Lecture 4 Impedance Matching (1)

1. What is the Impedance Matching?
2. Maximum Power Transfer Theorem
3. Reflectionless Matching
4. Conjugate Matching
5. Impedance Matching of Two-port Networks
6. Coding Example

1. What is the Impedance Matching?

- Impedance matching

- To deliver the maximum power to a load from a source

- The source impedance has a specified value.

- The load impedance can be varying.

- Find a network that transforms the load impedance for maximum power transfer from the source to the load.

- Applications

- Antennas and receiver front-ends

- Amplifiers

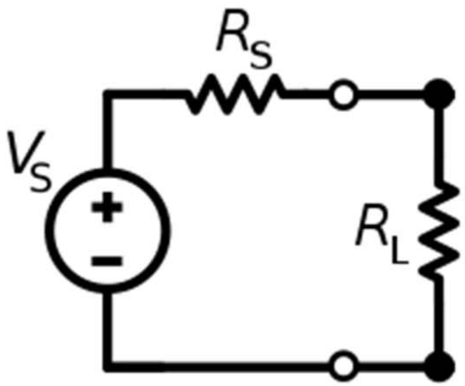
- Sensors and transducer

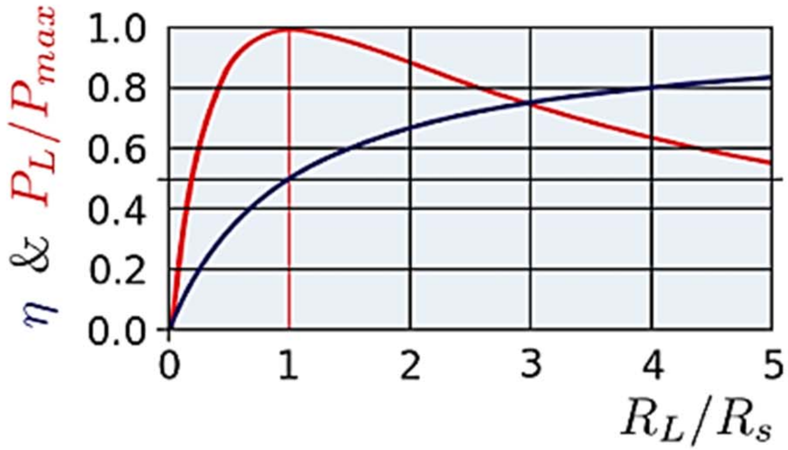
- Related topics

- Network synthesis

2. Maximum Power Transfer Theorem

- Maximum power transfer for DC circuits





$$\eta = \frac{R_L}{R_S + R_L}$$

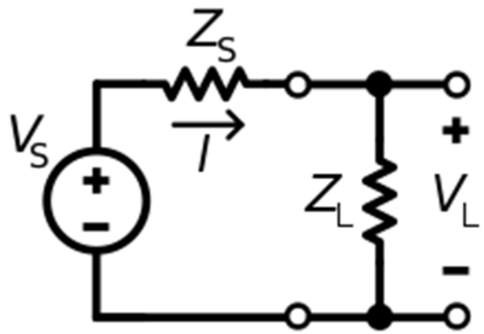
$$P_L = I^2 R_L = \left(\frac{V_S}{R_S + R_L} \right)^2 R_L = \frac{R_L}{(R_S + R_L)^2} V_S^2$$

$$\frac{\partial P_L}{\partial R_L} = 0 \rightarrow \boxed{R_L = R_S}$$

$$P_L = P_{L,\max} = \frac{V_S^2}{4R_S} = \frac{1}{2} P_S = \frac{1}{2} V_S I$$

$$P_S = P_{R_S} + P_{R_L} = 2P_L$$

- Maximum power transfer for AC networks



$$P_L = \mathrm{Re}(V_L I^*) = \mathrm{Re}\left[\frac{Z_L}{Z_S + Z_L} V_S \frac{1}{(Z_S + Z_L)^*} V_S^*\right] = \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} |V_S|^2$$

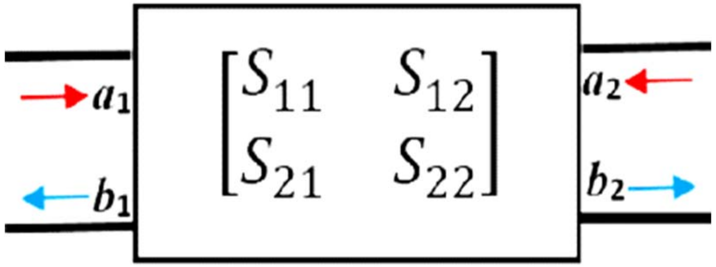
$$\frac{\partial P_L}{\partial R_L} = 0 \quad \text{and} \quad \frac{\partial P_L}{\partial X_L} = 0 \rightarrow \boxed{R_L = R_S \quad \text{and} \quad X_L = -X_S}$$

$$Z_L = Z_S^*$$

$$P_L = P_{L,\max} = \frac{1}{2}\frac{R_L}{(R_S + R_L)^2} |V_S|^2 = \frac{1}{2} P_S$$

3. Scattering Parameters

- Scattering parameter, S-paramter



$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$a_n = \frac{V_{0n}^{+}}{\sqrt{Z_{0n}}} \text{ : incident wave at port } n$$

$$b_n = \frac{V_{0n}^{-}}{\sqrt{Z_{0n}}} \text{ : reflected wave at port } n$$

$$S_{mn} = \frac{b_m}{a_n} = \frac{V_{0m}^{-}}{V_{0n}^{+}} \frac{\sqrt{Z_{0n}}}{\sqrt{Z_{0m}}} \quad (a_k = 0 \text{ for all } k \neq n) \text{ : scattering parameter}$$

- Powert at port n

$$P_n^+ = \frac{|V_{0n}^+|^2}{2Z_{0n}} = \frac{|a_n|^2}{2}$$: incident power at port n

$$P_n^- = \frac{|V_{0n}^-|^2}{2Z_{0n}} = \frac{|b_n|^2}{2}$$: reflected power at port n

$$P_n = P_n^+ - P_n^-$$: power delivered to port n

- Lossless networt: unitary condition

$$\sum_{k=1}^{N} |S_{nk}|^2 = 1 \quad \text{for} \;\; n = 1, \cdots, N \quad \text{(power conservation law)}$$

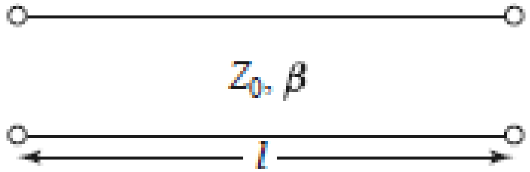
- Reciprocal network:

$$S_{mn} = S_{nm}$$

▪ Scattering parameters of simple networks

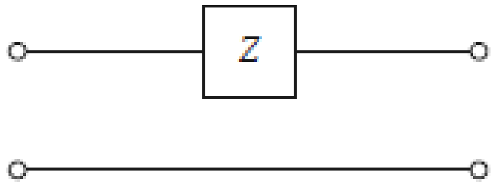
- Transmission line

$$[S]=\begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}$$



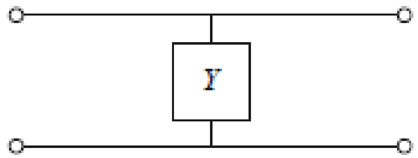
- Series element on transmission line

$$[S]=\begin{bmatrix} \dfrac{Z}{2Z_0+Z} & \dfrac{Z_0+Z}{2Z_0+Z} \\ \dfrac{Z_0+Z}{2Z_0+Z} & \dfrac{Z}{2Z_0+Z} \end{bmatrix}$$



- Shunt element on transmission line

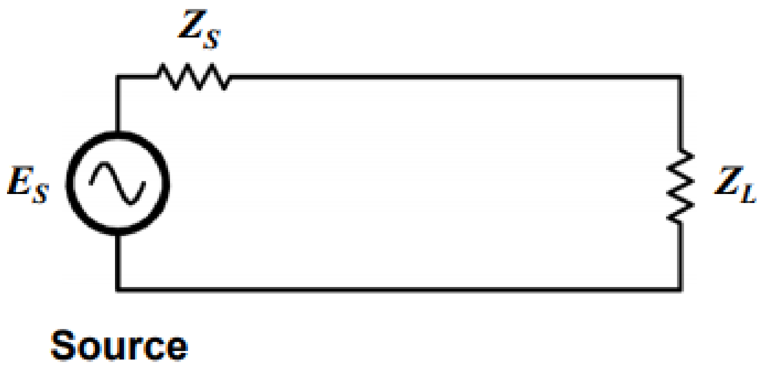
$$[S]=\begin{bmatrix} \dfrac{-Y}{2Y_0+Y} & \dfrac{2Y_0}{2Y_0+Y} \\ \dfrac{2Y_0}{2Y_0+Y} & \dfrac{-Y}{2Y_0+Y} \end{bmatrix}$$



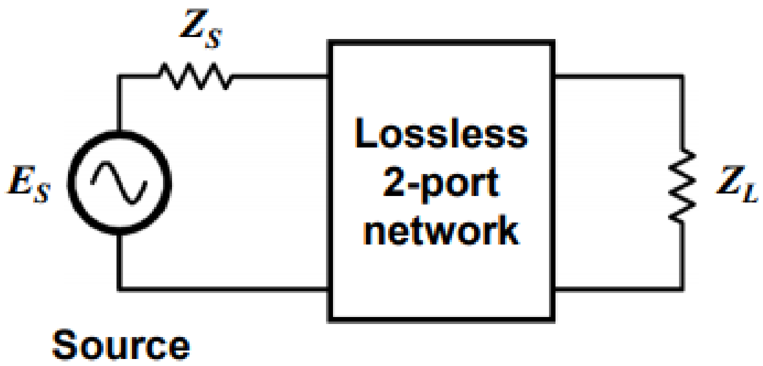
4. Conjugate Matching

- Maximum power transfer theorem for two-ports

- S. D. Stearns, ARRL Pacificon Antenna Seminar, 2011



$$Z_L = Z_s^*$$ (conjugate matching)

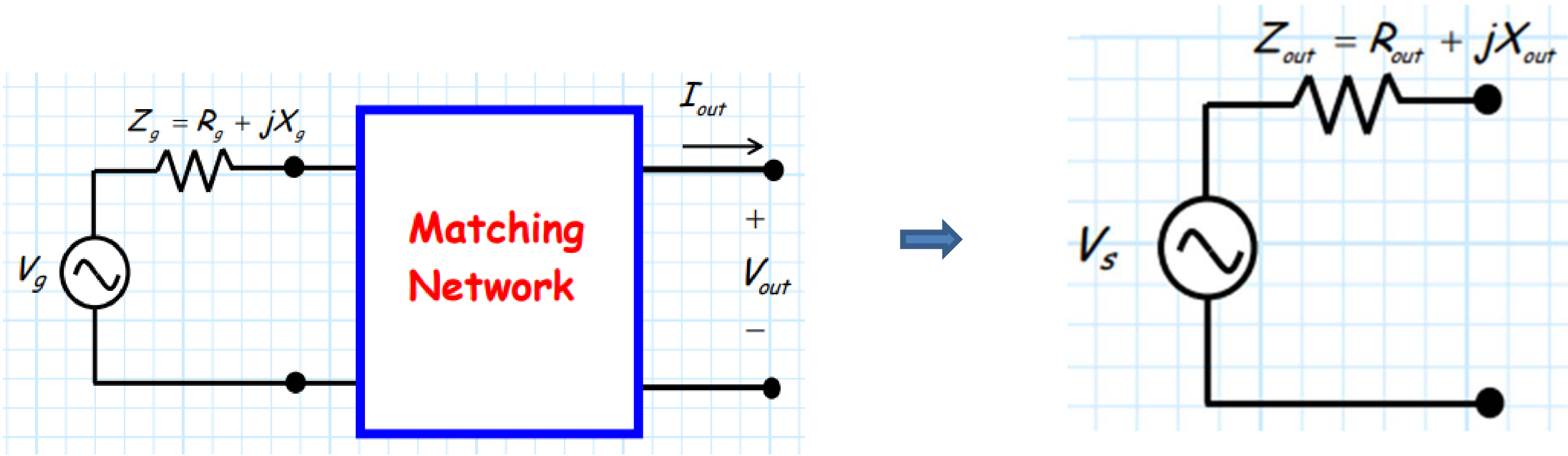


$$Z_{\text{in}} = Z_s^* \quad \Leftrightarrow \quad Z_{\text{out}} = Z_L^*$$ (simultaneous conjugate matching)

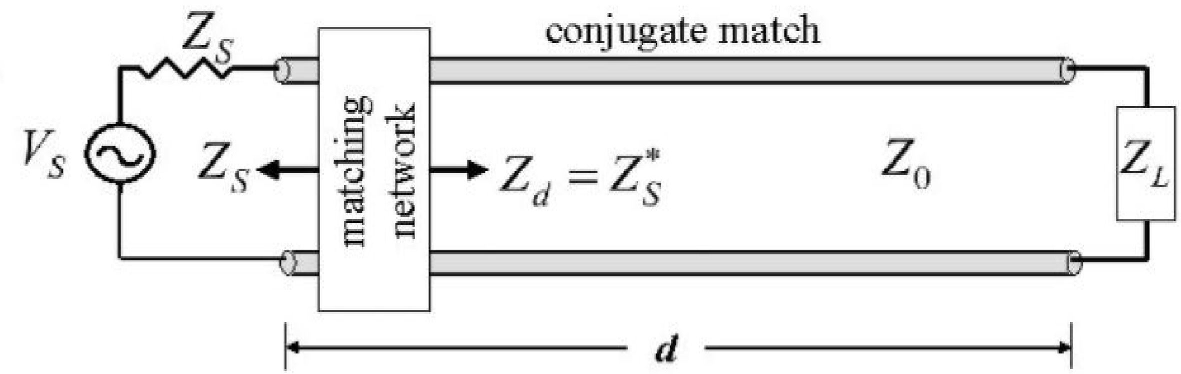
- Two-port matching network design [www.ittc.ku.edu/~jstiles/723/]

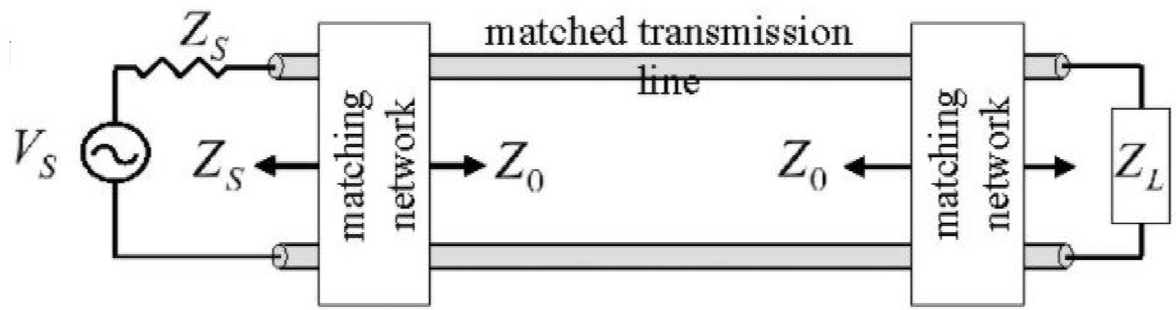
- Find the equivalent circuit at the output of the two-port network using the open-circuit and short-circuit method.
- Design a two-port network for the conjugate matching at the load.

$$Z_{\text{out}} = Z_L^*$$



- Source-load connected with a transmission line





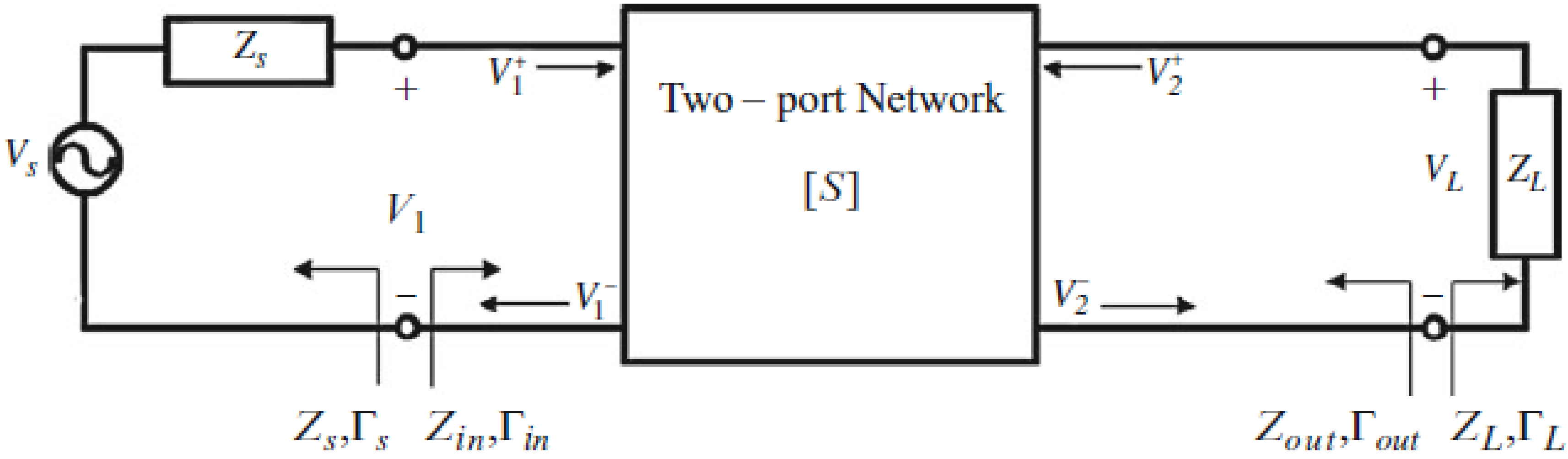
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = 0 \rightarrow \boxed{Z_L = Z_0}$$

- Case 1: Load not matched to Z_0 and input conjugate-matched → standing wave and additional loss with standing wave

- Case 2: Load matched to Z_0 and input matched to Z_0 → VSWR =1 and no additional loss

- Two-port network power gains



$$\Gamma_{\text{in}} = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{\text{out}} = \frac{Z_{\text{out}} - Z_0}{Z_{\text{out}} + Z_0} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

$[S]$: scattering matrix of a two-port network

- Operating power gain G_P:

$$G_P = \frac{P_L}{P_{\text{in}}}$$

P_L : power delivered to the load from the two-port network

P_{in} : input power to the two-port network

- Avaialable power gain G_A:

$$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}}$$

P_{avn} : maximum power available from network and delivered to load (when network output is conjugate-matched)

P_{avs} : maximum power available from source and delivered to network input (when network input is conjugate-matched)

- Transducer power gain G_T :

$$G_T = \frac{P_L}{P_{\text{avs}}}$$

- When input and output are conjugate-matched:

$$G_P = G_A = G_T$$

- Operating power gain G_P: source & load not matched
- Available power gain G_A: source & load matched
- Transducer power gain G_T: source matched, load not matched

$$\boxed{G_P \leq G_T \leq G_A}$$

- For conjugate-match at input and output:

$$G_P = G_T = G_A$$

- Gain in dB:

$$G\ (\text{dB}) = 10\log_{10} G$$

- Two-port network power gains

$$P_{\text{in}} = \frac{1}{2Z_0}\left|V_1^+\right|^2\left(1-|\Gamma_{\text{in}}|^2\right) = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{|1-\Gamma_S\Gamma_{\text{in}}|^2}\left(1-|\Gamma_{\text{in}}|^2\right), \text{ (power input at port 1)}$$

$$P_L = \frac{\left|V_1^+\right|^2}{2Z_0}\frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-S_{22}\Gamma_L|^2} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_L|^2\,|1-\Gamma_S\Gamma_{\text{in}}|^2} \text{ (power input at load)}$$

$\max(P_{\text{in}}) = P_{avs}$

P_{in} : maximum if $\Gamma_{\text{in}} = \Gamma_S^*$ or $Z_{\text{in}} = Z_S^*$

⇒

$$P_{avs} = P_{in}\Big|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{\left(1-|\Gamma_S|^2\right)}$$

$\max(P_L) = P_{\text{avn}}$

P_L : maximum if $\Gamma_L = \Gamma_{\text{out}}^*$ or $Z_L = Z_{\text{out}}^*$

⇒

$$P_{avn} = \frac{|V_S|^2}{8Z_0}|S_{21}|^2\frac{|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{out}|^2\right)}$$

Similarly, the power available from the network, P_{avn}, is the maximum power that can be delivered to the load. Thus, from (12.7),

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2\left(1-|\Gamma_{out}|^2\right)|1-\Gamma_S|^2}{\left|1-S_{22}\Gamma_{out}^*\right|^2|1-\Gamma_S\Gamma_{in}|^2}\Bigg|_{\Gamma_L=\Gamma_{out}^*}. \tag{12.10}$$

In (12.10), Γ_{in} must be evaluated for $\Gamma_L = \Gamma_{out}^*$. From (12.3a), it can be shown that

$$|1-\Gamma_S\Gamma_{in}|^2\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{out}|^2\right)^2}{\left|1-S_{22}\Gamma_{out}^*\right|^2},$$

which reduces (12.10) to

$$P_{avn} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{out}|^2\right)}. \tag{12.11}$$

$$G_P = \frac{P_L}{P_{\text{in}}} = \frac{(1-|\Gamma_L|^2)|S_{21}|^2}{|1-S_{22}\Gamma_L|^2(1-|\Gamma_{\text{in}}|^2)} \quad \text{: operating gain}$$

$$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} = \frac{(1-|\Gamma_S|^2)|S_{21}|^2}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{\text{out}}|^2)} \quad \text{: available gain}$$

$$G_T = \frac{P_L}{P_{\text{avs}}} = \frac{(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)|S_{21}|^2}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{12}S_{21}\Gamma_S\Gamma_L|^2} \quad \text{: transducer gain}$$

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{\text{in}}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{\text{out}}\Gamma_L|^2}$$

$$G_{\text{mag}} = \frac{|S_{21}|}{|S_{12}|}\left(K-\sqrt{K^2-1}\right) \quad \text{: maximum available gain} \quad (K>1)$$

$$G_{\text{msg}} = \frac{|S_{21}|}{|S_{12}|} \quad \text{: maximum stable gain} \quad (K=1)$$

$$K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{12}S_{21}|}$$

$$\Gamma_L = \Gamma_S = 0, \longrightarrow \quad G_T = |S_{21}|^2.$$

$$\Gamma_{in} = S_{11} \text{ when } S_{12} = 0$$

$$G_{TU} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - S_{11}\Gamma_S|^2 \, |1 - S_{22}\Gamma_L|^2}. \qquad \text{(Unilateral transducer gain)}$$

$$G_{TU} = G_S \, |S_{21}|^2 \, G_L$$

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} \quad \text{(source-match gain)}$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad \text{(load-match gain)}$$

$$M_{in} = \frac{P_{in}}{P_{avs}} = 1 - \left|\frac{\Gamma_{in} - \Gamma_S^*}{1 - \Gamma_{in}\Gamma_S}\right|^2 \quad \text{(source mismatch factor)}$$

$$M_{out} = \frac{P_L}{P_{avn}} = 1 - \left|\frac{\Gamma_{out} - \Gamma_L^*}{1 - \Gamma_{out}\Gamma_L}\right|^2 \quad \text{(load mismatch factor)}$$

- Example

A silicon bipolar junction transistor has the following scattering parameters at 1.0 GHz, with a 50 Ω reference impedance:

$$S_{11} = 0.38\angle{-158^\circ}$$
$$S_{12} = 0.11\angle{54^\circ}$$
$$S_{21} = 3.50\angle{80^\circ}$$
$$S_{22} = 0.40\angle{-43^\circ}$$

The source impedance is $Z_S = 25\ \Omega$ and the load impedance is $Z_L = 40\ \Omega$. Compute the power gain, the available power gain, and the transducer power gain.

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{25 - 50}{25 + 50} = -0.333,$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{40 - 50}{40 + 50} = -0.111.$$

$$\Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.365\angle - 152°,$$

$$\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.545\angle - 43°.$$

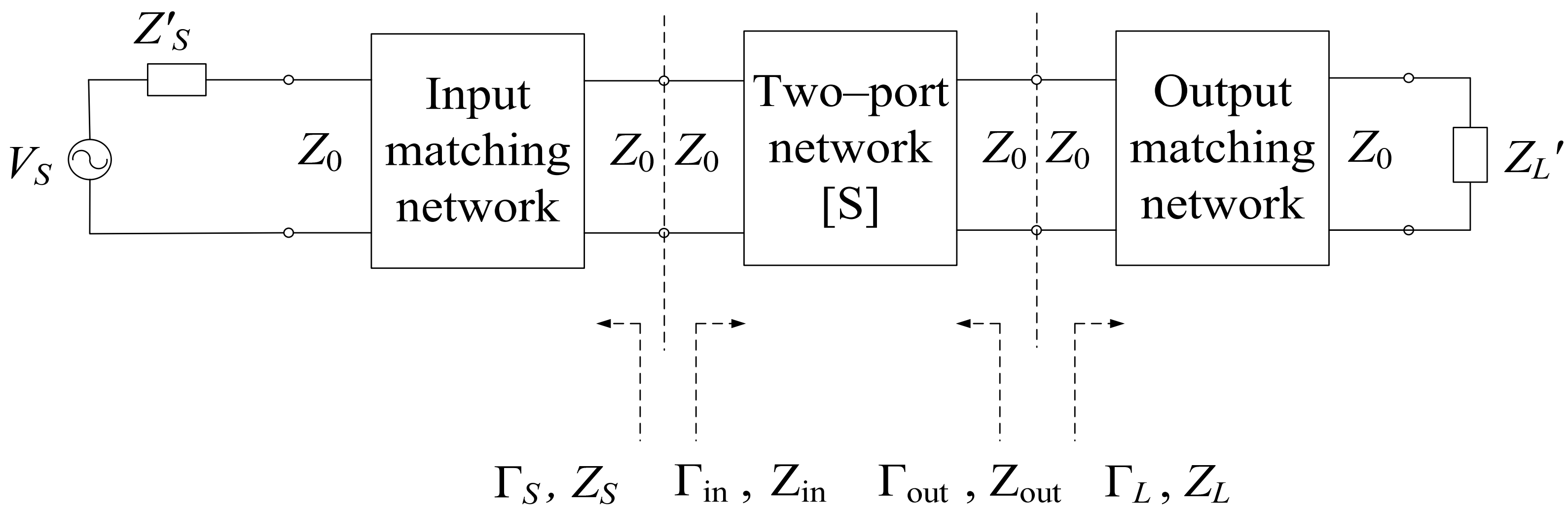
$$G = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{\text{in}}|^2\right)|1 - S_{22}\Gamma_L|^2} = 13.1.$$

$$G_A = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{|1 - S_{11}\Gamma_S|^2 \left(1 - |\Gamma_{\text{out}}|^2\right)} = 19.8.$$

$$G_T = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_S\Gamma_{\text{in}}|^2 \, |1 - S_{22}\Gamma_L|^2} = 12.6.$$

▪ Input and output matching of two-port networks

- Input and output simultaneous conjugate matching: solution to (1) and (2)



$$Z_{in} = Z_S^* \rightarrow \Gamma_{in} = \Gamma_S^* \quad \text{if } Z_0 \text{ is real.}$$

$$Z_{out} = Z_L^* \rightarrow \Gamma_{out} = \Gamma_L^* \quad \text{if } Z_0 \text{ is real.}$$

- Simultaneous conjutage matching:

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \Gamma_S^* \quad \text{and} \quad \Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = \Gamma_L^*$$

- Input and output matching of two-port networks

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$\Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$\Delta = \det(S) = S_{11}S_{22} - S_{12}S_{21}$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$: Rollett stability factor

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - S_{22}^*\Delta, \quad D_1 = |S_{11}|^2 - |\Delta|^2$$

$$C_2 = S_{22} - S_{11}^*\Delta, \quad D_2 = |S_{22}|^2 - |\Delta|^2$$

$$Z_S = \frac{1+\Gamma_S}{1-\Gamma_S} Z_0$$

$$Z_L = \frac{1+\Gamma_L}{1-\Gamma_L} Z_0$$

Input matching network: $Z'_S \rightarrow Z_S$

Output matching network: $Z'_L \rightarrow Z_L$

$K \geq 1$ (necessary and sufficient condition for simultaneous matching)

$M_{\text{in}} = 1$ and $M_{out} = 1$

$G_P = G_T = G_A = G_{\text{MAG}}$ (maximum available gain)

$$G_{\text{MAG}} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right)$$

$$G_{\text{MAG}} \leq G_{\text{MSG}} = \frac{|S_{21}|}{|S_{12}|} \text{ (maximum stable gain) } (K = 1)$$

- Simultaneous conjugate matching for the unilateral case:

$$S_{12} = 0$$

$$\Gamma_S = \Gamma_{\text{in}}^* = S_{11}^* \quad \text{and} \quad \Gamma_L = \Gamma_{\text{out}}^* = S_{22}^*$$

- Example

- With $Z_0 = Z_S = Z_L = 50\ \Omega$

S-Parameters	Real	Imaginary
S_{11}	0.599858625	-0.53991373
S_{21}	-0.219423779	1.14183461
S_{12}	0.067523223	0.03730980
S_{22}	0.116580879	-0.40044436

Γ and Z	Values
Γ_{in}	0.577503798 - j0.577166827
Γ_{out}	-0.096502369 - j0.402419084
Γ_G	0.577503798 + j0.577166827
Γ_L	-0.096502369 + j0.402419084
Z_{in}	32.57933879 - j112.810612 Ω
Z_{out}	30.37350084 - j29.4972738 Ω
Z_G	32.57933879 + j112.810612 Ω
Z_L	30.37350084 + j29.4972738 Ω

- Stability: two-port network's immunity to unwanted spurious oscillation

- Unconditional stability:

- Two-port stable for all passive source and load impedances

- Unconditionally stable two-port network:

If $|\Gamma_{\text{in}}| \leq 1$ and $|\Gamma_{\text{out}}| \leq 1$ for all passive loads ($|\Gamma_S| \leq 1$ and $|\Gamma_L| \leq 1$)

- Conditional stability:

- Two-port stable for a range of source and load impedances

- Stability criteria

- Rollet stability factor:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1 \text{ and } |\Delta| < 1 \rightarrow \text{unconditionally stable}$$

Otherwise $\rightarrow$ conditionally stable

$$\Delta = \det[S] = S_{11}S_{22} - S_{12}S_{21}$$

- Edwards-Sinsky stability parameters:

$$\mu_1 = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^*\Delta| + |S_{12}S_{21}|} > 1 \text{ (unconditionally stable): load stability}$$

$\mu_1 \leq 1$ (conditionally stable)

$$\mu_2 = \frac{1 - |S_{22}|^2}{|S_{11} - S_{22}^*\Delta| + |S_{12}S_{21}|} > 1 \text{ (unconditionally stable): source stability}$$

$\mu_2 \leq 1$ (conditionally stable)

- Relative measure of stability

- Rollett's K-factor: does not provide

- Edwards-Sinsky μ-factor: does provide

- Load stability region

$$|\Gamma_L - c_L| > r_L \text{ if } D_2 > 0$$

$$|\Gamma_L - c_L| < r_L \text{ if } D_2 < 0$$

- Source stability region

$$|\Gamma_S - c_S| > r_S \text{ if } D_1 > 0$$

$$|\Gamma_S - c_S| < r_S \text{ if } D_1 < 0$$

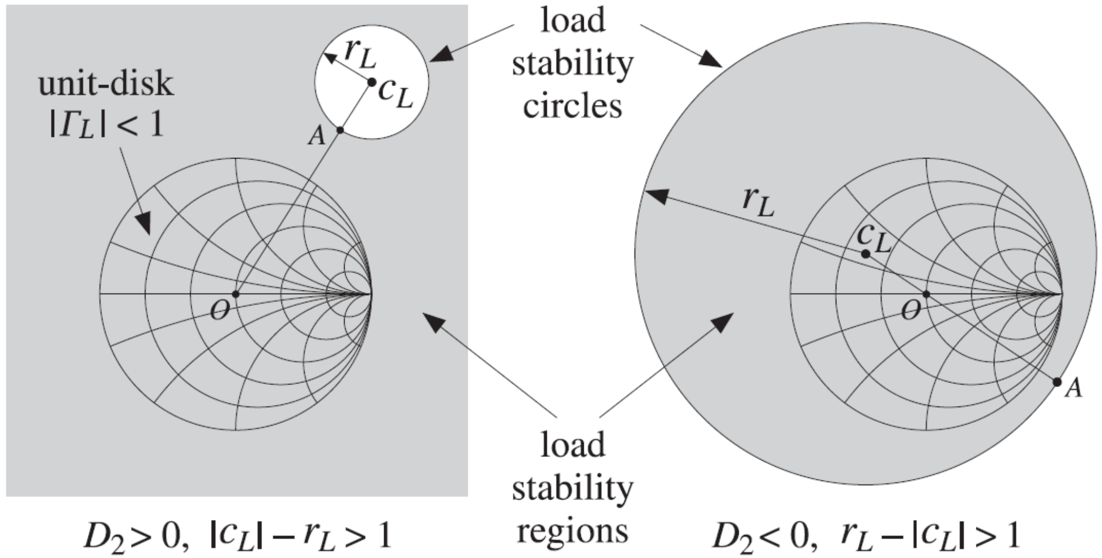


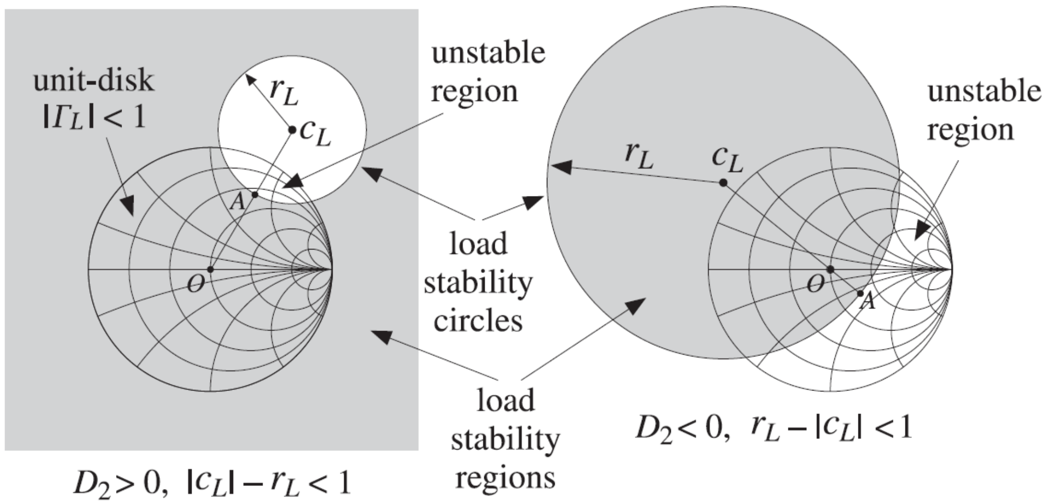
Figure: Load stability regions in the unconditionally stable case [Orfandis]

$$c_L = (S_{22}^* - S_{11}\Delta^*)/(|S_{22}|^2 - |\Delta|^2), \; r_L = |S_{12}S_{21}/(|S_{22}|^2 - |\Delta|^2)|$$

$$c_S = (S_{11}^* - S_{22}\Delta^*)/(|S_{11}|^2 - |\Delta|^2), \; r_L = |S_{12}S_{21}/(|S_{11}|^2 - |\Delta|^2)|$$

$$\Delta = \det[S] = S_{11}S_{22} - S_{12}S_{21}$$

Figure: Load stability regions in the potentially unstable case [Orfandis]



6. Coding Example

- Two-port network input & output matching

- Input data:

System reference impedance Z_0 (Ω) (real)

Source impedance Z_S' (Ω) in real/imaginary form

Load impedance Z_L' (Ω) in real/imaginary form

Two-port scattering parameters S_{11}, S_{21}, S_{12}, S_{22} in polar or rectangular form

- Output results:

A) Check stability

Calculate Rollett stability factor K and $|\Delta|$ (for unconditional stability $K > 1$ and $|\Delta| < 1$)

Calculate Edwards-Sinsky stability parameters μ_1 and μ_2 (for unconditional stability $\mu_1 > 1$ or $\mu_2 > 1$)

B) Unmatched case

Power gain G in natural unit

Available power gain G_A in natural unit

Transducer power gain G_T in natural unit

C) Matched case

Check if a conjugate-matching is possible at the input of the two-port network.

If possible, find the transformed source impedance Z_S.

Check if a conjugate-matching is possible at the output of the two-port network.

If possible, find the transformed source impedance Z_L.

Power gain G in natural unit

Available power gain G_A in natural unit

Transducer power gain G_T in natural unit

- Python code

```
# MW-05-Python-Ex1
# Two-port network input & output matching
# Input data:
#  System reference impedance z0
#  Source impedance (zs'), load impedace (zL') in real/imaginary format
#  Scattering parameters (s11, s21, s12, s22) in polar form (magnitude in natural unit, phase in degrees)
#    or in rectangular form
# Output results:
# A) Cheak stability of two-port
#  Calculate k and delta (Rollett stability factor)
#  Calculate mu1 and mu2 (Edwards-Sinsky stability parameters)
# B) Unmatched case
#  Power gain g, available power gain ga, transducer power gain gt
# C) Matched case
#  Transformed source impedance zs in ohms
#  Transformed load impedance zL in ohms
#  Power gain g, available power gain ga, transducer power gain gt
```

```
import math
import cmath
rad=3.141593/180
while True:
  z0=float(input('System reference impedance Z0 (ohm, real)='))
  rsp,xsp=map(float,input('Source impedance Zs=Rs+jXs (ohm): Rs, Xs=').split())
  zsp=complex(rsp,xsp)
  rLp,xLp=map(float,input('Load impedance ZL=RL+jXL (ohm): RL, XL=').split())
  zLp=complex(rLp,xLp)
  ipolar=int(input('S-parameter form(1:polar, 2:real/imaginary)='))
  if ipolar == 1:
    s11m,s11p=map(float,input('Mag(S11), phase(S11)(deg)=').split())
    s12m,s12p=map(float,input('Mag(S12), phase(S12)(deg)=').split())
    s21m,s21p=map(float,input('Mag(S21), phase(S21)(deg)=').split())
    s22m,s22p=map(float,input('Mag(S22), phase(S22)(deg)=').split())
    s11=cmath.rect(s11m,s11p*rad)
    s12=cmath.rect(s12m,s12p*rad)
    s21=cmath.rect(s21m,s21p*rad)
    s22=cmath.rect(s22m,s22p*rad)
  elif ipolar == 2:
    s11r,s11i=map(float,input('Re(S11), Im(S11)=').split())
    s12r,s12i=map(float,input('Re(S12), Im(S12)=').split())
    s21r,s21i=map(float,input('Re(S21), Im(S21)=').split())
    s22r,s22i=map(float,input('Re(S22), Im(S22)=').split())
    s11=complex(s11r,s11i)
    s12=complex(s12r,s12i)
    s21=complex(s21r,s21i)
    s22=complex(s22r,s22i)
```

```
# STABILITY FACTOR:
  print(' ')
  d=s11*s22-s12*s21
  k=(1-abs(s11)**2-abs(s22)**2+abs(d)**2)/(2*abs(s12*s21))
  mu1=(1-abs(s11)**2)/(abs(s22-d*s11.conjugate())+abs(s12*s21)) # Edward stability factor
  mu2=(1-abs(s22)**2)/(abs(s11-d*s22.conjugate())+abs(s12*s21)) # Edward stability factor
  print('Rollett Stability factor: K, abs(det)=',k,abs(d))
  print('Edwards-Sinsky stability factor: mu1, mu2=',mu1,mu2)
# BEFORE MATCHING:
#  Reflection coefficient
  R_s=(zsp-z0)/(zsp+z0) # Source reflection coefficient
  R_L=(zLp-z0)/(zLp+z0) # Load reflection coefficient
  R_in=s11+s12*s21*R_L/(1-s22*R_L) # Input reflection looking into port 1 of the two-port network
  R_out=s22+s12*s21*R_s/(1-s11*R_s) # Output reflection looking into port 2 of the two-port network
#  Power gain
  gp=abs(s21)**2*(1-abs(R_L)**2)/(abs(1-s22*R_L)**2*(1-abs(R_in)**2)) # power gain
  ga=abs(s21)**2*(1-abs(R_s)**2)/ (abs(1-s11*R_s)**2*(1-abs(R_out)**2)) # available power gain
  gt= abs(s21)**2*(1-abs(R_s)**2)*(1-abs(R_L)**2)/ (abs(1-s22*R_L)**2*abs(1-R_s*R_in)**2)
  print('POWER GAIN BEFORE MATCHING:')
  print('  Operating power gain  Gp=',gp)
  print('  Available power gain  Ga=',ga)
  print('  Transducer power gain Gt=',gt)
# AFTER CONJUGATE-MATCHING:
  print('POWER GAIN AFTER MATCHING')
  b1=1+abs(s11)**2-abs(s22)**2-abs(d)**2
  b2=1+abs(s22)**2-abs(s11)**2-abs(d)**2
  c1=s11-d*s22.conjugate() # complex class conjugate function
  c2=s22-d*s11.conjugate()
```

```
#  Check if the input side can be matched.
  ds=b1**2-4*abs(c1)**2
  if ds > 0:
    R_s=(b1-math.sqrt(ds))/(2*c1) # Transformed source reflection coefficient
    print('  Source side can be conjugate-matched.')
    zs=(1+R_s)*z0/(1-R_s)
    print('    Transformed source impedance: Zg=',zs)
  else:
    print('  Source side cannot be conjucate-matched.')
    print('    Source impedance remains unchanged.')
#  Check if the output side can be matched.
  dL=b2**2-4*abs(c2)**2
  if dL > 0:
    R_L=(b2-math.sqrt(dL))/(2*c2) # Transformed load reflection coeffcient
    print('  Load side can be conjugate-matched.')
    zL=(1+R_L)*z0/(1-R_L)
    print('    Transformed load impedance:   ZL=',zL)
  else:
    print('  Load side can be conjugate-matched.')
    print('    Load impedance remains unchanged.')
#  Reflecton coefficient
  R_in=s11+s12*s21*R_L/(1-s22*R_L) # Input reflection looking into port 1 of the two-port network
  R_out=s22+s12*s21*R_s/(1-s11*R_s) # Output reflection looking into port 2 of the two-port network
```

```
#  Power gain
  gp=abs(s21)**2*(1-abs(R_L)**2)/(abs(1-s22*R_L)**2*(1-abs(R_in)**2)) # power gain
  ga=abs(s21)**2*(1-abs(R_s)**2)/ (abs(1-s11*R_s)**2*(1-abs(R_out)**2)) # available power gain
  gt= abs(s21)**2*(1-abs(R_s)**2)*(1-abs(R_L)**2)/ (abs(1-s22*R_L)**2*abs(1-R_s*R_in)**2)
  print('  Operating Power gain  Gp=',gp)
  print('  Available power gain  Ga=',ga)
  print('  Transducer power gain Gt=',gt)
  print('  ')
```

- Code Execution:

```
System reference impedance Z0 (ohm, real)=
50
Source impedance Zs=Rs+jXs (ohm): Rs, Xs=
20 -30
Load impedance ZL=RL+jXL (ohm): RL, XL=
200 1000
S-parameter form(1:polar, 2:real/imaginary)=
2
Re(S11), Im(S11)=
0.60 -0.54
Re(S12), Im(S12)=
0.068 0.037
Re(S21), Im(S21)=
-0.22 1.14
Re(S22), Im(S22)=
0.12 -0.40

Rollett Stability factor: K, abs(det)= 1.7887870198179 44 0.3841293167671533
Edwards-Sinsky stability factor: mu1, mu2= 1.5700180443335303 1.1138680355350339
POWER GAIN BEFORE MATCHING:
  Operating power gain  Gp= 0.12422985810190754
  Available power gain  Ga= 0.5637991381007336
  Transducer power gain Gt= 0.021884922225449903
```

```
POWER GAIN AFTER MATCHING
 Source side can be conjugate-matched.
  Transformed source impedance: Zg= (32.66202172271324+112.79263043640468j)
 Load side can be conjugate-matched.
  Transformed load impedance:   ZL= (30.63645680478217+29.551735448459848j)
 Operating Power gain  Gp= 4.5837059513206855
 Available power gain  Ga= 4.583705951320686
 Transducer power gain Gt= 4.5837059513206855

System reference impedance Z0 (ohm, real)=
50
Source impedance Zs=Rs+jXs (ohm): Rs, Xs=
20 -30
Load impedance ZL=RL+jXL (ohm): RL, XL=
200 1000
S-parameter form(1:polar, 2:real/imaginary)=
1
Mag(S11), phase(S11)(deg)=
0.81 -42
Mag(S12), phase(S12)(deg)=
0.077 -28.6
Mag(S21), phase(S21)(deg)=
1.16 100.9
Mag(S22), phase(S22)(deg)=
0.42 -73.3
```

```
Rollett Stability factor: K, abs(det)= 1.9673844975622021 0.428898084216416
Edwards-Sinsky stability factor: mu1, mu2= 2.1140811103219113 1.1450487524927206
POWER GAIN BEFORE MATCHING:
  Operating power gain  Gp= 0.16881040993356675
  Available power gain  Ga= 0.5791607049162761
  Transducer power gain Gt= 0.020806486149065 44
POWER GAIN AFTER MATCHING
  Source side can be conjugate-matched.
    Transformed source impedance: Zg= (41.33371075888385+116.13301647188283j)
  Load side can be conjugate-matched.
    Transformed load impedance:   ZL= (48.36785317487852+23.141966365978412j)
  Operating Power gain  Gp= 4.114225752357467
  Available power gain  Ga= 4.1142257523574655
  Transducer power gain Gt= 4.1142257523574655

System reference impedance Z0 (ohm, real)=** Process Stopped **

Press Enter to exit terminal
```

Fin

(End)