward and backward wave amplitudes; and

(*b*) the circuit parameters Z, Y, H, G, A, B which form the other class and are so called because they relate voltages and currents to each other.

The relationships presented below are then split into four sets. Set 1 contains the relationships between the wave parameters (e.g. S in terms of T). Set 2 contains the relationships between the circuit parameters (e.g. Y in terms of Z). Set 3 contains the wave parameters in terms of the circuit parameters (e.g. S in terms of Z). Set 4 contains the circuit parameters in terms of the wave parameters (e.g. Z in terms of S).

In conclusion, a comment on the need for this many sets of parameters is called for. The Z, Y and S parameters are widely used for their relationships to the familiar quantities of impedance, admittance and forward/backward wave ratios. The H and G parameters are widely used in the context of active devices, where the parameters are readily identifiable with familiar concepts. However, none of these matrices can be meaningfully multiplied together to give the overall matrix for n two-ports cascaded. For this purpose, the A, B, T and U matrices are required, since we can write

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = A_1 A_2 \ldots A_n \begin{bmatrix} v_n \\ -i_n \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ -i_n \end{bmatrix} = B_n \ldots B_2 B_1 \begin{bmatrix} v_1 \\ i_1 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = T_1 T_2 \ldots T_n \begin{bmatrix} a_n \\ b_n \end{bmatrix}$$

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = U_n \ldots U_2 U_1 \begin{bmatrix} b_1 \\ a_1 \end{bmatrix}$$

The need for these four sets of equations arises with non-reciprocal networks. In particular, if there is zero transmission in one direction, only one of each of $A/B, T/U$ exists.

There is no overriding criterion for choice between the use of Z, Y, G, H, A and B.

Reference

Balabian, N., and Bickart, T. A., 1968, *Electrical Network Theory* (New York: Wiley).

$$S = \frac{1}{t_{22}} \begin{bmatrix} t_{12} & \Delta_T \\ 1 & -t_{21} \end{bmatrix} = \frac{1}{u_{11}} \begin{bmatrix} -u_{12} & 1 \\ \Delta_U & u_{21} \end{bmatrix}$$

$$T = \frac{1}{s_{21}} \begin{bmatrix} -\Delta_S & s_{11} \\ -s_{22} & 1 \end{bmatrix} = \frac{1}{\Delta_U} \begin{bmatrix} u_{22} & -u_{12} \\ -u_{21} & u_{11} \end{bmatrix}$$

$$U = \frac{1}{s_{12}} \begin{bmatrix} 1 & -s_{11} \\ s_{22} & -\Delta_S \end{bmatrix} = \frac{1}{\Delta_T} \begin{bmatrix} t_{22} & -t_{12} \\ -t_{21} & t_{11} \end{bmatrix}$$

Set 1. Wave parameter relations.

$$Z = \frac{1}{\Delta_Y}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix} = \frac{1}{h_{22}}\begin{bmatrix} \Delta_H & h_{12} \\ -h_{21} & 1 \end{bmatrix} = \frac{1}{g_{11}}\begin{bmatrix} 1 & -g_{12} \\ g_{21} & \Delta_G \end{bmatrix}$$
$$= \frac{1}{a_{21}}\begin{bmatrix} a_{11} & \Delta_A \\ 1 & a_{22} \end{bmatrix} = \frac{-1}{b_{21}}\begin{bmatrix} b_{22} & 1 \\ \Delta_B & b_{11} \end{bmatrix}$$

$$Y = \frac{1}{\Delta_Z}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix} = \frac{1}{h_{11}}\begin{bmatrix} 1 & -h_{12} \\ h_{21} & \Delta_H \end{bmatrix} = \frac{1}{g_{22}}\begin{bmatrix} \Delta_G & g_{12} \\ -g_{21} & 1 \end{bmatrix}$$
$$= \frac{1}{a_{12}}\begin{bmatrix} a_{22} & -\Delta_A \\ -1 & a_{11} \end{bmatrix} = \frac{1}{b_{12}}\begin{bmatrix} -b_{11} & 1 \\ \Delta_B & -b_{22} \end{bmatrix}$$

$$H = \frac{1}{z_{22}}\begin{bmatrix} \Delta_Z & z_{12} \\ -z_{21} & 1 \end{bmatrix} = \frac{1}{y_{11}}\begin{bmatrix} 1 & -y_{12} \\ y_{21} & \Delta_Y \end{bmatrix} = \frac{1}{\Delta_G}\begin{bmatrix} g_{22} & -g_{12} \\ -g_{21} & g_{11} \end{bmatrix}$$
$$= \frac{1}{a_{22}}\begin{bmatrix} a_{12} & \Delta_A \\ -1 & a_{21} \end{bmatrix} = \frac{-1}{b_{11}}\begin{bmatrix} b_{12} & -1 \\ \Delta_B & b_{21} \end{bmatrix}$$

$$G = \frac{1}{z_{11}}\begin{bmatrix} 1 & -z_{12} \\ z_{21} & \Delta_Z \end{bmatrix} = \frac{1}{y_{22}}\begin{bmatrix} \Delta_Y & y_{12} \\ -y_{21} & 1 \end{bmatrix} = \frac{1}{\Delta_H}\begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$$
$$= \frac{1}{a_{11}}\begin{bmatrix} a_{21} & -\Delta_A \\ 1 & a_{12} \end{bmatrix} = \frac{-1}{b_{22}}\begin{bmatrix} b_{21} & 1 \\ -\Delta_B & b_{12} \end{bmatrix}$$

$$A = \frac{1}{z_{21}}\begin{bmatrix} z_{11} & \Delta_Z \\ 1 & z_{22} \end{bmatrix} = \frac{-1}{y_{21}}\begin{bmatrix} y_{22} & 1 \\ \Delta_Y & y_{11} \end{bmatrix} = \frac{-1}{h_{21}}\begin{bmatrix} \Delta_H & h_{11} \\ h_{22} & 1 \end{bmatrix}$$
$$= \frac{1}{g_{21}}\begin{bmatrix} 1 & g_{22} \\ g_{11} & \Delta_G \end{bmatrix} = \frac{1}{\Delta_B}\begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}$$

$$B = \frac{1}{z_{12}}\begin{bmatrix} z_{22} & -\Delta_Z \\ -1 & z_{11} \end{bmatrix} = \frac{1}{y_{12}}\begin{bmatrix} -y_{11} & 1 \\ \Delta_Y & -y_{22} \end{bmatrix} = \frac{1}{h_{12}}\begin{bmatrix} 1 & -h_{11} \\ -h_{22} & \Delta_H \end{bmatrix}$$
$$= \frac{1}{g_{12}}\begin{bmatrix} -\Delta_G & g_{22} \\ g_{11} & -1 \end{bmatrix} = \frac{1}{\Delta_A}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Set 2. Circuit parameter relations.

$$S = \frac{1}{(z_{11}+Z_{01})(z_{22}+Z_{02})-z_{12}z_{21}}\begin{bmatrix}(z_{11}-Z_{01})(z_{22}+Z_{02})-z_{12}z_{21} & 2z_{12}\sqrt{Z_{01}Z_{02}}\\ 2z_{21}\sqrt{Z_{01}Z_{02}} & (z_{11}+Z_{01})(z_{22}-Z_{02})-z_{12}z_{21}\end{bmatrix}$$

$$= \frac{1}{Z_{01}Z_{02}\Delta_Y+Z_{01}y_{11}+Z_{02}y_{22}+1}\begin{bmatrix}-Z_{01}Z_{02}\Delta_Y-Z_{01}y_{11}+Z_{02}y_{22}+1 & -2y_{12}\sqrt{Z_{01}Z_{02}}\\ -2y_{21}\sqrt{Z_{01}Z_{02}} & -Z_{01}Z_{02}\Delta_Y+Z_{01}y_{11}-Z_{02}y_{22}+1\end{bmatrix}$$

$$= \frac{1}{Z_{02}\Delta_H+Z_{01}(Z_{02}h_{22}+1)+h_{11}}\begin{bmatrix}Z_{02}\Delta_H-Z_{01}(Z_{02}h_{22}+1)+h_{11} & 2h_{12}\sqrt{Z_{01}Z_{02}}\\ -2h_{21}\sqrt{Z_{01}Z_{02}} & -Z_{02}\Delta_H-Z_{01}(Z_{02}h_{22}-1)+h_{11}\end{bmatrix}$$

$$= \frac{1}{Z_{01}\Delta_G+Z_{02}(Z_{01}g_{11}+1)+g_{22}}\begin{bmatrix}-Z_{01}\Delta_G-Z_{02}(Z_{01}g_{11}-1)+g_{22} & -2g_{12}\sqrt{Z_{01}Z_{02}}\\ 2g_{21}\sqrt{Z_{01}Z_{02}} & Z_{01}\Delta_G-Z_{02}(Z_{01}g_{11}+1)+g_{22}\end{bmatrix}$$

$$= \frac{1}{Z_{02}(Z_{01}a_{21}+a_{11})+Z_{01}a_{22}+a_{12}}\begin{bmatrix}-Z_{02}(Z_{01}a_{21}-a_{11})-Z_{01}a_{22}+a_{12} & 2\Delta_A\sqrt{Z_{01}Z_{02}}\\ 2\sqrt{Z_{01}Z_{02}} & -Z_{02}(Z_{01}a_{21}+a_{11})+Z_{01}a_{22}+a_{12}\end{bmatrix}$$

$$= \frac{1}{Z_{02}(Z_{01}b_{21}-b_{22})-Z_{01}b_{11}+b_{12}}\begin{bmatrix}-Z_{02}(Z_{01}b_{21}+b_{22})+Z_{01}b_{11}+b_{12} & -2\sqrt{Z_{01}Z_{02}}\\ -2\Delta_B\sqrt{Z_{01}Z_{02}} & -Z_{02}(Z_{01}b_{21}-b_{22})-Z_{01}b_{11}+b_{12}\end{bmatrix}$$

(Set 3 continued overleaf)

$$
\begin{aligned}
T &= \frac{1}{2z_{21}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} -\Delta_Z + Z_{01}z_{22} + Z_{02}z_{11} - Z_{01}Z_{02} & \Delta_Z - Z_{01}z_{22} + Z_{02}z_{11} - Z_{01}Z_{02} \\ -\Delta_Z - Z_{01}z_{22} + Z_{02}z_{11} + Z_{01}Z_{02} & \Delta_Z + Z_{01}z_{22} + Z_{02}z_{11} + Z_{01}Z_{02} \end{bmatrix} \\
&= \frac{1}{2y_{21}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} Z_{01}Z_{02}\Delta_Y - Z_{01}y_{11} - Z_{02}y_{22} + 1 & Z_{01}Z_{02}\Delta_Y + Z_{01}y_{11} - Z_{02}y_{22} - 1 \\ -Z_{01}Z_{02}\Delta_Y + Z_{01}y_{11} - Z_{02}y_{22} + 1 & -Z_{01}Z_{02}\Delta_Y - Z_{01}y_{11} - Z_{02}y_{22} - 1 \end{bmatrix} \\
&= \frac{1}{2h_{21}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} -Z_{02}\Delta_H + Z_{01}Z_{02}h_{22} - Z_{01} + h_{11} & -Z_{02}\Delta_H + Z_{01}Z_{02}h_{22} + Z_{01} - h_{11} \\ -Z_{02}\Delta_H - Z_{01}Z_{02}h_{22} + Z_{01} + h_{11} & -Z_{02}\Delta_H - Z_{01}Z_{02}h_{22} - Z_{01} - h_{11} \end{bmatrix} \\
&= \frac{1}{2g_{21}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} Z_{01}\Delta_G - Z_{01}Z_{02}g_{11} + Z_{02} - g_{22} & -Z_{01}\Delta_G - Z_{01}Z_{02}g_{11} + Z_{02} + g_{22} \\ -Z_{01}\Delta_G + Z_{01}Z_{02}g_{11} + Z_{02} - g_{22} & Z_{01}\Delta_G + Z_{01}Z_{02}g_{11} + Z_{02} + g_{22} \end{bmatrix} \\
&= \frac{1}{2\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} -Z_{01}Z_{02}a_{21} + Z_{02}a_{11} + Z_{01}a_{22} - a_{12} & -Z_{01}Z_{02}a_{21} + Z_{02}a_{11} - Z_{01}a_{22} + a_{12} \\ Z_{01}Z_{02}a_{21} + Z_{02}a_{11} - Z_{01}a_{22} - a_{12} & Z_{01}Z_{02}a_{21} + Z_{02}a_{11} + Z_{01}a_{22} + a_{12} \end{bmatrix} \\
&= \frac{1}{2\Delta_B\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} Z_{01}Z_{02}b_{21} + Z_{02}b_{22} + Z_{01}b_{11} + b_{12} & Z_{01}Z_{02}b_{21} + Z_{02}b_{22} - Z_{01}b_{11} - b_{12} \\ -Z_{01}Z_{02}b_{21} + Z_{02}b_{22} - Z_{01}b_{11} + b_{12} & -Z_{01}Z_{02}b_{21} + Z_{02}b_{22} + Z_{01}b_{11} - b_{12} \end{bmatrix}
\end{aligned}
$$

$$U = \frac{1}{2z_{12}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix}\Delta_Z + Z_{01}z_{22} + Z_{02}z_{11} + Z_{01}Z_{02} & -\Delta_Z + Z_{01}z_{22} - Z_{02}z_{11} + Z_{01}Z_{02}\\ \Delta_Z + Z_{01}z_{22} - Z_{02}z_{11} - Z_{01}Z_{02} & -\Delta_Z + Z_{01}z_{22} + Z_{02}z_{11} - Z_{01}Z_{02}\end{bmatrix}$$

$$= \frac{1}{2y_{12}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix}-Z_{01}Z_{02}\Delta_Y - Z_{01}y_{11} - Z_{02}y_{22} - 1 & -Z_{01}Z_{02}\Delta_Y - Z_{01}y_{11} + Z_{02}y_{22} + 1\\ Z_{01}Z_{02}\Delta_Y - Z_{01}y_{11} + Z_{02}y_{22} - 1 & Z_{01}Z_{02}\Delta_Y - Z_{01}y_{11} - Z_{02}y_{22} + 1\end{bmatrix}$$

$$= \frac{1}{2h_{12}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix}Z_{02}\Delta_H + Z_{01}Z_{02}h_{22} + Z_{01} + h_{11} & -Z_{02}\Delta_H + Z_{01}Z_{02}h_{22} + Z_{01} - h_{11}\\ -Z_{02}\Delta_H - Z_{01}Z_{02}h_{22} + Z_{01} + h_{11} & Z_{02}\Delta_H - Z_{01}Z_{02}h_{22} + Z_{01} - h_{11}\end{bmatrix}$$

$$= \frac{1}{2g_{12}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix}-Z_{01}\Delta_G - Z_{01}Z_{02}g_{11} - Z_{02} - g_{22} & -Z_{01}\Delta_G - Z_{01}Z_{02}g_{11} + Z_{02} + g_{22}\\ -Z_{01}\Delta_G + Z_{01}Z_{02}g_{11} + Z_{02} - g_{22} & -Z_{01}\Delta_G + Z_{01}Z_{02}g_{11} - Z_{02} + g_{22}\end{bmatrix}$$

$$= \frac{1}{2\Delta_A\sqrt{Z_{01}Z_{02}}}\begin{bmatrix}Z_{01}Z_{02}a_{21} + Z_{02}a_{11} + Z_{01}a_{22} + a_{12} & Z_{01}Z_{02}a_{21} - Z_{02}a_{11} + Z_{01}a_{22} - a_{12}\\ -Z_{01}Z_{02}a_{21} - Z_{02}a_{11} + Z_{01}a_{22} + a_{12} & -Z_{01}Z_{02}a_{21} + Z_{02}a_{11} + Z_{01}a_{22} - a_{12}\end{bmatrix}$$

$$= \frac{1}{2\sqrt{Z_{01}Z_{02}}}\begin{bmatrix}-Z_{01}Z_{02}b_{21} + Z_{02}b_{22} + Z_{01}b_{11} - b_{12} & -Z_{01}Z_{02}b_{21} - Z_{02}b_{22} + Z_{01}b_{11} + b_{12}\\ Z_{01}Z_{02}b_{21} - Z_{02}b_{22} + Z_{01}b_{11} - b_{12} & Z_{01}Z_{02}b_{21} + Z_{02}b_{22} + Z_{01}b_{11} + b_{12}\end{bmatrix}$$

Set 3. Wave parameters in terms of circuit parameters.

$$Z = \frac{1}{\Delta_S - s_{11} - s_{22} + 1}\begin{bmatrix} -Z_{01}(\Delta_S - s_{11} + s_{22} - 1) & 2s_{12}\sqrt{Z_{01}Z_{02}} \\ 2s_{21}\sqrt{Z_{01}Z_{02}} & -Z_{02}(\Delta_S + s_{11} - s_{22} - 1) \end{bmatrix}$$

$$= \frac{1}{-t_{11} - t_{12} + t_{21} + t_{22}}\begin{bmatrix} Z_{01}(t_{11} + t_{12} + t_{21} + t_{22}) & 2\Delta_T\sqrt{Z_{01}Z_{02}} \\ 2\sqrt{Z_{01}Z_{02}} & Z_{02}(t_{11} - t_{12} - t_{21} + t_{22}) \end{bmatrix}$$

$$= \frac{1}{-u_{11} - u_{12} + u_{21} + u_{22}}\begin{bmatrix} -Z_{01}(u_{11} - u_{12} - u_{21} + u_{22}) & -2\sqrt{Z_{01}Z_{02}} \\ -2\Delta_U\sqrt{Z_{01}Z_{02}} & -Z_{02}(u_{11} + u_{12} + u_{21} + u_{22}) \end{bmatrix}$$

$$Y = \frac{1}{Z_{01}Z_{02}(\Delta_S + s_{11} + s_{22} + 1)}\begin{bmatrix} Z_{02}(-\Delta_S - s_{11} + s_{22} + 1) & -2s_{12}\sqrt{Z_{01}Z_{02}} \\ -2s_{21}\sqrt{Z_{01}Z_{02}} & Z_{01}(-\Delta_S + s_{11} - s_{22} + 1) \end{bmatrix}$$

$$= \frac{1}{Z_{01}Z_{02}(-t_{11} + t_{12} - t_{21} + t_{22})}\begin{bmatrix} Z_{02}(t_{11} - t_{12} - t_{21} + t_{22}) & -2\Delta_T\sqrt{Z_{01}Z_{02}} \\ -2\sqrt{Z_{01}Z_{02}} & Z_{01}(t_{11} + t_{12} + t_{21} + t_{22}) \end{bmatrix}$$

$$= \frac{1}{Z_{01}Z_{02}(-u_{11} + u_{12} - u_{21} + u_{22})}\begin{bmatrix} -Z_{02}(u_{11} + u_{12} + u_{21} + u_{22}) & 2\sqrt{Z_{01}Z_{02}} \\ 2\Delta_U\sqrt{Z_{01}Z_{02}} & -Z_{01}(u_{11} - u_{12} - u_{21} + u_{22}) \end{bmatrix}$$

$$H = \frac{1}{Z_{02}(\Delta_S + s_{11} - s_{22} - 1)} \begin{bmatrix} -Z_{01}Z_{02}(\Delta_S + s_{11} + s_{22} + 1) & -2s_{12}\sqrt{Z_{01}Z_{02}} \\ 2s_{21}\sqrt{Z_{01}Z_{02}} & -\Delta_S + s_{11} + s_{22} - 1 \end{bmatrix}$$

$$= \frac{1}{Z_{02}(t_{11} - t_{12} - t_{21} + t_{22})} \begin{bmatrix} Z_{01}Z_{02}(-t_{11} + t_{12} - t_{21} + t_{22}) & 2\Delta_T\sqrt{Z_{01}Z_{02}} \\ -2\sqrt{Z_{01}Z_{02}} & -t_{11} - t_{12} + t_{21} + t_{22} \end{bmatrix}$$

$$= \frac{1}{Z_{02}(u_{11} + u_{12} + u_{21} + u_{22})} \begin{bmatrix} Z_{01}Z_{02}(u_{11} - u_{12} + u_{21} - u_{22}) & 2\sqrt{Z_{01}Z_{02}} \\ -2\Delta_U\sqrt{Z_{01}Z_{02}} & u_{11} + u_{12} - u_{21} - u_{22} \end{bmatrix}$$

$$G = \frac{1}{Z_{01}(\Delta_S - s_{11} + s_{22} - 1)} \begin{bmatrix} -\Delta_S + s_{11} + s_{22} - 1 & 2s_{12}\sqrt{Z_{01}Z_{02}} \\ -2s_{21}\sqrt{Z_{01}Z_{02}} & -Z_{01}Z_{02}(\Delta_S + s_{11} + s_{22} + 1) \end{bmatrix}$$

$$= \frac{1}{Z_{01}(t_{11} + t_{12} + t_{21} + t_{22})} \begin{bmatrix} -t_{11} - t_{12} + t_{21} + t_{22} & -2\Delta_T\sqrt{Z_{01}Z_{02}} \\ 2\sqrt{Z_{01}Z_{02}} & Z_{01}Z_{02}(-t_{11} + t_{12} - t_{21} + t_{22}) \end{bmatrix}$$

$$= \frac{1}{Z_{01}(u_{11} - u_{12} - u_{21} + u_{22})} \begin{bmatrix} u_{11} + u_{12} - u_{21} - u_{22} & -2\sqrt{Z_{01}Z_{02}} \\ 2\Delta_U\sqrt{Z_{01}Z_{02}} & Z_{01}Z_{02}(u_{11} - u_{12} + u_{21} - u_{22}) \end{bmatrix}$$

(Set 4 continued overleaf)

$$A = \frac{1}{2s_{21}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} -Z_{01}(\Delta_S - s_{11} + s_{22} - 1) & Z_{01}Z_{02}(\Delta_S + s_{11} + s_{22} + 1) \\ \Delta_S - s_{11} - s_{22} + 1 & Z_{02}(-\Delta_S - s_{11} + s_{22} + 1) \end{bmatrix}$$

$$= \frac{1}{2\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} Z_{01}(t_{11} + t_{12} + t_{21} + t_{22}) & Z_{01}Z_{02}(-t_{11} + t_{12} - t_{21} + t_{22}) \\ -t_{11} - t_{12} + t_{21} + t_{22} & Z_{02}(t_{11} - t_{12} - t_{21} + t_{22}) \end{bmatrix}$$

$$= \frac{1}{2\Delta_U\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} Z_{01}(u_{11} - u_{12} - u_{21} + u_{22}) & Z_{01}Z_{02}(u_{11} - u_{12} + u_{21} - u_{22}) \\ u_{11} + u_{12} - u_{21} - u_{22} & Z_{02}(u_{11} + u_{12} + u_{21} + u_{22}) \end{bmatrix}$$

$$B = \frac{1}{2s_{12}\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} -Z_{02}(\Delta_S + s_{11} - s_{22} - 1) & -Z_{01}Z_{02}(\Delta_S + s_{11} + s_{22} + 1) \\ -\Delta_S + s_{11} + s_{22} - 1 & Z_{01}(-\Delta_S + s_{11} - s_{22} + 1) \end{bmatrix}$$

$$= \frac{1}{2\Delta_T\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} Z_{02}(t_{11} - t_{12} - t_{21} + t_{22}) & Z_{01}Z_{02}(t_{11} - t_{12} + t_{21} - t_{22}) \\ t_{11} + t_{12} - t_{21} - t_{22} & Z_{01}(t_{11} + t_{12} + t_{21} + t_{22}) \end{bmatrix}$$

$$= \frac{1}{2\sqrt{Z_{01}Z_{02}}}\begin{bmatrix} Z_{02}(u_{11} + u_{12} + u_{21} + u_{22}) & Z_{01}Z_{02}(-u_{11} + u_{12} - u_{21} + u_{22}) \\ -u_{11} - u_{12} + u_{21} + u_{22} & Z_{01}(u_{11} - u_{12} - u_{21} + u_{22}) \end{bmatrix}$$

Set 4. Circuit parameters in terms of wave parameters.

GENERALIZED TWO-PORT PERFORMANCE EVALUATION

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ABSTRACT

A generalized approach to linear two-port performance evaluation and determination of two-port performance characteristics is presented in this paper. These are analytically defined and derived in terms of general hybrid parameters, covering z-, y-, h-, and g- two-port parameters. This analysis reveals significant interrelation properties which allow the derivation of any two-port performance characteristic from just two of them.

1. INTRODUCTION

Two-port representation of linear networks, [1,2,3], offers the advantages of generality, conceptual simplicity and physical content, since two-port parameters provide full information about input and output variables' physical relation. In literature, two-port performance, in the form of driving point and transfer or network functions, [2,3,4], is separately evaluated for either z-, y-, h- or g- circuit parameters, [1,2,5]. In addition, performance evaluation relations, in the form of network functions, are often derived by inspection and comparison to some general theory relations, e.g., feedback theory, [1,5]. The resemblance among separately derived expressions, i.e., through different parameter representation, leads to a generalized notation met only in advanced circuit theory, [3,4,6], although a complete set of generalized performance characteristics has not been presented, yet. Therefore, the field of systematic and generalized linear two-port performance evaluation for all possible forward and reverse characteristics such as driving point, forward, and reverse transfer functions is still open.

In this work, we define in a strict manner and derive a complete set of forward and reverse performance characteristics in terms of a generalized set of circuit parameters covering z-, y-, h-, and g- parameters. As it is shown, the morphological symmetry between the performance characteristics of opposite direction as well as the relations among those of the same direction reveal a kind of interrelation. Thus, based on knowledge of any two - not straightforwardly related - performance characteristics, expressed in any kind of two-port parameters, we can derive any performance characteristic, in terms of z-, y-, h-, or g-parameters, i.e.,, from just two performance characteristics, forty of them can be determined.

2. GENERAL HYBRID TWO-PORT PARAMETERS

The resemblance among matrix representations of z-, y-, h-, and g- two-port parameters leads to the idea of using a "general hybrid matrix" **H** representation with "general hybrid parameters", k, [4], although, it has never, to our knowledge, been strictly defined as a model:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (1)$$

The correspondence of this generalized notation to the matrix notation of z-, y-, h-, and g- parameters is given in Table I, while, it should be mentioned that k-parameters notation corresponds to that, alternatively used, for m-, k-, or γ- parameters, [4,6,7].

	z- parameters		y-parameters		h-parameters		g-parameters	
Variables	"u"	"y"	"u"	"y"	"u"	"y"	"u"	"y"
subscript "1"	I_1	V_1	V_1	I_1	I_1	V_1	V_1	I_1
subscript "2"	I_2	V_2	V_2	I_2	V_2	I_2	I_2	V_2

Table I. Notation correspondence among general hybrid to z-, y-, h-, and g- parameters.

3. TWO-PORT PERFORMANCE EVALUATION

Two-port performance evaluation refers to determination of its performance characteristics, i.e., its input / output variables ratios. These are also termed as forward (or reverse) ratios and correspond to a specific (forward or reverse) signal flow direction, i.e., input-to-output or vice-versa.

When the performance characteristics of a two-port are strictly defined and derived based on the k-parameter representation, a designated "symmetry" between forward and reverse ones will be revealed. In this case, the term "symmetry" indicates a morphological resemblance between performance characteristics of opposite signal direction that leads to a kind of interrelation. As a result, knowledge of one of them can lead to the derivation of it's "symmetrical" (opposite direction) by, simply, following specific "symmetry transition rules", i.e., subscript substitutions. Furthermore, an "interrelation" among performance characteristics of the same "direction" can readily be identified. Hence, knowledge of the expressions of two performance characteristics of the same direction can lead to the determination of all the rest of the specific direction. In addition, the interrelation among forward performance characteristics can lead to the interrelation among the reverses, by just following the already mentioned symmetry transition rules.

Conclusively, based on "interrelation" and "symmetry transition rules", knowledge of any two "non-symmetrical" performance characteristics expressions, in terms of general hybrid k-parameters (regardless to direction), is adequate for the derivation of all other performance characteristics expressions, in terms of general hybrid parameters. The physical interpretation of these performance characteristics, defined in the following, depends on the correspondence of k-parameters to z-, y-, h-, or g- parameters. In general, X_i, X_o are input and output immittances (impedances or admitances) respectively, while two of A, B, Γ and Δ performance characteristics indicate voltage and current amplification, and two indicate "transimmittances" (one transimpedance and one transadmitance). The specific physical content of these, for the cases of z-, y-, h-, and g- parameters, is presented in Table II.

Performance Characteristic	**A**	**B**	**Γ**	**Δ**	**X**
Parameters	**Direction:** Forward, Subscript "f"(or "i" for X)				
z	current gain	voltage gain	transimped.	transadmit.	input imped.
y	voltage gain	current gain	transadmit.	transimped.	input admit.
h	transimped.	transadmit.	current gain	voltage gain	input imped.
g	transadmit.	transimped.	voltage gain	current gain	input admit.
Parameters	**Direction:** Reverse, Subscript "r"(or "o" for X)				
z	current gain	voltage gain	transadmit.	transadmit.	output imped.
y	voltage gain	current gain	transimped.	transadmit.	output admit.
h	transadmit.	transimped.	current gain	voltage gain	output admit.
g	transimped.	transadmit.	voltage gain	current gain	output imped.

Table II. Physical interpretation of the generalized performance characteristics

4. DEFINITION AND DETERMINATION OF GENERALIZED PERFORMANCE CHARACTERISTICS

a) Statement. When a linear two-port network is represented through general hybrid k-parameters, the following statement will be valid:

"The forward A_f, B_f, Γ_f, Δ_f, X_f, and reverse A_r, B_r, Γ_r, Δ_r, X_r performance characteristic ratios are defined in terms of the "y" and "u" generalized variables as:

$$A_f = \frac{u_2}{u_1},\ B_f = \frac{y_2}{y_1},\ \Gamma_f = \frac{y_2}{u_1},\ \Delta_f = \frac{u_2}{y_1},\ X_i = \frac{y_1}{u_1}$$

and
$$A_r = \left.\frac{u_1}{u_2}\right|_{\substack{y_s=0\\ k_s\neq 0}},\ B_r = \left.\frac{y_1}{y_2}\right|_{\substack{y_s=0\\ k_s\neq 0}},\ \Gamma_r = \left.\frac{u_1}{y_2}\right|_{\substack{y_s=0\\ k_s\neq 0}},$$

$$\Delta_r = \left.\frac{y_1}{u_2}\right|_{\substack{y_s=0\\ k_s\neq 0}},\ X_o = \left.\frac{y_2}{u_2}\right|_{\substack{y_s=0\\ k_s\neq 0}},$$

where (y_S, k_S) represent a real input source. Then, their expressions in terms of the general hybrid k-parameters will be :

$$A_f = -\frac{k_{21}}{k_{22}+k_L}, \qquad A_r = -\frac{k_{12}}{k_{11}+k_S},$$

$$B_f = \frac{1}{\frac{k_{11}}{k_{21}k_L}\left(k_{22}+k_L\right)-\frac{k_{12}}{k_L}},$$

$$B_r = \frac{1}{\frac{k_{22}}{k_{12}k_S}\left(k_{11}+k_S\right)-\frac{k_{21}}{k_S}}, \Gamma_f = \frac{k_{21}k_L}{k_{22}+k_L},$$

$$\Gamma_r = \frac{1}{k_{21}-\frac{k_{22}}{k_{12}}\left(k_{11}+k_S\right)},$$

$$\Delta_f = \frac{1}{k_{12}-\frac{k_{11}}{k_{21}}\left(k_{22}+k_L\right)},\ \Delta_r = \frac{k_{12}k_S}{k_{11}+k_S},$$

$$X_i = k_{11}-\frac{k_{12}k_{21}}{k_{22}+k_L},\ X_o = k_{22}-\frac{k_{21}k_{12}}{k_{11}+k_S},$$

where, k_S and k_L are source and load, impedances or admittances depending on the case considered, respectively.

b) Verification

For verification of the above statement, forward and reverse performance characteristics are separately expressed in terms of general hybrid k-parameters.

For deriving forward "f" ratios expressions, let us consider a real source connected at the input port and a load at the output. In addition, for establishing a generalized approach that could take into account all cases for the general hybrid k-parameters and reveal the previously mentioned "symmetry", we will consider either a real Thevenin source (V_S, z_S) or a Norton equivalent real source (I_S, y_S), whether the input variable "y_1" is voltage or current. Similarly, we will consider either an impedance load z_L or an admittance load y_L, whether the output variable "y_2" is voltage or current. The corresponding configuration, with the four possible cases of terminated k- equivalent two-ports for derivation of forward performance characteristics, is shown in Fig.1, where $(y_1,u_1)\in\{(I_1,V_1),(V_1,I_1)\}$, $(y_2,u_2)\in\{(I_2,V_2),(V_2,I_2)\}$, and $(k_S,k_L)\in\{(z_S,z_L),(z_S,y_L),(y_S,y_L),(y_S,z_L)\}$

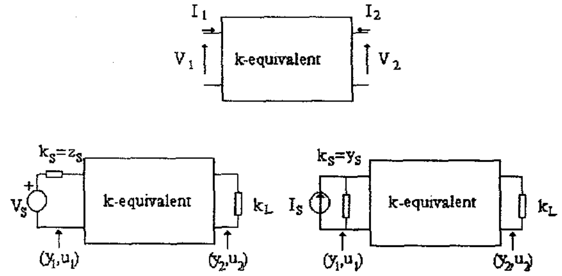

Fig. 1. Open and terminated k-equivalent two-ports for derivation of forward ratios.

For the sources mentioned above, a general notation (y_S, k_S) should be used in deriving "total" performance ratios (that is, forward performance characteristics defined as variables ratios of the two-port output to the voltage / current part of the real Thevenin / Norton source) and y_S should substitute y_1 in the statement definitions. However, to reveal the forward-reverse symmetry, according to statement **ii** of next section 5, we should derive the defined performance characteristics, instead of the "total" ones, thus, abolishing the need of y_S notation. Hence, since k_L satisfies $y_2=-u_2k_L$, eq.(1) leads to:

$$\left\{\begin{array}{c} y_1 = k_{11}u_1 + k_{12}u_2 \\ y_2 = k_{21}u_1 + k_{22}u_2 = -k_L u_2 \end{array}\right\} \quad (2)$$

yielding: $A_f = \frac{u_2}{u_1} = -\frac{k_{21}}{k_{22}+k_L}$ (3)

Using eqs (2) and (3), all forward ratios can easily be derived in terms of general hybrid k-parameters. The complete set of forward performance characteristics, expres-sed in terms of general hybrid k-parameters, has already be given with the statement.

For deriving reverse "r" performance characteristics, the two-port network must be excited from its output port while input sources should be nullified. However, instead that, for short-circuiting real Thevenin sources or open-circuiting real Norton sources, we can eliminate the ideal part of the source only, so that k_S to remain connected. This approach preserves symmetry (as mentioned in statement **ii** of section 5) since k_S can be considered symmetrical to load immittance of the reverse performance case. Note that, following the disconnection of k_L, either an ideal voltage or current source will be considered connected at the output port depending on whether variable y_2 is voltage or current, respectively. This is shown in Fig. 2.

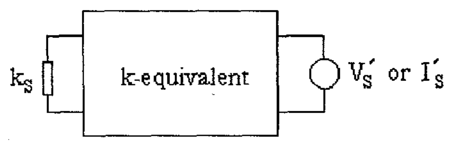

Fig. 2. Configuration for derivation of reverse performance characteristics.

Hence, since k_S satisfies $y_1=-u_1k_S$, eq.(1) leads to:

$$\left\{\begin{array}{c} y_1 = k_{11}u_1 + k_{12}u_2 = -u_1 k_S \\ y_2 = k_{21}u_1 + k_{22}u_2 \end{array}\right\} \quad (4)$$

which, finally, yields A_r in terms of general hybrid k-parameters:

$$A_r = \left.\frac{u_1}{u_2}\right|_{\substack{y_s=0 \\ k_s\neq 0}} = -\frac{k_{12}}{k_{11}+k_S} \quad (5)$$

In direct analogy to the previous derivation of forward performance characteristics, any reverse ratio can be derived, using eqs (4) and (5). The complete set of reverse performance characteristics, expressed in terms of general hybrid k-parameters has already be given with the statement.

Finally, the not examined "total" (forward) performance characteristics can be determined by simply substituting, in the already derived expressions, k_{11} with $k_{11}+k_S$. However, although this is possible, it must be clarified that knowledge of the "total" (forward) ratios cannot lead to the corresponding reverses, because "symmetry" is destroyed due to the fact that, real sources are not used in evaluating the reverses.

5. INTERRELATION PROPERTIES OF GENERALIZED PERFORMANCE CHARACTERISTICS

a) Statements

i. There is a symmetry between the following forward-reverse pairs of performance characteristics : (A_f, A_r), (B_f, B_r), (Γ_f, Δ_r), (Δ_f, Γ_r).

This is a morphological symmetry, which allows "transitions" between the performance characteristics of any of the above pairs, meaning that one performance characteristic can readily be specified, by just knowing its "symmetrical" one. These are performed according to specific and simple "transition rules" such as subscript replacement : L → S, 1 → 2, and 2 → 1, for a transition from a forward ratio to it's symmetrical one, and S → L, 1 → 2, and 2 →1, for reverse transitions.

ii. Forward performance characteristics are interrelated through general hybrid k-parameters. Similarly, reverse characteristics are also interrelated through general hybrid k-parameters. Assuming, arbitrarily and without damage of generality, that (A_f, B_f) and (A_r, B_r) are known, these interrelations can be expressed in the form:

$$\Gamma_f = -k_L A_f, \Delta_f = -\frac{1}{k_L}B_f, \quad X_i = -k_L\frac{A_f}{B_f}$$

and $\Gamma_r = -\frac{1}{k_S}B_r$, $\Delta_r = -k_S A_r$, $X_o = -k_S\frac{A_r}{B_r}$

Although forward and reverse ratios are given as functions of A_f, B_f and A_r, B_r, respectively, in general, any two ratios of the same direction can be considered known so that the remaining three ratios of this direction to be derived as their functions. Finally, the denoted, by statement **i**, symmetry transition rules between forward and reverse ratios lead to the corresponding relations among the extracted reverse ratios.

The above two statements imply that:

a) Several symmetry transition rules are engaging both directions performance characteristics. Knowing the expressions for one direction ratios, the expressions of the other direction ratios are easily derived using the symmetry transition rules.

b) The performance characteristics of each direction are interrelated through general hybrid k-parameters relations.

c) Combining the above results, the knowledge of any two non-symmetrical performance characteristics ratios expressed in terms of general hybrid k-parameters (regardless to direction) is adequate for deriving all other ratios, by simply following the symmetry transition rules.

b) Verification

Statement **i** is easily verified because symmetry between the (A_f,A_r), (B_f,B_r), (Γ_f,Δ_r) and (Δ_f,Γ_r) forward-reverse pairs is obvious just by inspection of the performance characteristics expressions, given in section 4. It is also obvious that transition between the performance characteristics of a pair is performed following the symmetry transition rules denoted above. This means that, a forward ratio transition to it's symmetrical one is performed by the subscripts replacement $L\rightarrow S$, $1\rightarrow 2$, and $2\rightarrow 1$. Reverse transitions are, similarly, performed following the corresponding rules.

Statement **ii** can readily be verified by using the performance characteristics definitions in terms of y and u general hybrid variables and the additional relation, which holds, depending upon the performance direction, i.e., $y_2=-k_L u_2$, for forward calculations and $y_1=-k_S u_1$, for reverse calculations.

Inspecting the resulting relations of statement **ii**, it is apparent that the same symmetry (as defined in statement **i**) between forward and reverse relations is present and the same transition rules (denoted also in statement **i**) are valid. It is also fairly easy to verify that each direction ratios can be expressed as functions of any two known ratios of the same direction and not as functions of only A and B.

Finally, combining the two statements of this section, due to the symmetry and the relations among the generalized performance characteristics of the same direction, it is obvious that knowledge of any two non-symmetrical performance characteristics expressed in terms of general hybrid k-parameters, regardless to direction, is adequate for deriving the rest of the ratios expressions. This means that, knowledge of only two non-symmetrical performance characteristics, in terms of a parameters kind belonging to the k-parameters class, leads to the knowledge of all forty performance characteristics (in terms of the four parameters kinds belonging to the k-parameters class).

6. CONCLUSION

An analytical and generalized approach to two-port performance evaluation is presented in this paper. Generalized notation of a general hybrid k-parameter class, covering z-, y-, h-, and g- parameters is adopted. Despite the analytical definition and determination of a complete two-port performance characteristics set, in terms of k-parameters, important interrelation properties among them are revealed. Using the results of this approach, it is finally proved that knowing just two non-symmetrical performance characteristics, expressed in terms of any of the z-, y-, h-, or g- parameters, all forty performance characteristics can be derived in terms of any of the these parameters.

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Conversion Rules Between X-Parameters and Linearized Two-Port Network Parameters for Large-Signal Operating Conditions

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***Abstract*— This paper presents conversion rules between X-parameters, linearized impedance parameters, linearized admittance parameters, linearized cascade parameters, linearized scattering transfer parameters, linearized hybrid parameters, and linearized inverse hybrid parameters of a two-port network under large-signal operating conditions. The rules have been developed along with a set of equations that allow obtaining the expressions of each linearized parameter from the remaining ones. The proposed approach has been evaluated, and very good agreement has been obtained between calculated parameters and simulated ones.**

***Index Terms*— Conversion rules, linearized admittance, linearized cascade parameters (*ABCD*-parameters), linearized hybrid parameters (*G*-parameters), linearized impedance, linearized inverse hybrid parameters (*H*-parameters), linearized scattering transfer parameters (*T*-parameters), network parameters, nonlinear, two-port, *X*-parameters.**

I. Introduction

X-PARAMETERS were introduced in 2005 [1–[3], and they represent new nonlinear scattering parameters, applicable to passive and active circuits under small- and large-signal excitation [4]. The main limitation is that X-parameters are not directly suitable for the analytic analysis of different network configurations. For example, they are not cascadable in their original form. For this reason, linearize scattering transfer parameters (T-parameters) are defined which are cascadable in their matrix format. This means that multiplying the individual T matrices of cascaded two-port networks leads to the overall resultan T matrix of the cascade system. The description of a nonlinear network havin ports and m harmonics can be described using $n \times m$ equations.

Manuscript received October 24, 2017; revised April 16, 2018 and July 14, 2018; accepted July 17, 2018. Date of publication September 10, 2018; date of current version November 5, 2018. *(Corresponding author: Riadh Essaadali.)*

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Digital Object Identifier 10.1109/TMTT.2018.2863227

In addition to X-parameters and T-parameters, linearized two-port networks can be described by their linearized impedance parameters (Z-parameters), linearized admittance parameters (Y-parameters), linearized cascade parameters (*ABCD*-parameters), linearized hybrid parameters (G-parameters), and linearized inverse hybrid parameters (H-parameters). The resulting linearized two-port parameters define the terminal voltage/current relationships of the two-port under specific operating conditions, such as dc bias, input power level, and temperature. With these linearized parameters, voltages and currents are used to define the independent and dependent variables instead of incident and reflected waves. At high frequencies, these linearized parameters cannot be measured directly and accurately at high frequency. However, they are useful to nonlinear circuit modeling and design. For example, they describe different network topologies such as series, parallel, series-to-parallel, parallel-to-series, and cascade. This was the motivation to introduce a new set of equations in this paper that allow determining the values of these linearized network parameters from X-parameter measurements. Developin conversion rules between these parameters is very important. These will be useful, for example, to exploit X-parameters to more accurately and quickly build a nonlinear equivalent circuit-based model that captures the device behavior at the fundamental and harmonic frequencies. The conversion from X-parameters to linearized impedance, admittance, and *ABCD* parameters is essential for model de-embedding and parameter extraction.

A first attempt to define conversion rules implicating X-parameters was in [5] and [6]. Only two equations were presented that allow determining linearized Z-parameters and linearized Y-parameters from X-parameters. Also, it is assumed that the phase of a_{11} is zero, which makes the model treat a special case or a nontime-invariant system in the general case. The equations presented in [5] are limited to only one port with two harmonics or to only two-port with fundamental. In [6], in order to analyze a mixed-series connection topology of nonlinear and linear component, i.e., a transistor operating in large-signal mode with a feedback capacitor, the expression between scattered and incident harmonic traveling voltage waves through X-parameters is transformed to a relation between harmonic voltage and current. Nonlinear Z-parameters are not explicitly presented.

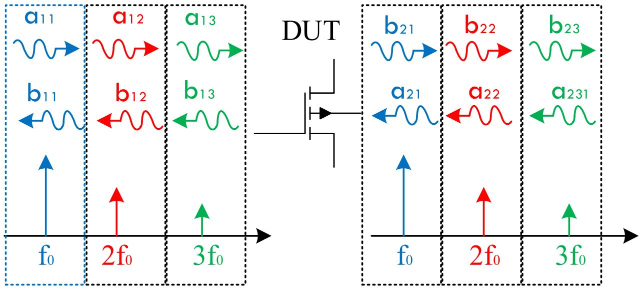

Fig. 1. Incident and scattered waves of a nonlinear two-port device.

In this paper, all conversion rules between the different linearized two-port network parameters such as X-, Z-, Y-, $ABCD$-, T-, G-, and H-parameters are presented and, then, validated in both linear and nonlinear operating modes. Some of these rules were used to develop an accurate large-signal equivalent circuit based model that is extracted from X-parameter measurements [7]. Another application example of linearized network parameters and their conversion rules is the development of a link between X-parameters and charge-controlled quasi-static model [8].

We believe that these conversion rules will be useful for other applications in the area of nonlinear circuit modeling, design, analysis, and measurements.This paper is organized as follows. Section II outlines the concept of X-parameter and defines the expressions of linearized two-port network parameters. Section III presents all conversion rules between the new linearized two-port network parameters. In Section IV, the conversion rules are validated in linear and nonlinear operation modes.

II. Linearized Two-Port Network Parameters

X-parameters are the result of the linearization of the multivariate complex functions that correlate all of the relevant input spectral components with the output spectral components for a two-port device [1], [2]

$$b_{ik} = X^F_{ik}(|a_{11}|)P^k + \sum_{(j,l)\neq(1,1)} X^S_{ik,jl}(|a_{11}|)P^{k-l}a_{jl} + \sum_{(j,l)\neq(1,1)} X^T_{ik,jl}(|a_{11}|)P^{k+l}a^*_{jl}. \quad (1)$$

where $b_{\rm pm}$ and a_{qj} ($a*_{qj}$ *is* the conjugate) are, respectively, the scattered and incident traveling voltage waves. Indices p and q range from one to the number of signal ports. Indices m and j range from one to the highest harmonic index. Term $P = \exp(j\varphi(a_{11}))$ is the unity length phasor having the same phase as a_{11}. X^F_{ik}, $X^S_{ik,jl}$, and $X^T_{ik,jl}$ are the X-parameters and depend on the amplitude of the incident wave. The graphical representation in Fig. 1 illustrates the basic concept of X-parameters. Multiple frequencies are present in the input and output spectrum. In each frequency, there are incident and scattered waves. And X-parameters define a linearized spectral map relating incident to scattered traveling voltage waves. In X-parameters, term $X^F_{ik}(|a_{11}|)$ is complex. Thus, it can be rewritten as $X^F_{ik}(|a_{11}|)P^k = X^S_{ik11}(|a_{11}|)P^{k-1}a_{11} = S_{ik,11}(|a_{11}|)a_{11}$, since the input drive a_{11} is always different than 0. It is mathematically correct, and this relation is well verified while measuring with the NVNA and simulating with ADS a nonlinear component. Parameters $S_{pmqj}(|a_{11}|)$ and $T_{pmqj}(|a_{11}|)$ can be set equal to $X^S_{pmqj}(|a_{11}|)P^{m-j}$ and $X^T_{pmqj}(|a_{11}|)P^{m+j}$, respectively. Terms T_{pm11} are equals to 0. For a two-port device and only n harmonics are considered, (1) can be rewritten as

$$b_{\rm pm} = \sum_{\substack{j=1..n\\ q=1,2}} S_{pmqj}\left(|a_{11}|\right)P^{m-j}a_{qj} + \sum_{\substack{j=1..n\\ q=1,2\\ (q,j)\neq(1,1)}} T_{pmqj}\left(|a_{11}|\right)P^{m+j}a^*_{qj}. \quad (2)$$

In addition to X-parameters, a nonlinear component can be described by linearized Z-, Y-, T-, $ABCD$-, H- and G-parameters. Y-parameters relate harmonic voltage components to harmonic current components at both ports.

Linearized Y-parameters include two-term categories, Y^α_{pmqj} and Y^β_{pmqj}, which are associated with harmonic component voltage υ_{qj} and its conjugate υ^*_{qj}, respectively. Linearized T-parameters include two-term categories T^α_{pmqj} and T^β_{pmqj} associated, respectively, to a harmonic component and its conjugate of a waves and b waves present at output port. Like in linear case, linearized $ABCD$ matrix includes four parameter categories: A, B, C, and D terms. A terms relate input harmonic voltages to output harmonic voltages. B terms relate input harmonic voltages to output harmonic currents. C terms relate input harmonic currents to output harmonic voltages. D terms relate input harmonic currents to output harmonic currents. Each $ABCD$-parameters include two-term categories associated, respectively, with a harmonic component and its conjugate of voltage and current present at second port. G-parameters use the input harmonic current components at port 1 and the output harmonic voltage components at port 2 as independent variables. Linearized H-parameters include two-term categories H^α_{pmqj} and H^β_{pmqj} associated with a harmonic component and its conjugate of voltage and current. Linearized G-parameters use the input harmonic voltage components and the output harmonic current components as independent variables.

The formulation of the nonlinear model based on linearized Z-, Y-, T-, $ABCD$-, G- and H-parameters are presented in the following equations. Analogous to X-parameters, linearized network parameters can be derived by using Taylor series. They are the result of the series expansion based on the partial derivatives. The resulting dependent parameters is composed of signal operating point (LSOP) term, plus contributions due to the small-signal stimulus on each individual frequency component [1], [2]. It is assumed that there is only one large-signal plus small remaining spectral components. For example, in linearized Y-parameters, $Y^\alpha_{pm,11}$ refers to the large-signal part of $i_{\rm pm}$ which is a nonlinear function of $|a_{11}|$, to be consistent with X-parameters measurements. $Y^\alpha_{{\rm pm},jl}$ and $Y^\beta_{{\rm pm},jl}$ with $(j,l) \neq (1,1)$ are the coefficients associated with the contribution of linear components to $i_{\rm pm}$ and of the conjugate of the linear components υ_{jl}. The conjugate term models the self-mixing behavior of nonlinear systems: when incident signals at different frequencies are injected simultaneously to a nonlinear system, they will generate signals that are the sum and difference of their frequencies. The complex conjugate term allows modeling the difference.

In this paper, it is assumed that the independent variables are considered as the sum of one large RF signal with small spectral components. X-parameters model include only one large input tone in the LSOP. Therefore, the characterized nonlinear device should be input matched for the fundamental frequency. For a DUT, the reflections at the fundamental and the harmonics at the output and the reflections at the harmonic at its input are considered as small perturbations. The validity of this assumption depends on whether the contribution of these perturbations is small enough such that the considered variable can be assumed linear. If the DUT is mismatched, load-dependent X-parameters are used. Load-dependent X-parameters are just a set of X-parameters measured at different load conditions [9]. Each term in X-parameters and in linearized network parameters depends nonlinearly on the dc bias power, fundamental frequency, input RF power, and source and load fundamental and harmonic terminations. In summary, since the proposed linearized network parameters are extracted using X-parameters, they automatically inherent the limitation of X-parameters and bear the same LSOP as that of X-parameters. For simplification purposes, dependence on only a_{11} is considered in the following equations:

$$v_{\text{pm}} = \sum_{\substack{j=1..n\\q=1,2}} Z^{\alpha}_{pmqj}(|a_{11}|)P^{m-j}i_{qj} + \sum_{\substack{j=1..n\\q=1,2\\(q,j)\neq(1,1)}} Z^{\beta}_{pmqj}(|a_{11}|)P^{m+j}i^{*}_{qj} \tag{3}$$

$$i_{\text{pm}} = \sum_{\substack{j=1..n\\q=1,2}} Y^{\alpha}_{pmqj}(|a_{11}|)P^{m-j}v_{qj} + \sum_{\substack{j=1..n\\q=1,2\\(q,j)\neq(1,1)}} Y^{\beta}_{pmqj}(|a_{11}|)P^{m+j}v^{*}_{qj} \tag{4}$$

$$b_{1m} = \sum_{j=1..n} T^{\alpha}_{1m1j}(|a_{11}|)P^{m-j}a_{2j} + T^{\beta}_{1m1j}(|a_{11}|)P^{m+j}a^{*}_{2j} + \sum_{j=1..n} T^{\alpha}_{1m2j}(|a_{11}|)P^{m-j}b_{2j} + T^{\beta}_{1m2j}(|a_{11}|)P^{m+j}b^{*}_{2j} \tag{5}$$

$$a_{1m} = \sum_{j=1..n} T^{\alpha}_{2m1j}(|a_{11}|)P^{m-j}b_{2j} + T^{\beta}_{2m1j}(|a_{11}|)P^{m+j}b^{*}_{2j} + \sum_{j=1..n} T^{\alpha}_{2m2j}(|a_{11}|)P^{m-j}a_{2j} + T^{\beta}_{2m2j}(|a_{11}|)P^{m+j}a^{*}_{2j} \tag{6}$$

$$v_{1m} = \sum_{j=1..n} A^{\alpha}_{1m2j}(|a_{11}|)P^{m-j}v_{2j} + A^{\beta}_{1m2j}(|a_{11}|)P^{m+j}v^{*}_{2j} + \sum_{j=1..n} B^{\alpha}_{1m2j}(|a_{11}|)P^{m-j}i_{2j} + B^{\beta}_{1m2j}(|a_{11}|)P^{m+j}i^{*}_{2j} \tag{7}$$

$$i_{1m} = \sum_{j=1..n} C^{\alpha}_{1m2j}(|a_{11}|)P^{m-j}v_{2j} + C^{\beta}_{1m2j}(|a_{11}|)P^{m+j}v^{*}_{2j} + \sum_{j=1..n} D^{\alpha}_{1m2j}(|a_{11}|)P^{m-j}i_{2j} + D^{\beta}_{1m2j}(|a_{11}|)P^{m+j}i^{*}_{2j} \tag{8}$$

$$v_{1m} = \sum_{j=1..n} H^{\alpha}_{1m1j}(|a_{11}|)P^{m-j}i_{1j} + \sum_{j=2..n} H^{\beta}_{1m1j}(|a_{11}|)P^{m+j}i^{*}_{1j} + \sum_{j=1..n} H^{\alpha}_{1m2j}(|a_{11}|)P^{m-j}v_{2j} + H^{\beta}_{1m2j}(|a_{11}|)P^{m+j}v^{*}_{2j} \tag{9}$$

$$i_{2m} = \sum_{j=1..n} H^{\alpha}_{2m1j}(|a_{11}|)P^{m-j}i_{1j} + \sum_{j=2..n} H^{\beta}_{2m1j}(|a_{11}|)P^{m+j}i^{*}_{1j} + \sum_{j=1..n} H^{\alpha}_{2m2j}(|a_{11}|)P^{m-j}v_{2j} + H^{\beta}_{2m2j}(|a_{11}|)P^{m+j}v^{*}_{2j} \tag{10}$$

$$i_{1m} = \sum_{j=1..n} G^{\alpha}_{1m1j}(|a_{11}|)P^{m-j}v_{1j} + \sum_{j=2..n} G^{\beta}_{1m1j}(|a_{11}|)P^{m+j}v^{*}_{1j} + \sum_{j=1..n} G^{\alpha}_{1m2j}(|a_{11}|)P^{m-j}i_{2j} + G^{\beta}_{1m2j}(|a_{11}|)P^{m+j}i^{*}_{2j} \tag{11}$$

$$v_{2m} = \sum_{j=1..n} G^{\alpha}_{2m1j}(|a_{11}|)P^{m-j}v_{1j} + \sum_{j=2..n} G^{\beta}_{2m1j}(|a_{11}|)P^{m+j}v^{*}_{1j} + \sum_{j=1..n} G^{\alpha}_{2m2j}(|a_{11}|)P^{m-j}i_{2j} + G^{\beta}_{2m2j}(|a_{11}|)P^{m+j}i^{*}_{2j}. \tag{12}$$

The P terms are added to take into account the time-invariance characteristic. A time-invariant system exhibits that if the excitation is time shifted by τ, the response should be time shifted with the same value τ. As shown in the following equation, if a delay τ is applied at the fundamental frequency current, the voltage or current at $n\omega_1$ will be shifted by $n\tau$

$$I_{\text{in}}e^{jn\omega_1(t-\tau)} = I_{\text{in}}e^{jn\omega_1 t}e^{-jn\omega_1\tau}. \tag{13}$$

For example, in linearized Z-parameter case, applying a delay $P = e^{-j\omega\tau}$ leads to

$$v_{\text{pm}}P^{-m} = \sum_{\substack{j=1..n\\q=1,2}} Z^{\alpha}_{pmqj}\,(|a_{11}|)\,P^{-j}i_{qj} + \sum_{\substack{j=1..n\\q=1,2\\(q,j)\neq(1,1)}} Z^{\beta}_{pmqj}\,(|a_{11}|)\,P^{j}i^{*}_{qj} \tag{14}$$

which is equivalent to (3). In measurement, P is set as the phasor of a_{11}.

Linearized two-port network parameters are not only derived from X-parameters formalism, but also they can be derived by using the first-order approximation of the Taylor series of the input–output nonlinear functional relationship. For example, the voltage–current functional relationship that can characterize the nonlinear steady-state behavior of a DUT can be formulated as

$$i_{\text{pm}} = K_{\text{pm}}\big(|a_{11}|, v_{12}P^{-2}, v^{*}_{12}P^{+2}, \ldots, v_{1n}P^{-n}, v^{*}_{1n}P^{+n}\; v_{21}P^{-1}, v^{*}_{21}P^{+1}, \ldots, v_{2n}P^{-n}, v^{*}_{2n}P^{+n}\big)P^{+m}. \tag{15}$$

The function K_{pm} is linearized around $|a_{11}|$ to be consistent with X-parameter measurement. This yields to the series expansion in

$$i_{pm} = K^F_{pm}(|a_{11}|)\,P^{+m} + \sum_{\substack{q=1\\ j=1\\ (q,j)\\ \neq(1,1)}}^{\substack{q=2\\ j=n}} \frac{\partial K_{pm}}{\partial(v_{qj}P^{-j})}(|a_{11}|)P^{m-j}v_{qj} + \sum_{\substack{q=1\\ j=1\\ (q,j)\\ \neq(1,1)}}^{\substack{q=2\\ j=n}} \frac{\partial K_{pm}}{\partial(v^*_{qj}P^{j})}(|a_{11}|)P^{m+j}v^*_{qj}. \quad (16)$$

Substituting $K_{pm}(|a_{11}|)P^{+m}$ by $Y^{\alpha}_{pm11}(|a_{11}|)P^{m-1}v_{11}$, $\partial K_{pm}/\partial(i_{qj}P^{-j})(|a_{11}|)P^{m-j}v_{qj}$ by $Y^{\alpha}_{pmqj}(|a_{11}|)P^{m-j}v_{qj}$ and for (*q*,*j*) different to (1,1) $\partial K_{pm}/\partial(i^*_{qj}P^{j})(|a_{11}|)P^{m+j}v^*_{qj}$ by $Y^{\beta}_{pmqj}(|a_{11}|)P^{m+j}v^*_{qj}$, (20) is equivalent to (4). The independent and dependent variables related by the linearized parameters can be defined in the following vector form: $[K] = [[K_1],[K_2]]^T$ where the subvectors are defined as: $[K_{i=1,2}] = [\,k_{i1}\;\; k^*_{i1}\;\; \ldots\;\; k_{in}\; k^*_{in}\,]^T$. The relation between B-wave and A-wave vectors through X- and linearized T-matrices are in (17) and (18), respectively. And, the relation between voltage and current vectors through linearized Z-, Y-, $ABCD$-, G-, and H-matrices are in (19), (20), (21), (22), and (23), respectively. The dependent and independent vectors have the same length, which allows getting a square matrix. The only issue that can happen is the inversion of the matrices and submatrices to calculate the linearized network parameters. Thanks to its type, the matrices of linearized parameters will not lose their full rank. And its inversion will be accurate and quickly calculated. For the rest of this paper, in X-parameters and linearized network matrices, the dependence on the LSOP is omitted to reduce the size of equations

$$[B] = [X][A] \quad (17)$$

$$\begin{bmatrix}[B_1]\\ [A_1]\end{bmatrix} = [T_{nonlin}]\begin{bmatrix}[A_2]\\ [B_2]\end{bmatrix} \quad (18)$$

$$[V] = [Z_{nonlin}]\,[I] \quad (19)$$

$$[I] = [Y_{nonlin}]\,[V] \quad (20)$$

$$\begin{bmatrix}[V_1]\\ [I_1]\end{bmatrix} = [ABCD_{nonlin}]$$

$$\begin{bmatrix}[V_2]\\ [I_2]\end{bmatrix} = \begin{bmatrix}[A] & [B]\\ [C] & [D]\end{bmatrix}\begin{bmatrix}[V_2]\\ [I_2]\end{bmatrix} \quad (21)$$

$$\begin{bmatrix}[I_1]\\ [V_2]\end{bmatrix} = [G_{nonlin}]\begin{bmatrix}[V_1]\\ [I_2]\end{bmatrix} \quad (22)$$

$$\begin{bmatrix}[V_1]\\ [I_2]\end{bmatrix} = [H_{nonlin}]\begin{bmatrix}[I_1]\\ [V_2]\end{bmatrix}. \quad (23)$$

where the X matrix can be represented by

$$[X] = \begin{bmatrix}[X_{11}] & [X_{12}]\\ [X_{21}] & [X_{22}]\end{bmatrix} \quad (24)$$

and (25), shown at the bottom of the next page.

The linearized Z-, Y-, $ABCD$-, G-, or H-parameters can be represented by the general matrix $[R_{nonlin}]$. If N harmonics are considered and for a two-port circuit, matrix $[R]$ has the size of $4N \times 4N$

$$[R_{nonlin}] = \begin{bmatrix}[R_{11}] & [R_{12}]\\ [R_{21}] & [R_{22}]\end{bmatrix} \quad (26)$$

and (27), shown at the bottom of the next page.

Actually, terms S_{pmqj} and T_{pmqj} do not depend only on $|a_{11}|$, but they are complex functions of the frequency, the magnitude of the excitation at the fundamental $|a_{11}|$, and terminations at source Γ_S and load Γ_L. The load-dependent X-parameters were introduced to circumvent the limited accuracy of the poly-harmonic distortion model. And it consists of a set of X-parameters measured at different source and load terminations. At this time, there is no measurement capability to gather directly the linearized network parameters values. The NVNA can be used together with a source and load tuner to measure load-dependent X-parameters across the Smith Chart. The linearized network parameters are extracted from X-parameter measurement under the same conditions: same input power, same dc voltages, same fundamental frequency, and same source and load terminations. Using the conversion rules listed below, linearized network parameters data can be extracted and will be ready for use. To reduce data size, source and load grid can be optimized. The grid can be defined by the error between measured- and predicted-dependent variable waveforms and by the used interpolation and extrapolation algorithms.

III. Conversion Between Linearized Two-Port Network Parameters

In this section, a transparent matricial transformation between linearized parameters will be presented. In this paper, it is assumed that the noise level is very low in a way that can be neglected and does not affect the conversion rules. Generally, low-noise setup and configuration is used to measure X-parameters.

A. Conversion From X- to Linearized Z-, Y-, ABCD-, G-, H-, and T-Parameters

The relation between A waves and B waves through X-parameters are represented by (17). The A-wave and B-wave vectors are defined as: $[B] = [[B_1],[B_2]]^T$ and $[A] = [[A_1],[A_2]]^T$ and $[B_i] = [\,b_{i1}\;\; b^*_{i1}\;\; \ldots\;\; b_{in}\; b^*_{in}\,]^T$ and $[A_i] = [\,a_{i1}\;\; a^*_{i1}\;\; \ldots\;\; a_{in}\; a^*_{in}\,]^T$.

The basic quantities used for X-parameters are traveling voltage waves [10]. The waves are defined as linear combinations of the signal port voltage v and the signal port current i [9]. The incident waves are called the a waves and the scattered waves are called the b waves. They are defined in (28) and (29), as in [11]. Each spectral component has an associated harmonic index, which denotes the ratio between the associated frequency and the fundamental tone. The harmonic index is indicated by the last subscript k. The first subscript i indicates the respective DUT signal port. Port 1 typically corresponds to the input and port 2 to the output of the DUT. The incident waves a_{ik} and scattered waves b_{ik} are defined as functions of the spectral components of voltage υ_{ik}, current i_{ik}, and the reference impedance Z_c that is assumed to be a real constant in this paper

$$a_{ik} = \frac{\upsilon_{ik} + Z_c i_{ik}}{2} \tag{28}$$

$$b_{ik} = \frac{\upsilon_{ik} - Z_c i_{ik}}{2}. \tag{29}$$

Harmonic voltages and currents can be expressed, respectively, in the following equations as a function of spectral components of incident and scattered waves

$$\upsilon_{ik} = a_{ik} + b_{ik} \tag{30}$$

$$i_{ik} = \frac{a_{ik} - b_{ik}}{Z_c}. \tag{31}$$

Substituting a_{jl} and b_{ik} by their expression in (28) and (29), linearized Z-parameters, Y-parameters, G-parameters, H-parameters, and $ABCD$-parameters can be derived in terms of X-parameters. Equation (17) is equivalent to

$$[V] - Z_c[I] = [X]\{[V] + Z_c[I]\} \tag{32}$$

where $[V] = [[V_1], [V_2]]^T$ and $[I] = [[I_1], [I_2]]^T$ are, respectively, the voltage and current vectors. The subvector $[V_i] = [\,\upsilon_{i1} \;\; \upsilon_{i1}^* \;\; \cdots \;\; \upsilon_{\text{in}} \; \upsilon_{\text{in}}^*\,]^T$ is expressed in terms of n harmonic voltage components. The current subvector is $[I_i] = [\,i_{i1} \;\; i_{i1}^* \;\; \cdots \;\; i_{\text{in}} \; i_{\text{in}}^*\,]^T$. Equation (32) is equivalent to

$$[V] = Z_c[[I_d] - [X]]^{-1}[[I_d] + [X]][I] \tag{33}$$

where $[I_d]$ is the identity matrix. Therefore, the linearized impedance matrix can be defined to describe the relationship between harmonic voltages and harmonic current components

$$[Z_{\text{nonlin}}] = Z_c \left[[Id] - [X]\right]^{-1} \left[[Id] + [X]\right]. \tag{34}$$

From (33), the expression of current vector in terms of voltage vector and X-parameters can be derived and its expression is

$$[I] = \frac{1}{Z_c} \left[[I_d] + [X]\right]^{-1} \left[[I_d] - [X]\right] [V]. \tag{35}$$

Thus, the linearized admittance matrix can be defined in (36) to describe the relationship between harmonic currents and harmonic voltages in

$$[Y_{\text{nonlin}}] = \frac{1}{Z_c} \{[Id] + [X]\}^{-1} \{[Id] - [X]\}. \tag{36}$$

To determine linearized $[A]$ and $[C]$ submatrices in terms of X-parameters, vector $[I_2]$ should be assumed to be a null vector [0]. In this case, (17) is equivalent to

$$\begin{bmatrix} [V_1] - Z_c[I_1] \\ [V_2] \end{bmatrix} = \begin{bmatrix} [X_{11}] & [X_{12}] \\ [X_{21}] & [X_{22}] \end{bmatrix} \begin{bmatrix} [V_1] + Z_c[I_1] \\ [V_2] \end{bmatrix}. \tag{37}$$

The manipulation of (37) leads to the expression of $[A]$ and $[C]$ submatrices in

$$\begin{aligned} [A] = {} & [I_d + [I_d - X_{11}]^{-1}[I_d + X_{11}]]^{-1} \\ & \cdot \{[I_d - X_{11}]^{-1}[I_d + X_{11}][X_{21}]^{-1}[I_d - X_{22}] \\ & + [I_d - X_{11}]^{-1}[X_{12}]\} \end{aligned} \tag{38}$$

$$\begin{aligned} [C] = {} & \frac{1}{Z_c}[X_{22}]^{-1}[I_d - X_{22}] \\ & - \frac{1}{Z_c}[I_d + [I_d - X_{11}]^{-1}[I_d + X_{11}]]^{-1} \\ & \{[I_d - X_{11}]^{-1}[I_d + X_{11}][X_{21}]^{-1}[I_d - X_{22}] \\ & + [I_d - X_{11}]^{-1}[X_{12}]\}. \end{aligned} \tag{39}$$

Similarly, to determine linearized $[B]$ and $[D]$ submatrices in terms of X-parameters, $[V_2]$ is assumed to be equal to [0]. In this case, (17) is equivalent to

$$\begin{bmatrix} [V_1] - Z_c[I_1] \\ -Z_c[I_2] \end{bmatrix} = \begin{bmatrix} [X_{11}] & [X_{12}] \\ [X_{21}] & [X_{22}] \end{bmatrix} \begin{bmatrix} [V_1] + Z_c[I_1] \\ Z_c[I_2] \end{bmatrix}. \tag{40}$$

The manipulation of (40) leads to the expression of $[B]$ and $[D]$ in

$$[B] = \frac{Z_c}{2}([X_{12}] - [I_d + X_{11}][X_{21}]^{-1}[I_d + X_{22}]) \tag{41}$$

$$[D] = \frac{1}{2}([[X_{11}] - [I_d]][X_{21}]^{-1}[[I_d] + [X_{22}]] - [X_{12}]). \tag{42}$$

To derive the expression of linearized G-parameters in terms of linearized X-parameters, the expression of voltage waves

$$\left[X_{ij}\right]_{\substack{i=1,2 \\ j=1,2}} = \begin{bmatrix} S_{i1j1}P^{1-1} & T_{i1j1}P^{1+1} & \dots & S_{i1jn}P^{1-n} & T_{i1jn}P^{1+n} \\ T^*_{i1j1}P^{-(1+1)} & S^*_{i1j1}P^{-(1-1)} & \dots & T^*_{i1jn}P^{-(1+n)} & S^*_{i1jn}P^{-(1-n)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ S_{injl}P^{n-1} & T_{inj1}P^{n+1} & \dots & S_{injn}P^{n-n} & T_{injn}P^{n+n} \\ T^*_{inj1}P^{-(n+1)} & S^*_{inj1}P^{-(n-1)} & \dots & T^*_{injn}P^{-(n+n)} & S^*_{injn}P^{-(n-n)} \end{bmatrix} \tag{25}$$

$$\left[R_{ij}\right]_{\substack{i=1,2 \\ j=1,2}} = \begin{bmatrix} R^{\alpha}_{i1j1}P^{1-1} & R^{\beta}_{i1j1}P^{1+1} & \dots & R^{\alpha}_{i1jn}P^{1-n} & R^{\beta}_{i1jn}P^{1+n} \\ R^{\beta*}_{i1j1}P^{-(1+1)} & R^{\alpha*}_{i1j1}P^{-(1-1)} & \dots & R^{\beta*}_{i1jn}P^{-(1+n)} & R^{\alpha*}_{i1jn}P^{-(1-n)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ R^{\alpha}_{inj1}P^{n-1} & R^{\beta}_{inj1}P^{n+1} & \dots & R^{\alpha}_{injn}P^{n-n} & R^{\beta}_{injn}P^{n+n} \\ R^{\beta*}_{inj1}P^{-(n+1)} & R^{\alpha*}_{inj1}P^{-(n-1)} & \dots & R^{\beta*}_{injn}P^{-(n+n)} & R^{\alpha*}_{injn}P^{-(n-n)} \end{bmatrix} \tag{27}$$

in terms of harmonic voltages (28), (29) is used. Then, the expressions of $[G_{11}]$, $[G_{12}]$, $[G_{21}]$ and $[G_{22}]$ are, respectively, in the following equations:

$$[G_{11}] = \frac{-1}{Z_c}[[I_d]+[X_{11}]+[X_{12}][[I_d]-[X_{22}]]^{-1}[X_{21}]]^{-1} \cdot [[X_{11}]-[I_d]+[X_{12}][[I_d]-[X_{22}]]^{-1}[X_{21}]] \quad (43)$$

$$[G_{21}] = 2[[X_{12}]+[[X_{11}]+[I_d]][X_{21}]^{-1}[[I_d]-[X_{22}]]]^{-1} \quad (44)$$

$$[G_{12}] = 2[[[X_{22}]-[I_d]][X_{12}]^{-1}[[X_{11}]+[I_d]]-[X_{21}]]^{-1} \quad (45)$$

$$[G_{22}] = Z_c[[X_{21}]-[[X_{22}]+[I_d]][X_{12}]^{-1}[[X_{11}]+[I_d]]] \cdot [[[X_{22}]-[I_d]][X_{12}]^{-1}[[X_{11}]+[I_d]]-[X_{21}]]^{-1}. \quad (46)$$

In (37), assuming $[V_2] = 0$ allows calculating $[H_{11}]$ and $[H_{21}]$ and assuming $[I_1] = 0$ allows calculating $[H_{12}]$ and $[H_{22}]$. Then, the expressions of $[H_{11}]$, $[H_{21}]$, $[H_{12}]$, and $[H_{22}]$ are, respectively, in the following equations:

$$[H_{11}] = Z_c[[I_d]-[X_{11}]+[X_{12}][I_d+X_{22}]^{-1}[X_{21}]]^{-1} \cdot [[I_d]+[X_{11}]-[X_{12}][I_d+X_{22}]^{-1}[X_{21}]] \quad (47)$$

$$[H_{21}] = -[I_d+X_{22}]^{-1}[X_{21}] \cdot \{[I_d]+[[I_d]-[X_{11}]+[X_{12}][I_d+X_{22}]^{-1}[X_{21}]]^{-1}. \cdot [[I_d]+[X_{11}]-[X_{12}][I_d+X_{22}]^{-1}[X_{21}]]\} \quad (48)$$

$$[H_{12}] = [[I_d]-[X_{11}]+[X_{12}][I_d+X_{22}]^{-1}[X_{21}]]^{-1} \cdot [[X_{12}]-[X_{12}][I_d+X_{22}]^{-1}[X_{22}-I_d]] \quad (49)$$

$$[H_{22}] = \frac{1}{Z_c}[[I_d]+[X_{22}]+[X_{21}][I_d-X_{11}]^{-1}[X_{12}]]^{-1} \cdot [[I_d]-[X_{22}]-[X_{21}][I_d-X_{11}]^{-1}[X_{12}]]. \quad (50)$$

The expressions of $[T_{11}]$ and $[T_{21}]$ in terms of X-parameters are determined in (51) and (52) by assuming that $[B_2] = [0]$. The expressions of $[T_{12}]$ and $[T_{22}]$ in terms of X-parameters are determined in (53) and (54) by assuming that $[A_2] = [0]$

$$[T_{11}] = [X_{12}]-[X_{11}][X_{21}]^{-1}[X_{22}] \quad (51)$$

$$[T_{21}] = -[X_{21}]^{-1}[X_{22}] \quad (52)$$

$$[T_{12}] = [X_{11}][X_{21}]^{-1} \quad (53)$$

$$[T_{22}] = [X_{21}]^{-1}. \quad (54)$$

Conversion rules are independent of the definition of the a and b waves. Whether traveling voltage waves or power waves or pseudowaves are used, the expressions of the nonlinear parameters are the same. By substituting A- and B-power waves by their expression in terms of voltages and current in (55) and (56) [12], and by simplifying by $1/2\sqrt{Z_c}$, X-parameter equation is equivalent to (57)

$$a_{jl} = \frac{v_{jl}+Z_c i_{jl}}{2\sqrt{Z_c}} \quad (55)$$

$$b_{jl} = \frac{v_{jl}-Z_c i_{jl}}{2\sqrt{Z_c}} \quad (56)$$

$$[V]-Z_c[I] = [X]\{[V]+Z_c[I]\}. \quad (57)$$

Equation (57) is equivalent to

$$[I] = \frac{1}{Z_c}[[I_d]+[X]]^{-1}[[I_d]-[X]][V]. \quad (58)$$

Therefore, the nonlinear admittance matrix can be defined to describe the relationship between harmonic currents and harmonic voltage components

$$[Y_{\text{nonlin}}] = \frac{1}{Z_c}[[I_d]+[X]]^{-1}[[I_d]-[X]][V]. \quad (59)$$

If pseudowave definition is used [12], term $(Re(Z_{\text{ref}}))^{1/2}/(2|Z_{\text{ref}}|)$ is simplified, and the same expression of linearized two-port network parameters is obtained.

B. Conversion From Linearized Z- to X-, linearized T-, H-, G-, ABCD-, and Y-Parameters

Linearized Z-parameters use voltages and currents, whereas X- and linearized T-parameters use traveling waves. To derive the expression of X- and linearized T-parameters in terms of linearized Z-parameters, (30) and (31) are used. Equation (19) is equivalent to

$$[A]+[B] = \frac{1}{Z_c}[Z_{\text{nonlin}}]\{[A]-[B]\}. \quad (60)$$

The expression of the B vector in terms of the A vector will lead to the expression of X-parameters in terms of linearized Z-parameters

$$[X] = \left[[I_d]+\frac{1}{Z_c}[Z_{\text{nonlin}}]\right]^{-1}\left[\frac{1}{Z_c}[Z_{\text{nonlin}}]-[I_d]\right]. \quad (61)$$

The expressions of linearized T-submatrices $[T_{11}]$, $[T_{21}]$, $[T_{12}]$, and $[T_{22}]$ in terms of linearized Z-parameters are, respectively, derived in (72)–(75). $[T_{11}]$ and $[T_{21}]$ are obtained by assuming $[B_2] = [0]$. $[T_{12}]$ and $[T_{22}]$ are obtained by assuming $[A_2] = [0]$

$$[T_{11}] = \frac{[Z_{12}]}{2Z_c}+\frac{1}{2}\left(\frac{[Z_{11}]}{Z_c}-[I_d]\right)(Z_c[Z_{21}]^{-1}-[Z_{21}]^{-1}[Z_{22}]) \quad (62)$$

$$[T_{21}] = \frac{[Z_{12}]}{2Z_c}+\frac{1}{2}\left(\frac{[Z_{11}]}{Z_c}+[I_d]\right)(Z_c[Z_{21}]^{-1}-[Z_{21}]^{-1}[Z_{22}]) \quad (63)$$

$$[T_{12}] = \frac{1}{2}([Z_{11}]-Z_c[I_d])[Z_{21}]^{-1}\left(\frac{[Z_{22}]}{Z_c}+[I_d]\right)-\frac{[Z_{12}]}{2Z_c} \quad (64)$$

$$[T_{12}] = \frac{1}{2}([Z_{11}]-Z_c[I_d])[Z_{21}]^{-1}\left(\frac{[Z_{22}]}{Z_c}+[I_d]\right)-\frac{[Z_{12}]}{2Z_c}. \quad (65)$$

The expressions of linearized H-, G-, $ABCD$-, and Y-parameters in terms of linearized Z-parameters are derived and presented, respectively, in the following equations:

$$[H_{11}] = [Z_{11}]-[Z_{12}][Z_{22}]^{-1}[Z_{21}] \quad (66)$$

$$[H_{21}] = -[Z_{22}]^{-1}[Z_{21}] \quad (67)$$

$$[H_{12}] = [Z_{12}][Z_{22}]^{-1} \quad (68)$$

$$[H_{22}] = [Z_{22}]^{-1} \quad (69)$$

$$[G_{11}] = [Z_{11}]^{-1} \quad (70)$$

$$[G_{21}] = [Z_{21}][Z_{11}]^{-1} \tag{71}$$
$$[G_{12}] = -[Z_{11}]^{-1}[Z_{12}] \tag{72}$$
$$[G_{22}] = [Z_{22}] - [Z_{21}][Z_{11}]^{-1}[Z_{12}] \tag{73}$$
$$[A] = [Z_{11}][Z_{21}]^{-1} \tag{74}$$
$$[C] = [Z_{21}]^{-1} \tag{75}$$
$$[B] = [Z_{12}] - [Z_{11}][Z_{21}]^{-1}[Z_{22}] \tag{76}$$
$$[D] = -[Z_{21}]^{-1}[Z_{22}] \tag{77}$$
$$[Y_{\text{nonlin}}] = [Z_{\text{nonlin}}]^{-1}. \tag{78}$$

For the remaining conversion rules, to derive wave-based linearized network parameters ($[X]$ or $[T]$) $[A_1]$, $[A_2]$, $[B_1]$ or $[B_2]$ have to be set to [0]. And, to derive voltage-/current-based linearized network parameters, $[V_1]$, $[V_2]$, $[I_1]$, or $[I_2]$ have to be set to [0].

C. Conversion From Linearized Y- to X-, Linearized T-, H-, G-, ABCD- and Z-Parameters

The expressions of X-, linearized T-, H-, G-, $ABCD$- and Z-parameters in terms of linearized Y-parameters are formulated in the following expressions:

$$[X] = [[Id] - Z_c[Y_{\text{nonlin}}]][Z_c[Y_{\text{nonlin}}] + [Id]]^{-1} \tag{79}$$
$$[T_{11}] = \frac{1}{2}\left([I_d] - Z_c[Y_{11}]\right)[Y_{21}]^{-1}\left(\frac{[I_d]}{Z_c} - [Y_{22}]\right) - \frac{1}{2}Z_c[Y_{12}]. \tag{80}$$
$$[T_{21}] = \frac{1}{2}\left(Z_c[Y_{11}] + [I_d]\right)[Y_{21}]^{-1}\left(\frac{[I_d]}{Z_c} - [Y_{22}]\right) + \frac{1}{2}Z_c[Y_{12}] \tag{81}$$
$$[T_{12}] = \frac{1}{2}\left\{\left(Z_c[Y_{11}] - [I_d]\right)[Y_{21}]^{-1} \times \left(\frac{[I_d]}{Z_c} + [Y_{22}]\right) - Z_c[Y_{12}]\right\} \tag{82}$$
$$[T_{22}] = \frac{1}{2}\left\{Z_c[Y_{12}] - \left(Z_c[Y_{11}] + [I_d]\right)[Y_{21}]^{-1} \times \left(\frac{[I_d]}{Z_c} + [Y_{22}]\right)\right\} \tag{83}$$
$$[H_{11}] = [Y_{11}]^{-1} \tag{84}$$
$$[H_{21}] = [Y_{21}][Y_{11}]^{-1} \tag{85}$$
$$[H_{12}] = -[Y_{11}]^{-1}[Y_{12}] \tag{86}$$
$$[H_{22}] = [Y_{22}] - [Y_{21}][Y_{11}]^{-1}[Y_{12}] \tag{87}$$
$$[G_{11}] = [Y_{11}] - [Y_{12}][Y_{22}]^{-1}[Y_{21}] \tag{88}$$
$$[G_{21}] = -[Y_{22}]^{-1}[Y_{21}] \tag{89}$$
$$[G_{12}] = [Y_{12}][Y_{22}]^{-1} \tag{90}$$
$$[G_{22}] = [Y_{22}]^{-1} \tag{91}$$
$$[A] = -[Y_{21}]^{-1}[Y_{22}] \tag{92}$$
$$[C] = [Y_{12}] - [Y_{11}][Y_{21}]^{-1}[Y_{22}] \tag{93}$$
$$[B] = [Y_{21}]^{-1} \tag{94}$$
$$[D] = [Y_{11}][Y_{21}]^{-1} \tag{95}$$
$$[Z_{\text{nonlin}}] = [Y_{\text{nonlin}}]^{-1}. \tag{96}$$

D. Conversion From Linearized ABCD- to X-, Linearized T-, Z-, Y-, H-, and G-Parameters

The expressions of X, linearized T-, Z-, Y-, H-, and G-parameters in terms of linearized $ABCD$-parameters are derived in the following equations:

$$[X_{11}] = 2\left[[A] - \frac{[B]}{Z_c}\right]\left[[A] - \frac{[B]}{Z_c} + Z_c[C] - [D]\right]^{-1} - [I_d] \tag{97}$$
$$[X_{21}] = 2\left[[A] - \frac{[B]}{Z_c} + Z_c[C] - [D]\right]^{-1} \tag{98}$$
$$[X_{12}] = \left[[A] + \frac{[B]}{Z_c}\right] + \left[[A] - \frac{[B]}{Z_c}\right][X_{22}] \tag{99}$$
$$[X_{22}] = -\left[[A] - \frac{[B]}{Z_c} + Z_c[C] - [D]\right]^{-1} \cdot \left[[A] + \frac{[B]}{Z_c} + Z_c[C] + [D]\right] \tag{100}$$
$$[T_{11}] = \frac{1}{2}\left([A] + 1/Z_c[B] - Z_c[C] - [D]\right) \tag{101}$$
$$[T_{21}] = \frac{1}{2}\left([A] + 1/Z_c[B] + Z_c[C] + [D]\right) \tag{102}$$
$$[T_{12}] = \frac{1}{2}\left([A] - 1/Z_c[B] - Z_c[C] + [D]\right) \tag{103}$$
$$[T_{22}] = \frac{1}{2}\left([A] - 1/Z_c[B] + Z_c[C] - [D]\right) \tag{104}$$
$$[Z_{11}] = [A][C]^{-1} \tag{105}$$
$$[Z_{21}] = [C]^{-1} \tag{106}$$
$$[Z_{12}] = [B] - [A][C]^{-1}[D] \tag{107}$$
$$[Z_{22}] = -[C]^{-1}[D] \tag{108}$$
$$[Y_{11}] = [D][B]^{-1} \tag{109}$$
$$[Y_{21}] = [B]^{-1} \tag{110}$$
$$[Y_{12}] = [C] - [D][B]^{-1}[A] \tag{111}$$
$$[Y_{22}] = -[B]^{-1}[A] \tag{112}$$
$$[H_{11}] = [B][D]^{-1} \tag{113}$$
$$[H_{21}] = [D]^{-1} \tag{114}$$
$$[H_{12}] = [A] - [B][D]^{-1}[C] \tag{115}$$
$$[H_{22}] = -[D]^{-1}[C] \tag{116}$$
$$[G_{11}] = [C][A]^{-1} \tag{117}$$
$$[G_{21}] = [A]^{-1} \tag{118}$$
$$[G_{12}] = [D] - [C][A]^{-1}[B] \tag{119}$$
$$[G_{22}] = -[A]^{-1}[B]. \tag{120}$$

E. Conversion From Linearized T- to X-, Linearized ABCD-, Z- Y-, H-, and G-Parameters

The expressions of X, linearized $ABCD$-, Z-, Y-, H-, and G-parameters in terms of linearized T-parameters are derived in the following equations:

$$[X_{11}] = [T_{12}][T_{22}]^{-1} \tag{121}$$
$$[X_{12}] = [T_{11}] - [T_{12}][T_{22}]^{-1}[T_{21}] \tag{122}$$
$$[X_{21}] = [T_{22}]^{-1} \tag{123}$$

$$[X_{22}] = -[T_{22}]^{-1}[T_{21}] \quad (124)$$

$$[A] = \frac{1}{2}([T_{11}] + [T_{12}] + [T_{21}] + [T_{22}]) \quad (125)$$

$$[C] = \frac{1}{2Z_c}([T_{21}] + [T_{22}] - [T_{11}] - [T_{12}]) \quad (126)$$

$$[B] = \frac{Z_c}{2}([T_{11}] - [T_{12}] + [T_{21}] - [T_{22}]) \quad (127)$$

$$[D] = \frac{1}{2}([T_{21}] - [T_{22}] - [T_{11}] + [T_{12}]) \quad (128)$$

$$[Z_{11}] = Z_c([I_d] + 2[[T_{11}] + [T_{12}]] \cdot [[T_{21}] + [T_{22}] - [T_{11}] - [T_{12}]]^{-1}) \quad (129)$$

$$[Z_{21}] = 2Z_c[[T_{21}] + [T_{22}] - [T_{11}] - [T_{12}]]^{-1} \quad (130)$$

$$[Z_{12}] = Z_c[[T_{21}] - [T_{22}]] + Z_c[[T_{21}] + [T_{22}]] \cdot [[T_{11}] + [T_{12}] - [T_{21}] - [T_{22}]]^{-1} \cdot [[T_{21}] - [T_{22}] - [T_{11}] + [T_{12}]] \quad (131)$$

$$[Z_{22}] = Z_c[[T_{11}] + [T_{12}] - [T_{21}] - [T_{22}]]^{-1} \cdot [[T_{21}] - [T_{22}] - [T_{11}] + [T_{12}]] \quad (132)$$

$$[Y_{11}] = \frac{1}{Z_c}[[T_{21}] - [T_{22}] - [T_{11}] + [T_{12}]] \cdot [[T_{11}] - [T_{12}] + [T_{21}] - [T_{22}]]^{-1} \quad (133)$$

$$[Y_{22}] = -\frac{1}{Z_c}[[T_{11}] - [T_{12}] + [T_{21}] - [T_{22}]]^{-1} \cdot [[T_{11}] + [T_{12}] + [T_{21}] + [T_{22}]] \quad (134)$$

$$[Y_{21}] = \frac{2}{Z_c}[[T_{11}] - [T_{12}] + [T_{21}] - [T_{22}]]^{-1} \quad (135)$$

$$[Y_{12}] = \frac{1}{Z_c}([T_{21}] + [T_{22}]. - [[T_{21}] - [T_{22}]] \cdot [[T_{11}] - [T_{12}] + [T_{21}] - [T_{22}]]^{-1} \cdot .[[T_{11}] + [T_{12}] + [T_{21}] + [T_{22}]]) \quad (136)$$

$$[H_{11}] = Z_c[[T_{11}] + [T_{21}] - [T_{12}] - [T_{22}]] \cdot [[T_{21}] - [T_{22}] - [T_{11}] + [T_{12}]]^{-1} \quad (137)$$

$$[H_{21}] = 2[[T_{21}] - [T_{22}] - [T_{11}] + [T_{12}]]^{-1} \quad (138)$$

$$[H_{12}] = [T_{11}] + [T_{12}] + [[T_{11}] - [T_{12}]] \cdot [[T_{11}] - [T_{12}] - [T_{21}] + [T_{22}]]^{-1} \cdot [[T_{21}] + [T_{22}] - [T_{11}] - [T_{12}]] \quad (139)$$

$$[H_{22}] = \frac{1}{Z_c}[[T_{11}] - [T_{12}] - [T_{21}] + [T_{22}]]^{-1} \cdot [[T_{21}] + [T_{22}] - [T_{11}] - [T_{12}]] \quad (140)$$

$$[G_{11}] = \frac{1}{Z_c}[[T_{21}] + [T_{22}] - [T_{11}] - [T_{12}]] \cdot [[T_{11}] + [T_{12}] + [T_{21}] + [T_{22}]]^{-1} \quad (141)$$

$$[G_{21}] = 2[[T_{11}] + [T_{12}] + [T_{21}] + [T_{22}]]^{-1} \quad (142)$$

$$[G_{22}] = -Z_c[[T_{11}] + [T_{12}] + [T_{21}] + [T_{22}]]^{-1} \cdot [[T_{11}] + [T_{21}] - [T_{12}] - [T_{22}]] \quad (143)$$

$$[G_{12}] = [T_{21}] - [T_{22}] - [[T_{21}] + [T_{22}]] \cdot [[T_{11}] + [T_{12}] + [T_{21}] + [T_{22}]]^{-1} \cdot [[T_{11}] + [T_{21}] - [T_{12}] - [T_{22}]]. \quad (144)$$

F. Conversion From Linearized G- to X-, Linearized T-, Z-, Y- ABCD-, and H-Parameters

The expressions of X-, linearized T-, Z-, Y-, $ABCD$-, and H-parameters in terms of linearized G-parameters are derived in the following expressions:

$$[X_{11}] = \left[[G_{11}] + \frac{[I_d]}{Z_c} - \frac{[G_{12}]}{Z_c}\left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}[G_{21}]\right]^{-1} \cdot \left[\frac{[I_d]}{Z_c} - [G_{11}] + \frac{[G_{12}]}{Z_c}\left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}[G_{21}]\right] \quad (145)$$

$$[X_{21}] = \left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}[G_{21}] \times \left\{[I_d] + \cdot \left[[G_{11}] + \frac{[I_d]}{Z_c} - \frac{[G_{12}]}{Z_c} \times \left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}[G_{21}]\right]^{-1} \cdot \left[\frac{[I_d]}{Z_c} - [G_{11}] + \frac{[G_{12}]}{Z_c}\left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}[G_{21}]\right]\right\} \quad (146)$$

$$[X_{12}] = \left[[G_{11}] + \frac{[I_d]}{Z_c} - \frac{[G_{12}]}{Z_c}\left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}[G_{21}]\right]^{-1} \cdot \frac{[G_{12}]}{Z_c}\left[\left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}\left[\frac{[G_{22}]}{Z_c} - [I_d]\right] - [I_d]\right] \quad (147)$$

$$[X_{22}] = \left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1} \times \left\{[G_{21}] \cdot \left[[G_{11}] + \frac{[I_d]}{Z_c} - \frac{[G_{12}]}{Z_c} \times \left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}[G_{21}]\right]^{-1} \cdot \frac{[G_{12}]}{Z_c}\left[\left[[I_d] + \frac{[G_{22}]}{Z_c}\right]^{-1}\left[\frac{[G_{22}]}{Z_c} - [I_d]\right] - [I_d]\right] \cdot + \left[\frac{[G_{22}]}{Z_c} - [I_d]\right]\right\} \quad (148)$$

$$[T_{11}] = \frac{1}{2}\left\{[[I_d] - Z_c[G_{11}]]\,[G_{21}]^{-1}\left[[I_d] - \frac{[G_{22}]}{Z_c}\right] - [G_{12}]\right\} \quad (149)$$

$$[T_{21}] = \frac{1}{2}\left\{[[I_d] + Z_c[G_{11}]]\,[G_{21}]^{-1}\left[[I_d] - \frac{[G_{22}]}{Z_c}\right] + [G_{12}]\right\} \quad (150)$$

$$[T_{12}] = \frac{1}{2}\left\{[[I_d] - Z_c[G_{11}]]\,[G_{21}]^{-1}\left[[I_d] + \frac{[G_{22}]}{Z_c}\right] + [G_{12}]\right\} \quad (151)$$

$$[T_{22}] = \frac{1}{2}\left\{[[I_d] + Z_c[G_{11}]]\,[G_{21}]^{-1}\left[[I_d] + \frac{[G_{22}]}{Z_c}\right] - [G_{12}]\right\} \quad (152)$$

$$[Z_{11}] = [G_{11}]^{-1} \quad (153)$$
$$[Z_{21}] = [G_{21}][G_{11}]^{-1} \quad (154)$$
$$[Z_{12}] = -[G_{11}]^{-1}[G_{12}] \quad (155)$$
$$[Z_{22}] = [G_{22}] - [G_{21}][G_{11}]^{-1}[G_{12}] \quad (156)$$
$$[Y_{11}] = [G_{11}] - [G_{12}][G_{22}]^{-1}[G_{21}] \quad (157)$$
$$[Y_{21}] = -[G_{22}]^{-1}[G_{21}] \quad (158)$$
$$[Y_{12}] = [G_{12}][G_{22}]^{-1} \quad (159)$$
$$[Y_{22}] = [G_{22}]^{-1} \quad (160)$$
$$[A] = [G_{21}]^{-1} \quad (161)$$
$$[C] = [G_{11}][G_{21}]^{-1} \quad (162)$$
$$[B] = -[G_{21}]^{-1}[G_{22}] \quad (163)$$
$$[D] = [G_{12}] - [G_{11}][G_{21}]^{-1}[G_{22}] \quad (164)$$
$$[H_{\text{nonlin}}] = [G_{\text{nonlin}}]^{-1}. \quad (165)$$

G. Conversion From Linearized H- to X-, Linearized Z-, Y- ABCD-, T-, and G-Parameters

The expressions of X-, linearized Z-, Y-, $ABCD$-, T-, and G-parameters in terms of linearized H-parameters are derived in the following equations:

$$[X_{11}] = \left[[I_d] + \frac{[H_{11}]}{Z_c} - [H_{12}]\left[\frac{[I_d]}{Z_c} + [H_{22}]\right]^{-1}\frac{[H_{21}]}{Z_c}\right]^{-1} \cdot \left[\frac{[H_{11}]}{Z_c} - [I_d] - [H_{12}]\left[\frac{[I_d]}{Z_c} + [H_{22}]\right]^{-1}\frac{[H_{21}]}{Z_c}\right] \quad (166)$$

$$[X_{12}] = -\frac{2}{Z_c}\left[\frac{[H_{21}]}{Z_c} - \left[\frac{[I_d]}{Z_c} + [H_{22}]\right][H_{12}]^{-1} \times \left[[I_d] + \frac{[H_{11}]}{Z_c}\right]\right]^{-1} \quad (167)$$

$$[X_{21}] = \left[\left[\frac{I_d}{Z_c}\right] + [H_{22}]\right]^{-1}\frac{[H_{21}]}{Z_c} \cdot \left\{\left[[I_d] + \frac{[H_{11}]}{Z_c} - [H_{12}]\left[\frac{[I_d]}{Z_c} + [H_{22}]\right]^{-1}\frac{[H_{21}]}{Z_c}\right]^{-1} \cdot \left[\frac{[H_{11}]}{Z_c} - [I_d] - [H_{12}]\left[\frac{[I_d]}{Z_c} + [H_{22}]\right]^{-1}\frac{[H_{21}]}{Z_c}\right] - [I_d]\right\} \quad (168)$$

$$[X_{22}] = -\frac{2}{Z_c}[H_{12}]^{-1}\left[[I_d] + \frac{[H_{11}]}{Z_c}\right] \cdot \left[\frac{[H_{21}]}{Z_c} - \left[\frac{[I_d]}{Z_c} + [H_{22}]\right][H_{12}]^{-1} \times \left[[I_d] + \frac{[H_{11}]}{Z_c}\right]\right]^{-1} - [I_d] \quad (169)$$

$$[Z_{11}] = [H_{11}] - [H_{12}][H_{22}]^{-1}[H_{21}] \quad (170)$$
$$[Z_{21}] = -[H_{22}]^{-1}[H_{21}] \quad (171)$$
$$[Z_{12}] = [H_{12}][H_{22}]^{-1} \quad (172)$$
$$[Z_{22}] = [H_{22}]^{-1} \quad (173)$$
$$[Y_{11}] = [H_{11}]^{-1} \quad (174)$$
$$[Y_{21}] = [H_{21}][H_{11}]^{-1} \quad (175)$$
$$[Y_{12}] = -[H_{11}]^{-1}[H_{12}] \quad (176)$$
$$[Y_{22}] = [H_{22}] - [H_{21}][H_{11}]^{-1}[H_{12}] \quad (177)$$
$$[A] = [H_{12}] - [H_{11}][H_{21}]^{-1}[H_{22}] \quad (178)$$
$$[B] = [H_{11}][H_{21}]^{-1} \quad (179)$$
$$[C] = -[H_{21}]^{-1}[H_{22}] \quad (180)$$
$$[D] = [H_{21}]^{-1} \quad (181)$$

$$[T_{11}] = \frac{1}{2}\left[[H_{12}] + Z_c\left[\frac{[H_{11}]}{Z_c} - [I_d]\right][H_{21}]^{-1} \times \left[\frac{[I_d]}{Z_c} - [H_{22}]\right]\right] \quad (182)$$

$$[T_{21}] = \frac{1}{2}\left[[H_{12}] + \left[\frac{[H_{11}]}{Z_c} - [I_d]\right][H_{21}]^{-1}\left[[I_d] - Z_c[H_{22}]\right]\right] \quad (183)$$

$$[T_{12}] = \frac{1}{2}\left[[H_{12}] - \left[\frac{[H_{11}]}{Z_c} - [I_d]\right][H_{21}]^{-1}\left[[I_d] + Z_c[H_{22}]\right]\right] \quad (184)$$

$$[T_{22}] = \frac{1}{2}\left[[H_{12}] - \left[\frac{[H_{11}]}{Z_c} + [I_d]\right][H_{21}]^{-1}\left[[I_d] + Z_c[H_{22}]\right]\right] \quad (185)$$

$$[G_{\text{nonlin}}] = [H_{\text{nonlin}}]^{-1}. \quad (186)$$

IV. Linear and Linearized Operation Mode Validation of Conversion Rules Between the Linearized Two-Port Network Parameters

In order to not burden this paper with the validation of all conversion rules between the linearized network parameters, we will limit this section to the validation of the equations giving linearized Z-parameters. The validation procedure of the remaining conversion rules between X-parameters, linearized Y-parameters, linearized Z-parameters, linearized $ABCD$-parameters, linearized T-parameters, linearized G-parameters, and linearized H-parameters is done with the same manner and good results are obtained for all of them.

The process of the validation is explained by the flow graph illustrated in Fig. 2. To verify the conversion rules from the linearized two-port network parameters to Z-parameters, Fig. 3 illustrates the amplitude and phase of Z-parameters determined from X-, linearized Y-, linearized $ABCD$-, linearized T-, linearized G-, and linearized H-parameters. The validation procedure consists on generating X-parameters of the PA that is represented by its ADS model based on measurement file "ZX602522M_X2P.xnp" from Keysight Technologies Inc.

X-parameter data are then transformed, by using the conversion rules presented in Section III, to the remaining linearized network parameters.

Then, the obtained linearized network parameters data are transformed to Z-parameters. The linearized Z-parameters data determined from linearized Y-, Z-, G-, H-, $ABCD$-,

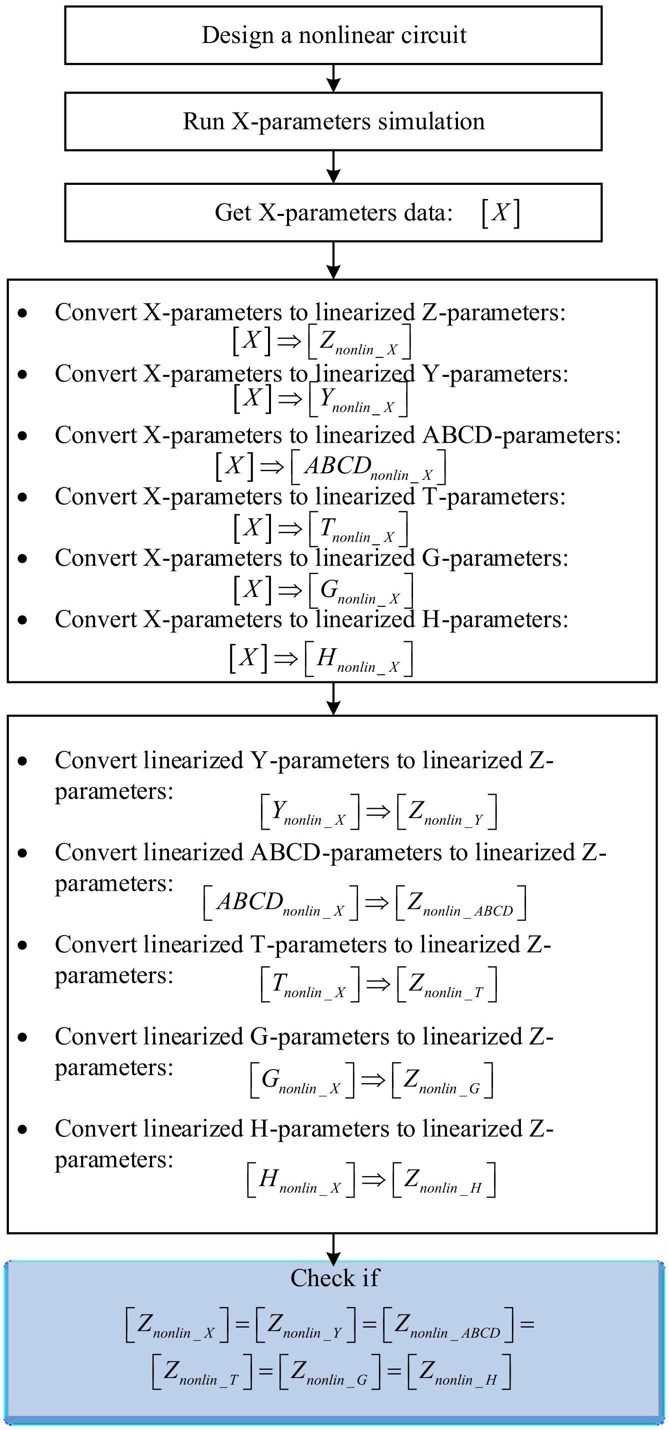

Fig. 2. Validation process of conversion rules between linearized two-port network parameters.

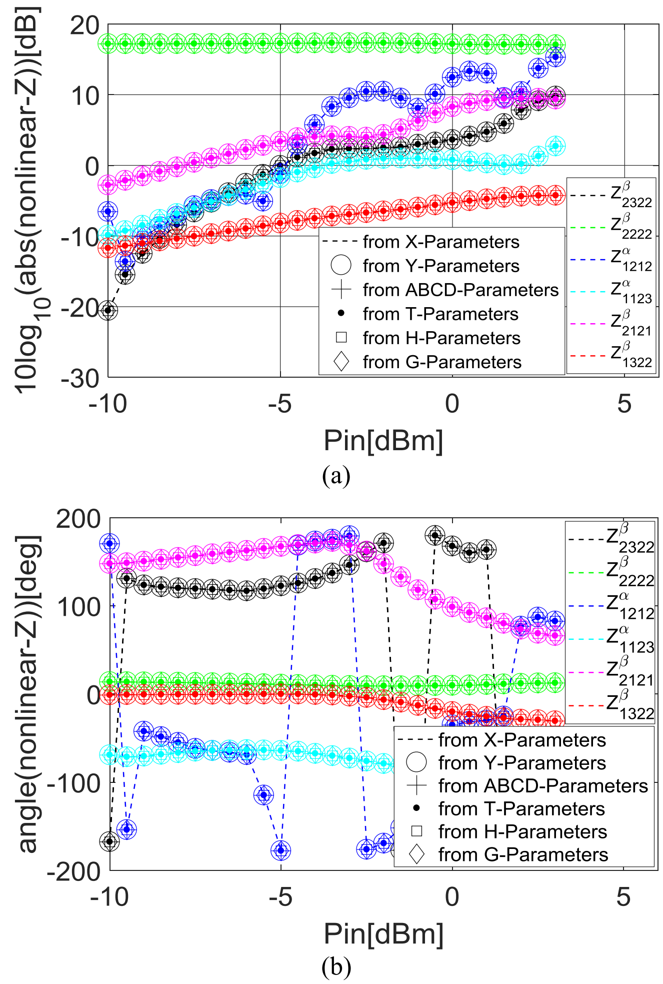

Fig. 3. (a) Amplitude and (b) phase of a set of Z-parameters converted from X-, linearized Y-, $ABCD$-, T-, H-, and G-parameters.

and T-parameters are then compared. For a 2-port circuit and considering the first three harmonics, 72 linearized impedance terms could be obtained. In order to keep the figures legible, only six arbitrary terms are presented: Z^{β}_{2322}, Z^{β}_{2222}, Z^{α}_{1212}, Z^{α}_{1123}, Z^{β}_{2121}, and Z^{α}_{1322}. Fig. 3 illustrates a very good agreement between Z-parameters calculated from X-parameters and the remaining linearized two-port network parameters. In the advanced design system (ADS), a frequency-domain defined (FDD) component enables the spectral values of current and voltage to be expressed in terms of other harmonic components of voltages and currents through algebraic relationships. FDD components can describe input and output voltage components or current components; therefore, the relation between the input or output spectral component voltages and currents is required. FDD components can describe input and output voltage components or current components; therefore, nonlinear network parameters can be implemented in ADS. A data access component is used to get access to the values of the nonlinear impedances that are extracted from X-parameter measurements file of an unpackaged CMOS transistor in a cascode configuration. Both of nonlinear parameters and X-parameters are extracted under the same conditions, i.e., bias, input power and impedance terminations at the fundamental and the harmonics in source and load sides.

Harmonic balance simulation is carried out to simulate and deduce the transducer gain of an unpackaged cascade device under different load terminations using both X- and linearized Z-parameters. Both X and linearized Z-parameters lead to the same values as shown in Fig. 4. X-parameters reduce to S-parameters when the device is operated in linear mode. For small $|a_{11}|$ (linear operation), the expressions of nonlinear parameters terms vanish to linear network parameters. Besides, in linear operation mode, conversion rules between nonlinear parameters are also reduced to their linear

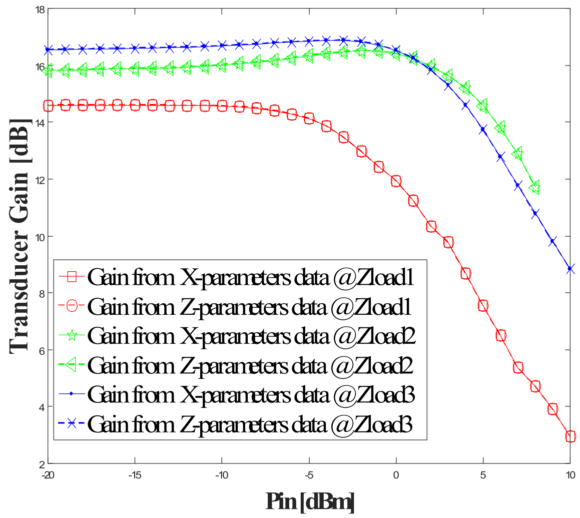

Fig. 4. Comparison between transducer gain obtained at different loads: $Z_{\text{load1}}(\text{fund}) = 6.25 + j0.198\Omega$, $Z_{\text{load2}}(\text{fund}) = 52.23 + j0.57\ \Omega$, and $Z_{\text{load3}}(\text{fund}) = 10.89 + j50.227\ \Omega$. The impedance at the second harmonic is $Z_{\text{load}}(2{*}\ \text{fund}) = 6.18\ \Omega$. The impedance at the higher harmonic components is arbitrary.

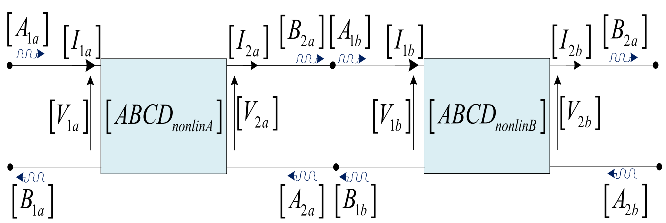

Fig. 5. Cascade connection of two nonlinear networks.

counterparts [13]. Nonlinear T-parameters can be used to model a cascaded configuration of nonlinear systems. The cascaded configuration of two nonlinear components is illustrated in Fig. 5. The relations between a and b waves through nonlinear T-parameters of the first and second nonlinear components are expressed as

$$\begin{bmatrix} [B_{1x;x=a,b}] \\ [A_{1x;x=a,b}] \end{bmatrix} = \left[T_{\text{nonlinx};x=A,B}\right] \begin{bmatrix} [A_{2x;x=a,b}] \\ [B_{2x;x=a,b}] \end{bmatrix}. \quad (187)$$

Multiharmonic wave vector $[B_{2a}]$ scattered at the output of the first nonlinear component is equal to multiharmonic wave vector $[A_{1b}]$ incident to the second nonlinear components. Moreover, multiharmonic wave vector $[B_{1b}]$ scattered at the input of the second nonlinear component is equal to multiharmonic wave vector $[A_{2a}]$ incident to the second nonlinear components at its output. Thus, $[[A_{2a}], [B_{2a}]]^T$ is equal to $[[B_{1b}], [A_{1b}]]^T$, and the relation between a and b waves of the whole system is given by

$$\begin{bmatrix} [B_{1a}] \\ [A_{1a}] \end{bmatrix} = [T_{\text{nonlinA}}] \times [T_{\text{nonlinB}}] \begin{bmatrix} [A_{2b}] \\ [B_{2b}] \end{bmatrix}. \quad (188)$$

The equivalent nonlinear T-parameters are the products of nonlinear T-parameters of both nonlinear components

$$\left[T_{\text{nonlin\_Eq}}\right] = [T_{\text{nonlinA}}] \times [T_{\text{nonlinB}}]. \quad (189)$$

The validity of analytical cascaded nonlinear T-parameters was evaluated by measuring the X-parameters of a nonlinear

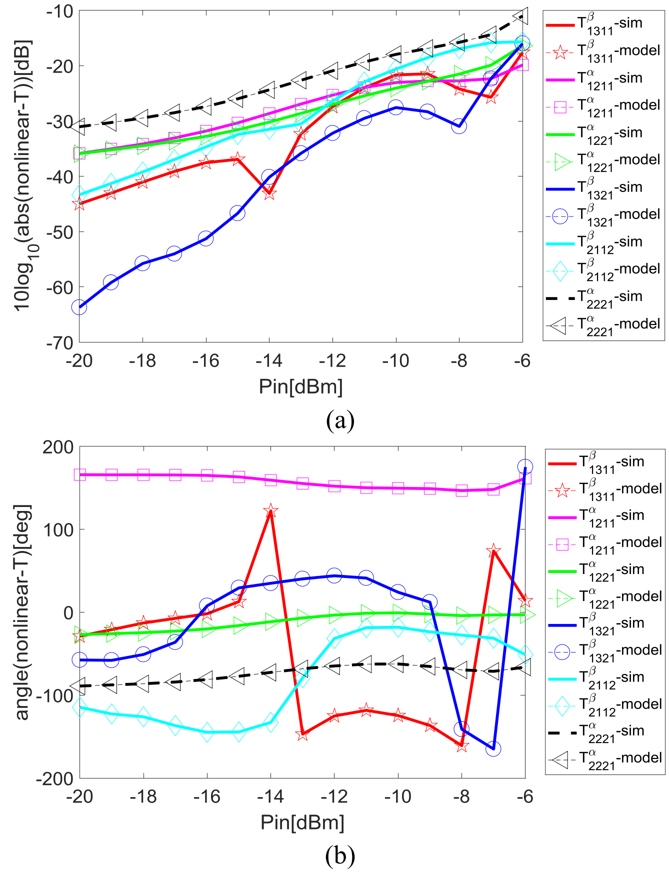

Fig. 6. Comparison between the (a) logarithmic amplitude and the (b) phase of a random set of nonlinear T-parameters calculated from the X-parameters of a cascaded system and those calculated through the cascaded nonlinear T expressions in (189).

cascaded system of two mini-circuits PAs, ZFL11AD and ZX602522M. X-parameters simulations of xnp measurement files in ADS simulator software could be sufficient to validate the cascade behavior. The .xnp files are in generic MDIF file format. They contain the multi-dimensional measured X-parameters. X-parameters simulations are used to generate X-parameters data of the whole cascaded system and of each individual component under the same conditions as when it is in the cascaded network. The comparison between the amplitudes and phases in Fig. 6 demonstrates a good agreement between the nonlinear T-parameters calculated from the X-parameters of the cascaded system and the results of the multiplication of nonlinear T matrices calculated from the X-parameters of each component. In Fig. 6, only seven arbitrary terms are considered: T^{β}_{1311}, T^{α}_{1211}, T^{α}_{1221}, T^{β}_{1321}, T^{β}_{2112}, and T^{α}_{2221}.

V. Conclusion

X-parameters are superset of S-parameters useful to characterize linear and nonlinear circuits operating in small- and large-signal regimes. However, in its form, X-parameters are not suitable for the analytic analysis of different network configurations. Other linearized network two-port parameters (linearized Z-parameters, linearized Y-parameters, linearized $ABCD$-parameters, linearized T-parameters, linearized

G- parameters, and linearized H-parameters are able to characterize any network topology. They can describe any topology of all linear, all nonlinear or a mixed of linear and nonlinear components. Most of these parameters use harmonic voltage and current as dependent and independent parameters. Therefore, it is impractical to measure them especially at higher frequency. The unique available measurement tool is a mixer-based NVNA [14] introduced by Keysight Technologies. It is a combination of NVNA with X-parameters [4]. To get the values of the linearized network parameters, measured or simulated X-parameters data can be used jointly to the conversion rules derived in this paper.

ACKNOWLEDGMENT

The authors would like to thank Dr. J. Verspecht from Keysight technologies, Santa Rosa, CA, USA, and Prof. J. E. Schutt-Ainé from the Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign, Champaign, IL, USA, for their fruitful discussions and recommendations.

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APPENDIX D

Two-Port Parameter Conversion

Conversion between the z, y, h, and g two-port voltage–current parameters is simply rearrangement of two linear equations relating voltages and currents at the two ports. Converting between these and the S parameters requires relating the voltage waves to voltages and currents. This latter relationship always includes the characteristic impedance, Z_0, by which the S parameters are referenced. Typically, this value is 50 Ω. Table D.1 shows this conversion. The program PARCONV is basically a code of many of the conversions in Table D.1.

The definitions of the various two-port parameters are described below. In each case, it is assumed that the current is flowing into the port terminal:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{D.1}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{D.2}$$

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \tag{D.3}$$

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \tag{D.4}$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} \tag{D.5}$$

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TABLE D.1 *S*-Parameter Conversion Chart

	S	z	y	$ABCD$
S_{11}	S_{11}	$\frac{(z_{11}-Z_0)(z_{22}+Z_0)-z_{12}z_{21}}{(z_{11}+Z_0)(z_{22}+Z_0)-z_{12}z_{21}}$	$\frac{(Y_0-y_{11})(Y_0+y_{22})+y_{12}y_{21}}{(Y_0+y_{11})(Y_0+y_{22})-y_{12}y_{21}}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\frac{2z_{12}Z_0}{(z_{11}+Z_0)(z_{22}+Z_0)-z_{12}z_{21}}$	$\frac{-2y_{12}Y_0}{(Y_0+y_{11})(Y_0+y_{22})-y_{12}y_{21}}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\frac{2z_{12}Z_0}{(z_{11}+Z_0)(z_{22}+Z_0)-z_{12}z_{21}}$	$\frac{-2y_{12}Y_0}{(Y_0+y_{11})(Y_0+y_{22})-y_{12}y_{21}}$	$\frac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\frac{(z_{11}+Z_0)(z_{22}-Z_0)-z_{12}z_{21}}{(z_{11}+Z_0)(z_{22}+Z_0)-z_{12}z_{21}}$	$\frac{(Y_0+y_{11})(Y_0-y_{22})+y_{12}y_{21}}{(Y_0+y_{11})(Y_0+y_{22})-y_{12}y_{21}}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	z_{11}	$\frac{y_{22}}{y_{11}y_{22}-y_{12}y_{21}}$	$\frac{A}{C}$
z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	z_{12}	$\frac{-y_{12}}{y_{11}y_{22}-y_{12}y_{21}}$	$\frac{AD-BC}{C}$
z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	z_{21}	$\frac{-y_{21}}{y_{11}y_{22}-y_{12}y_{21}}$	$\frac{1}{C}$
z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	z_{22}	$\frac{y_{11}}{y_{11}y_{22}-y_{12}y_{21}}$	$\frac{D}{C}$

y_{11}	$Y_0\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{z_{22}}{z_{11}z_{22}-z_{12}z_{21}}$	y_{11}	$\dfrac{D}{B}$
y_{12}	$Y_0\dfrac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-z_{12}}{z_{11}z_{22}-z_{12}z_{21}}$	y_{12}	$\dfrac{BC-AD}{B}$
y_{21}	$Y_0\dfrac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{-z_{21}}{z_{11}z_{22}-z_{12}z_{21}}$	y_{21}	$\dfrac{-1}{B}$
y_{22}	$Y_0\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\dfrac{z_{11}}{z_{11}z_{22}-z_{12}z_{21}}$	y_{22}	$\dfrac{A}{B}$
A	$\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{z_{11}}{z_{21}}$	$\dfrac{-y_{22}}{y_{21}}$	A
B	$Z_0\dfrac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{z_{11}z_{22}-z_{12}z_{21}}{z_{21}}$	$\dfrac{-1}{y_{21}}$	B
C	$\dfrac{1}{Z_0}\dfrac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\dfrac{1}{z_{21}}$	$-\dfrac{y_{11}y_{22}-y_{12}y_{21}}{y_{21}}$	C
D	$\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\dfrac{z_{22}}{z_{21}}$	$\dfrac{-y_{11}}{y_{21}}$	D

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{D.6}$$

For conversion to and from S parameters for circuits with more than two ports, the following formulas may be used [1]. Each variable is understood to be a matrix representing the S, z, or y parameters. The conversion formulas are

$$S = F(Z - G^*)(Z + G)^{-1} F^{-1} \tag{D.7}$$

$$Z = F^{-1}(I - S)^{-1}(SG + G^*)F \tag{D.8}$$

$$S = F(I - G^* Y)(I + GY)^{-1} F^{-1} \tag{D.9}$$

$$Y = F^{-1} G^{-1} (I + S)^{-1} (I - S) F \tag{D.10}$$

where

$$F = \begin{bmatrix} \frac{1}{2\sqrt{Z_{01}}} & 0 & \dots & 0 \\ 0 & \frac{1}{2\sqrt{Z_{02}}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{2\sqrt{Z_{0n}}} \end{bmatrix} \tag{D.11}$$

and

$$G = \begin{bmatrix} Z_{01} & 0 & \dots & 0 \\ 0 & Z_{02} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Z_{0n} \end{bmatrix} \tag{D.12}$$

The I in Eqs. (D.8) through (D.10) is the square identity matrix, and the Z_{0i}, $i = 1 \dots n$, are the characteristic impedances associated with each of the ports. An example of the usage of PARCONV is shown below. In using the program, make sure to include the decimals with the input data. Boldface values represent user inputs to the program. To exit the program use Ctrl. C.

```
TYPE SOURCE AND LOAD REFERENCE IMPEDANCE Z01,Z02 =
50., 50.Y --> S = YS OR S --> Y = SY
Z --> S = ZS OR S --> Z = SZ
ABCD --> S = AS OR S --> ABCD = SA
H --> S = HS OR S --> H = SH
H --> Z = HZ OR Z --> H =ZH
SY
INPUT S11, MAG. AND PHASE (deg)
.9, −80.
```

TABLE D.2 *S*-Parameter to Hybrid Parameter Conversion Chart

	S	h
S_{11}	S_{11}	$\frac{(h_{11}-Z_0)(h_{22}Z_0+1)-h_{12}h_{21}Z_0}{(h_{11}+Z_0)(h_{22}Z_0+1)-h_{12}h_{21}Z_0}$
S_{12}	S_{12}	$\frac{2h_{12}Z_0}{(h_{11}+Z_0)(h_{22}Z_0+1)-h_{12}h_{21}Z_0}$
S_{21}	S_{21}	$\frac{-2h_{12}Z_0}{(h_{11}+Z_0)(h_{22}Z_0+1)-h_{12}h_{21}Z_0}$
S_{22}	S_{22}	$\frac{(h_{11}+Z_0)(1-h_{22}Z_0)+h_{12}h_{21}Z_0}{(h_{11}+Z_0)(h_{22}Z_0+1)-h_{12}h_{21}Z_0}$
h_{11}	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{(1-S_{11})(1+S_{S22})+S_{12}S_{21}}$	h_{11}
h_{12}	$\frac{2S_{12}}{(1-S_{11})(1+S_{S22})+S_{12}S_{21}}$	h_{12}
h_{21}	$\frac{-2S_{21}}{(1-S_{11})(1+S_{S22})+S_{12}S_{21}}$	h_{21}
h_{22}	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{(1-S_{11})(1+S_{S22})+S_{12}S_{21}}$	h_{22}

```
INPUT S21, MAG. AND PHASE (deg)
1.9, 112.
INPUT S12, MAG. AND PHASE (deg)
0.043, 48.
INPUT S22, MAG. AND PHASE (deg)
0.7, −70.
Y(1,1) = .162912E-02 J .156482E-01
Y(1,2) = .304363E-03 J -.759390E-03
Y(2,1) = .360540E-01 J -.262179E-02
Y(2,2) = .483468E-02 J .123116E-01
Y --> S = YS OR S --> Y = SY
Z --> S = ZS OR S --> Z = SZ
ABCD --> S = AS OR S --> ABCD = SA
H --> S = HS OR S --> H = SH
H --> Z = HZ OR Z --> H =ZH
```

Table D.2 provides a direct conversion between two-port S parameters and two-port h parameters. This can be convenient with transistor models that are given in terms of h parameters.

REFERENCE

1. K. Kurokawa, "Power Waves and the Scattering Matrix," *IEEE Trans. Microwave Theory Tech.*, **MTT-11**, pp. 194–202, March 1965.