35-GHz Lens Antenna with Short Focal Length
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1. Specification
- Frequency: 34.0-35.8 GHz
- \( D = 140 \text{mm}, \text{Aperture to focal point} = 78 \text{mm} \)
Monopulse: sum, \( \Delta \text{-AZ}, \Delta \text{-EL} \)

2. Design
2.1 Structure
2.2. Feed design

2.2.1 Feed Illumination Angle

From the lens geometry one can obtain the feed's lens illumination angle:

\[ \theta_0 = \tan^{-1} \left( \frac{D/2}{F + T} \right) = \tan^{-1} \left( \frac{70}{78} \right) = 41.9^\circ \]

2.2.2 Feed Geometry
2.2.3 Feed Design Concept

- Five dielectric rods are employed.
  Center rod: for sum pattern
Right and left rods: for azimuth difference pattern
Top and bottom rods: for elevation difference pattern
- Design optimization
  Minimize the rod spacing for simultaneous optimal sum and difference patterns.
  Side rods (right/left and top/bottom rods) are shorter than the center rod by 25%.
  Side rods are not rotationally symmetric. Side rods

2.2.4 Feed Dimensions
- Dielectric rod material: Rexolite (er = 2.52, tand = 0.0007)
- Center element (dimensions from left):
  dia 1 = 1.86mm, dia 2 = 6.5mm; length from dia 1 to dia 2 = 27.00mm.
  dia 2 = 6.5mm, dia 3 = 6.5mm; length from dia 2 to dia 3 = 19.84mm.
- Side elements (4 places): dimensions from left
  dia 1 = 1.86mm, dia 2 = 6.5mm; length from dia 1 to dia 2 = 20.50mm.
  dia 2 = 6.5mm, dia 3 = 6.5mm; length from dia 2 to dia 3 = 19.84mm.
- Rod spacing, center to center = 5.68mm.
- Wave guide body: dia = 30.0mm, length = 19.84mm.
- Waveguide (square cross section): width = height = 5.64mm.
- Phase center (from center rod tip) = 9.84mm

2.2.5 Feed Reflection and Transmission Coefficients
- Electric field on the wave ports:
Port numbers:

Feed reflection coefficients:

S55 = center rod reflection coefficient
S44 = right/left rod reflection coefficient
S11 = top/bottom rod reflection coefficient

- Feed transmission coefficients:
  S45 = center-to-right/left rod coupling
  S15 = center-to-top/bottom rod coupling,
  S34 = right-to-left rod coupling
  S12 = top-to-bottom rod coupling
  S14 = top/bottom-to-right/left rod coupling

- E-plane co-pol pattern (phi = 0 deg):
  Gain pattern: sum, d-el
Sum pattern: max gain=14.2dB, gain at 42 deg=0.81dB, sidelobe level=-23.3dB

d-el pattern: max gain=13.1dB, null depth=-57.57dB, cross over angle=12.9deg. Gain at 42 deg = 5.26dB, sidelobe level=-20.0dB

Phase pattern: sum, d-el

Sum pattern: phase at 0 deg=203.0deg, phase at 42 deg=214.86deg

d-el pattern: phase at 5 deg=9.00deg, phase at 42 deg=-76deg

- H-plane co-pol pattern (phi = 90 deg):
Sum pattern: max gain=14.2dB, gain at 42 deg=2.99dB, sidelobe level= -11.2dB

d-az pattern: max gain=13.3dB, null depth=-57.57dB, cross over angle, gain at 42 deg = 0.09dB, sidelob level=-14.4

Phase pattern: sum, d-az

Sum pattern: phase at 0 deg=208.2deg, phase at 42 deg=195.9deg

d-az pattern: phase at 5 deg=8.89deg, phase at 42 deg=-31deg

- Diagonal plane pattern (phi = 45 deg)
  Co-pol gain pattern: sum
  Co-pol phase pattern: sum
X-pol gain pattern: sum, d-az, d-el

Minus 45-deg plane x-pol pattern (phi = -45 deg): not plotted. same as those of phi = +45 deg case

2.3 Lens Design

2.3.1 Design Requirements
- Shape: Convex-planar with convex surface in the feed side
- Lens diameter: 140mm
- Lens aperture-to-focal point: 78mm
2.3.2 Lens without Anti-Reflection Coating

A. Design Concept

- A spherical wave of the feed is transformed into a planewave after refraction at the first hyperbolic surface.
- The planewave is not refracted at the second planar surface.
- The center of the dielectric lens is thicker than the edge to compensate for the phase difference.
- A flat surface of the same material can be added to the second planar surface for mechanical strength.
- If the dielectric constant $\varepsilon_r$ is small, the lens will be thick and the incidence angle at the lens edge will be excessive leading to a high reflection at the hyperbolic lens surface.
- If the dielectric constant $\varepsilon_r$ is large, the lens will not be thick and the incidence angle at the lens edge will not be excessive. However, the reflection will be increased due to the mismatch in the intrinsic impedance of the air and the lens material. In this case, a quarter-wave matching layer may be needed to enhance the antenna performance.
Figure: Lens antenna coordinate

**B. Lens Equation**

- Hyperbolic lens surface (a textbook formula)

\[ y = \sqrt{(n^2 - 1)x^2 + 2(n - 1)Fx}, \quad n = \sqrt{\varepsilon_r} \]

C. Geometry
D. Lens Dimensions
- Lens material: er = 4, tand = 0
- Lens dimensions: \( D = 140\) mm; \( F+T = 78, T = 27.04 \) mm

E. Lens Gain Patterns
- E-plane co-pol pattern (Phi=90 deg)

Sum pattern: max gain=29.75dB, angular width(3dB)=5.3deg, sidelobe level= -25.7dB

d-el pattern: max gain=27.0dB, null depth= -29.17dB, cross over angle=2.84deg, sidelob level= -24.2dB

- H-plane co-pol pattern (Phi=0 deg)
Sum pattern: max gain=29.75dB, angular width(3dB)=4.9deg, sidelobe level=-25.1dB

d-az pattern: max gain=26.0dB, null depth=-23.67dB, cross over angle=2.98deg, , sidelobe level=-21.9dB

- Diagonal pattern (phi = 45 deg)
  
  Co-pol gain pattern: sum

X-pol gain patterns: sum, d-az, d-el

Sum-pattern: gain in main direction: -1.4dB
Difference pattern: gain in main direction: ΔEL=+3.45dB, ΔAZ=+5.56dB
2.3.3 Lens with Matching Layer

A. Matching Layer Design Concept
- The planewave reflection and transmission at a dielectric layer can be modeled as an equivalent transmission line.
- The lens match layer then works in the same as the quarter-wave transformer between two transmission lines.
- For wideband performance, the theory of the multi-step quarter-wave transformer can be applied.

B. Matching Layer Design Equation
- Represent a three-layer structure with an equivalent transmission line.
- Apply the theory of the quarter-wave impedance matching.

\[
\begin{align*}
\epsilon_1 & \quad \epsilon_2 & \quad \epsilon_3 \\
\theta_1 & \quad d & \quad \text{Figure: Lens quarter-wave matching layer}
\end{align*}
\]

- Matching layer design equations

\[
\begin{align*}
k &= \omega \sqrt{\mu \epsilon}, \quad k^2 = k_x^2 + k_z^2 \\
k_x &= k \sin \theta, \quad k_z = k \cos \theta \\
k_{x1} &= k_{x2} = k_{x3} \rightarrow \sqrt{\epsilon_1} \sin \theta_1 = \sqrt{\epsilon_2} \sin \theta_2 = \sqrt{\epsilon_3} \sin \theta_3 \\
\cos \theta_2 &= \sqrt{1 - (\epsilon_1 / \epsilon_2) \sin^2 \theta_1}, \quad \cos \theta_3 = \sqrt{1 - (\epsilon_1 / \epsilon_3) \sin^2 \theta_1} \\
k_{z2} &= k_0 \sqrt{\epsilon_2 - \epsilon_1 \sin^2 \theta_1} \\
k_{z2} d &= k_2 d \cos \theta_2 = k_0 d \sqrt{\epsilon_2} \cos \theta_2 = n\pi + \frac{\pi}{2} \quad (n = 0, 1, 2, \ldots)
\end{align*}
\]

\[
d = \left( \frac{n}{2} + \frac{1}{4} \right) \frac{\lambda_0}{\sqrt{\epsilon_2 / \epsilon_0 - (\epsilon_1 / \epsilon_0) \sin^2 \theta_1}} - \left( \frac{n}{2} + \frac{1}{4} \right) \frac{\lambda_2}{\sqrt{1 - (\epsilon_1 / \epsilon_2) \sin^2 \theta_1}}
\]

\[
Z_2 = \sqrt{Z_1 Z_3}
\]

\[
\begin{align*}
Z = \begin{cases} 
\frac{\omega \mu}{k_z}, & \text{TE} \\
\frac{k_z}{\omega \epsilon}, & \text{TE}
\end{cases}
\end{align*}
\]
\[ k_{22} = k_{21} k_{22} \rightarrow \varepsilon_2 - \varepsilon_1 \sin^2 \theta_1 = \sqrt{(\varepsilon_1 - \varepsilon_1 \sin^2 \theta_1)(\varepsilon_3 - \varepsilon_1 \sin^2 \theta_1)} \]

\[ \varepsilon_2 = \sqrt{(\varepsilon_1 - \varepsilon_1 \sin^2 \theta_1)(\varepsilon_3 - \varepsilon_1 \sin^2 \theta_1)} + \varepsilon_1 \sin^2 \theta_1 \]

\[ \varepsilon_2 = \begin{cases} \sqrt{\varepsilon_1 \varepsilon_3}, & \theta_1 = 0 \\ \varepsilon_1, & \theta_1 = \pi / 2 \end{cases} \]

- Design example:
  \[ \varepsilon_1 = \varepsilon_0, \varepsilon_3 = 4 \varepsilon_0 \]
  \[ \theta_1 = 0^\circ \rightarrow \varepsilon_2 = 2.000 \varepsilon_0, \ d = 0.250 \lambda_2 = 1.515 \text{ mm} \]
  \[ \theta_1 = 21^\circ \rightarrow \varepsilon_2 = 1.965 \varepsilon_0, \ d = 0.261 \lambda_2 = 1.581 \text{ mm} \]
  \[ \theta_1 = 42^\circ \rightarrow \varepsilon_2 = 1.848 \varepsilon_0, \ d = 0.298 \lambda_2 = 1.811 \text{ mm} \]

C. Geometry of the Designed Lens with Matching Layer
- Lens material: \( \epsilon_r = 4, \tan \delta = 0 \)
- Lens dimensions: \( D = 140\text{mm}; F+T = 78, T = 27.04\text{ mm} \)
- Matching layer on the planar surface:
  \( \epsilon_r = 2, \tan \delta = 0 \)
  Thickness = 1.51mm(0°) - 1.59mm(25°) - 1.80(42°)mm; piecewise linear approximation
- Matching layer on the hyperbolic surface:
  \( \epsilon_r = 2, \tan \delta = 0 \)
  Thickness: uniformly 1.51mm repeat shape of the lens

E. Lens Gain Patterns
- E-plane co-pol pattern (phi=90 deg)

Sum pattern: max gain = 30.70dB, angular width(3dB)=5.3deg, sidelobe level = -30.2dB
d-el pattern: max gain = 28.3dB, null depth = -23.77dB, cross over angle = 2.72deg, sidelob level = -28.7dB
- H-plane co-pol pattern (phi=0 deg)

Sum pattern: max gain=29.75dB, angular width(3dB)=4.9deg, sidelobe level= -25.1dB
d-az pattern: max gain=27.0dB, null depth=-20.14dB, cross over angle=3.03deg, sidelob level=-23.6dB

- Diagonal plane pattern (phi = 45 deg)
  
  Co-pol gain pattern: sum

X-pol gain patterns: sum, d-az, d-el
Sum-pattern: gain in main direction: $-2.7\text{dB}$

Difference pattern: gain in main direction: $\Delta EL=-1.58\text{dB}, \Delta AZ=+9.13$