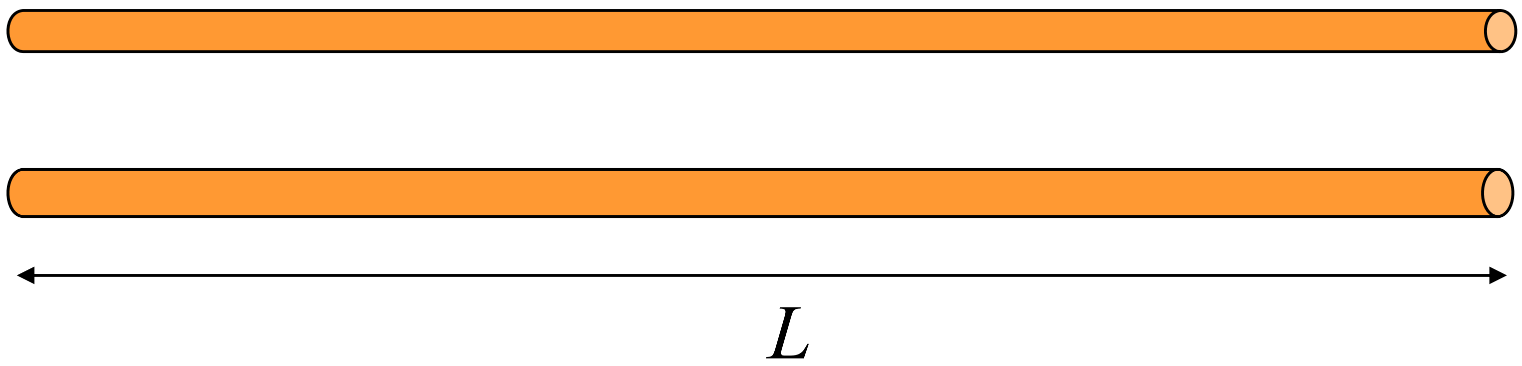
Transmission Lines 1

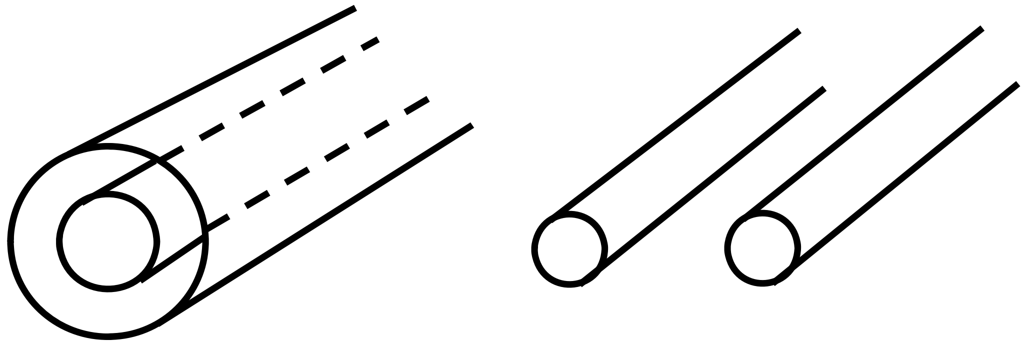
Transmission-Line Theory

We need transmission-line theory whenever the length of a line is significant compared to a wavelength.



Transmission Line

2 conductors: c1=signal, c2=ground



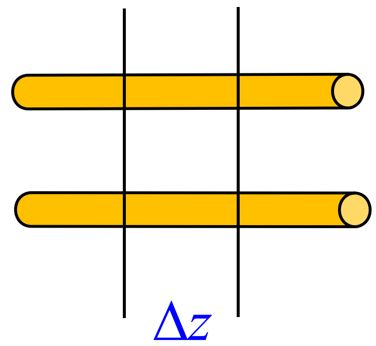
4 per-unit-length parameters:

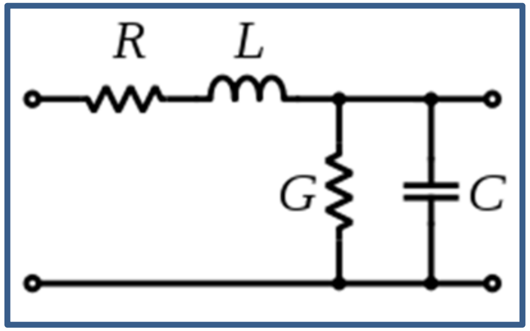
C = capacitance/length [F/m]

L = inductance/length [H/m]

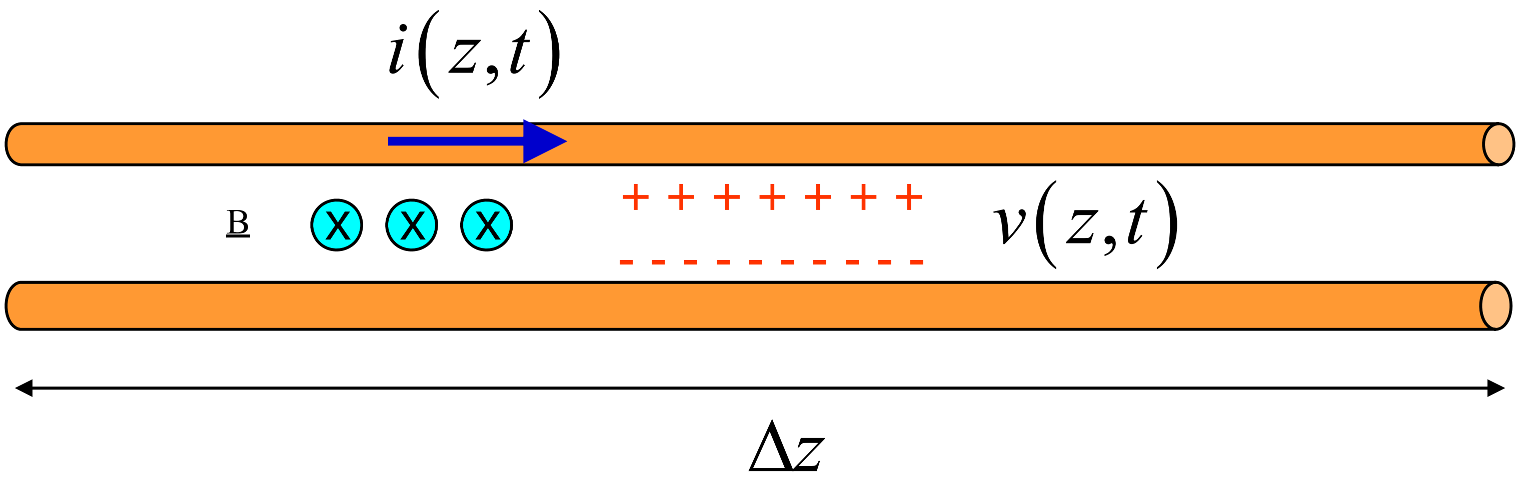
R = resistance/length [Ω/m]

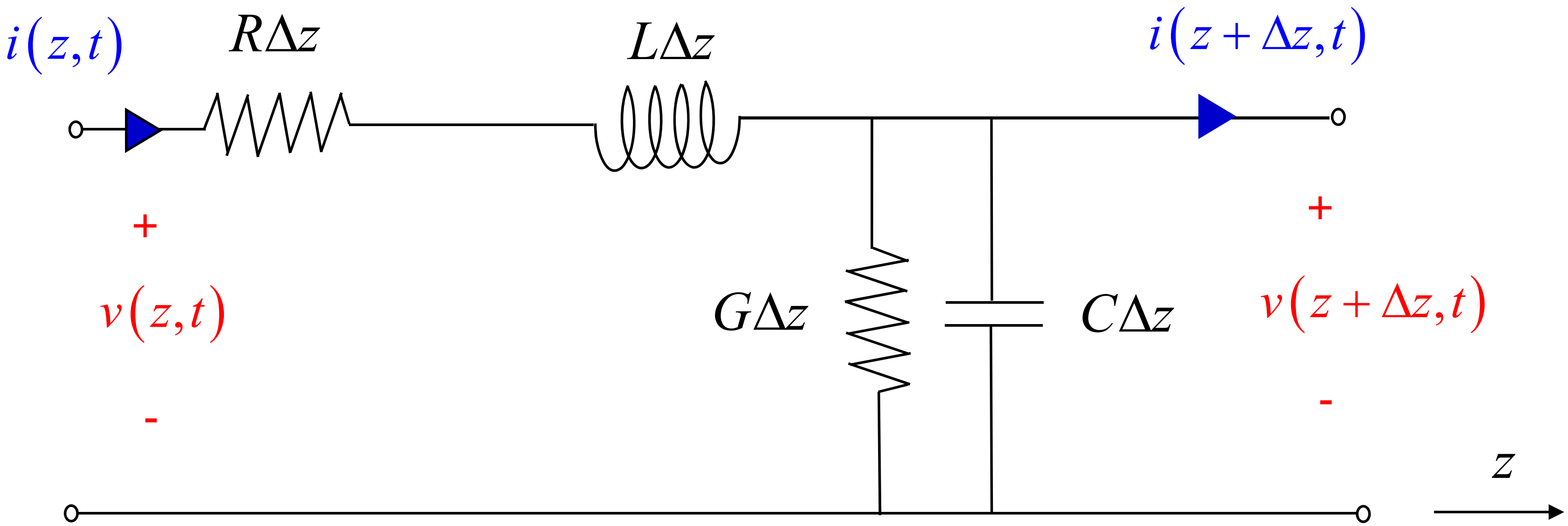
G = conductance/length [℧/m or S/m]





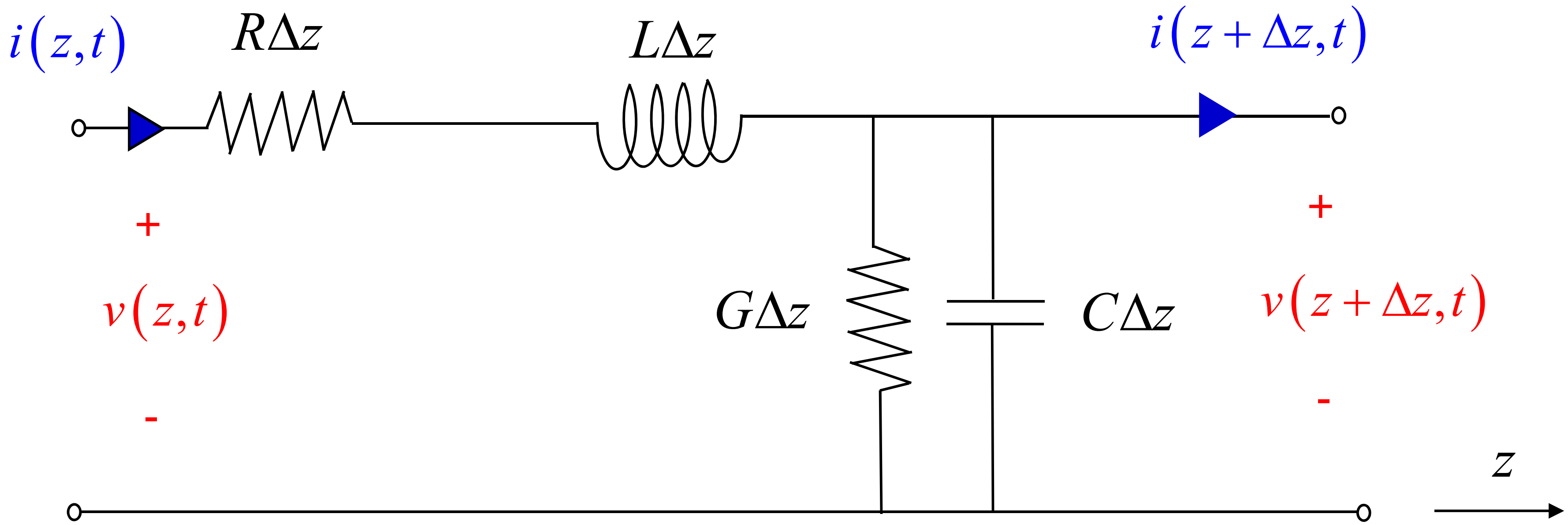
Transmission Line (cont.)





Note: There are equal and opposite currents on the two conductors.
(We only need to work with the current on the top conductor, since we have chosen to put all of the series elements there.)

Transmission Line (cont.)



$$v(z,t) = v(z+\Delta z,t) + i(z,t)R\Delta z + L\Delta z\frac{\partial i(z,t)}{\partial t}$$

$$i(z,t) = i(z+\Delta z,t) + v(z+\Delta z,t)G\Delta z + C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}$$

TEM Transmission Line (cont.)

Hence

$$\frac{v(z+\Delta z,t)-v(z,t)}{\Delta z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{i(z+\Delta z,t)-i(z,t)}{\Delta z} = -Gv(z+\Delta z,t) - C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

Now let $\Delta z \to 0$:

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z} = -Gv - C\frac{\partial v}{\partial t}$$

"Telegrapher's Equations"

TEM Transmission Line (cont.)

To combine these, take the derivative of the first one with respect to z:

$$\frac{\partial^2 v}{\partial z^2} = -R\frac{\partial i}{\partial z} - L\frac{\partial}{\partial z}\left(\frac{\partial i}{\partial t}\right)$$

$$= -R\frac{\partial i}{\partial z} - L\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right)$$

Switch the order of the derivatives.

$$= -R\left[-Gv - C\frac{\partial v}{\partial t}\right] - L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]$$

$$\frac{\partial i}{\partial z} = -Gv - C\frac{\partial v}{\partial t}$$

TEM Transmission Line (cont.)

$$\frac{\partial^2 v}{\partial z^2} = -R\left[-Gv - C\frac{\partial v}{\partial t}\right] - L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]$$

Hence, we have:

$$\frac{\partial^2 v}{\partial z^2} - (RG)v - (RC + LG)\frac{\partial v}{\partial t} - LC\left(\frac{\partial^2 v}{\partial t^2}\right) = 0$$

The same differential equation also holds for i.

Note: There is no exact solution in the time domain, in the lossy case.

TEM Transmission Line (cont.)

Time-Harmonic (=Sinusoidal) Waves:

$$\boxed{v(t) = A\cos(\omega t + \phi), \;\; V = Ae^{j\phi} \text{(phasor of } v)}, \quad \frac{\partial}{\partial t} \rightarrow j\omega$$

$$\frac{\partial^2 v}{\partial z^2} - (RG)v - (RC + LG)\frac{\partial v}{\partial t} - LC\left(\frac{\partial^2 v}{\partial t^2}\right) = 0$$

⇓

$$\frac{d^2 V}{dz^2} - (RG)V - (RC + LG)j\omega V - LC(-\omega^2)V = 0$$

TEM Transmission Line (cont.)

$$\frac{d^2V}{dz^2} = \left(RG\right)V + j\omega(RC + LG)V - \left(\omega^2 LC\right)V$$

Note that

$$RG + j\omega(RC + LG) - \omega^2 LC = (R + j\omega L)(G + j\omega C)$$

$$Z = R + j\omega L \;=\text{series impedance / unit length}$$

$$Y = G + j\omega C \;=\text{parallel admittance / unit length}$$

Then we can write:

$$\frac{d^2V}{dz^2} = (ZY)V$$

Phasor Domain

$$v(t) = V_0 \cos(\omega t + \phi)$$
$$= \Re\mathfrak{e}[\underbrace{V_0 e^{j\phi}} e^{j\omega t}]$$

Phasor counterpar $v(t)$

Time Domain		**Phasor Domain**
$v(t) = V_0 \cos \omega t$	$\leftrightarrow$	$\mathbf{V} = V_0$
$v(t) = V_0 \cos(\omega t + \phi)$	$\leftrightarrow$	$\mathbf{V} = V_0 e^{j\phi}.$

If $\phi = -\pi/2$,

$$v(t) = V_0 \cos(\omega t - \pi/2) \quad \leftrightarrow \quad \mathbf{V} = V_0 e^{-j\pi/2}.$$

Time and Phasor Domain

$x(t)$		$\mathbf{X}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j\phi}$
$-A\cos(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi\pm\pi)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}=-jA$
$A\sin(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi-\pi/2)}$
$-A\sin(\omega t+\phi)$	$\leftrightarrow$	$Ae^{j(\phi+\pi/2)}$
$\frac{d}{dt}(x(t))$	$\leftrightarrow$	$j\omega\mathbf{X}$
$\frac{d}{dt}[A\cos(\omega t+\phi)]$	$\leftrightarrow$	$j\omega Ae^{j\phi}$
$\int x(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\mathbf{X}$
$\int A\cos(\omega t+\phi)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j\phi}$

It is much easier to deal with exponentials in the phasor domain than sinusoidal relations in the time domain

Just need to track magnitude/phase, knowing that everything is at frequency ω

Complex Numbers

We will find it is useful to represent sinusoids as complex numbers

$$j = \sqrt{-1}$$

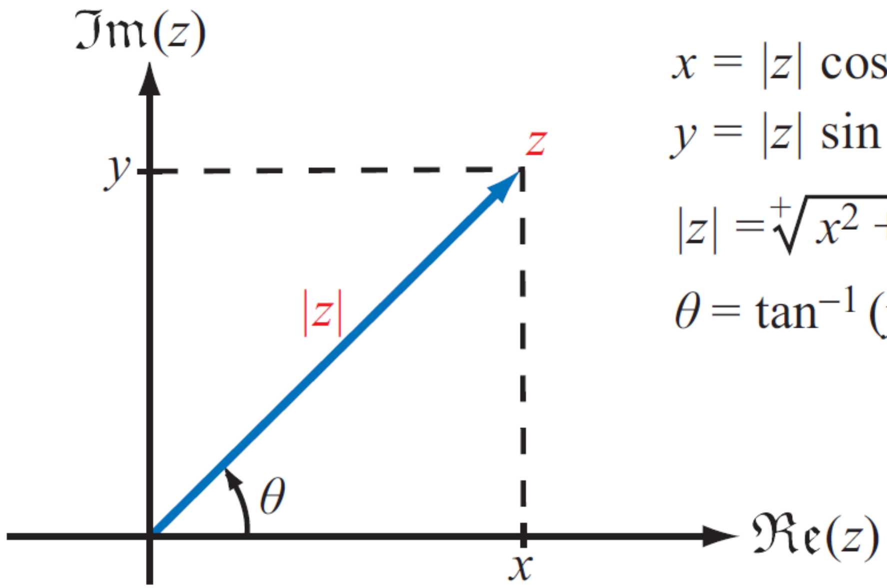
$$z = x + jy$$

Rectangular coordinates

$$z = |z|\angle\theta = |z|e^{j\theta}$$

Polar coordinates

$$\mathrm{Re}(z) = x$$
$$\mathrm{Im}(z) = y$$



$$x = |z| \cos \theta$$
$$y = |z| \sin \theta$$
$$|z| = \sqrt[+]{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$

Relations based on Euler's Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

TEM Transmission Line (cont.)

Define $\gamma^2 \equiv ZY$ Then $\dfrac{d^2V(z)}{dz^2} = \gamma^2 V(z)$

Solution:

$$V(z) = Ae^{-\gamma z} + Be^{+\gamma z}$$

$Ae^{-\gamma z}$: voltage wave propagating in $+z$ direction

$Be^{+\gamma z}$: voltage wave propagating in $-z$ direction

γ is called the "propagation constant".

We have:

$$\gamma = \left[(R + j\omega L)(G + j\omega C)\right]^{1/2}$$

Question: Which sign of the square root is correct?

TEM Transmission Line (cont.)

$$\gamma = \left[(R + j\omega L)(G + j\omega C) \right]^{1/2}$$

We choose the principal square root.

Principal square root: $z = re^{j\theta}$

$$\sqrt{z} = \sqrt{r}\, e^{j\theta/2} \quad -\pi < \theta \le \pi$$ (Note the square-root ("radical") symbol here.)

$$\Rightarrow \quad \text{Re}\left(\sqrt{z}\right) \ge 0$$

Hence

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad \text{Re}\,\gamma \ge 0$$

TEM Transmission Line (cont.)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Denote: $\gamma = \alpha + j\beta$

$\gamma =$ propagation constant $\left[1/\mathrm{m}\right]$

$\beta =$ phase constant $\left[\mathrm{rad/m}\right]$

$\alpha =$ attenuation contant $[\mathrm{np/m}]$

$$\alpha = \mathrm{Re}\,\gamma \geq 0$$

TEM Transmission Line (cont.)

Wave traveling in $+z$ direction:

$$\boxed{V(z) = Ae^{-\gamma z}} = Ae^{-\alpha z}e^{-j\beta z} \qquad (\gamma = \alpha + j\beta)$$

Wave is attenuating as it propagates.

Wave traveling in $-z$ direction:

$$\boxed{V(z) = Ae^{+\gamma z}} = Ae^{+\alpha z}e^{+j\beta z} \qquad (\gamma = \alpha + j\beta)$$

Wave is attenuating as it propagates.

TEM Transmission Line (cont.)

Attenuation in dB/m: $V(z) = Ae^{-\alpha z}e^{-j\beta z}$

$$\text{Attenuation in dB} = -20\log_{10}\left(\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right|\right)$$

$$= -20\log_{10}\left(\frac{|A|e^{-\alpha z}}{|A|}\right)$$

$$= -20(0.4343)(-\alpha z)$$

$$= 8.686\alpha z$$

Note: $\log_{10}(x) = 0.4343\ \ln(x)$

$$\alpha(\text{dB/m}) = 8.686\ \alpha(\text{Np/m})$$

$$\text{Np: neper,}\ \ 1\ \text{Np} = e^{-1} = 0.37$$

Wavenumber Notation

$$V(z) = Ae^{-\gamma z} = Ae^{-\alpha z}e^{-j\beta z} \qquad \gamma = \alpha + j\beta$$

$$V(z) = Ae^{-jk_z z} = Ae^{-\alpha z}e^{-j\beta z} \qquad k_z = \beta - j\alpha$$

$$\gamma = jk_z$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$ "(complex) propagation constant"

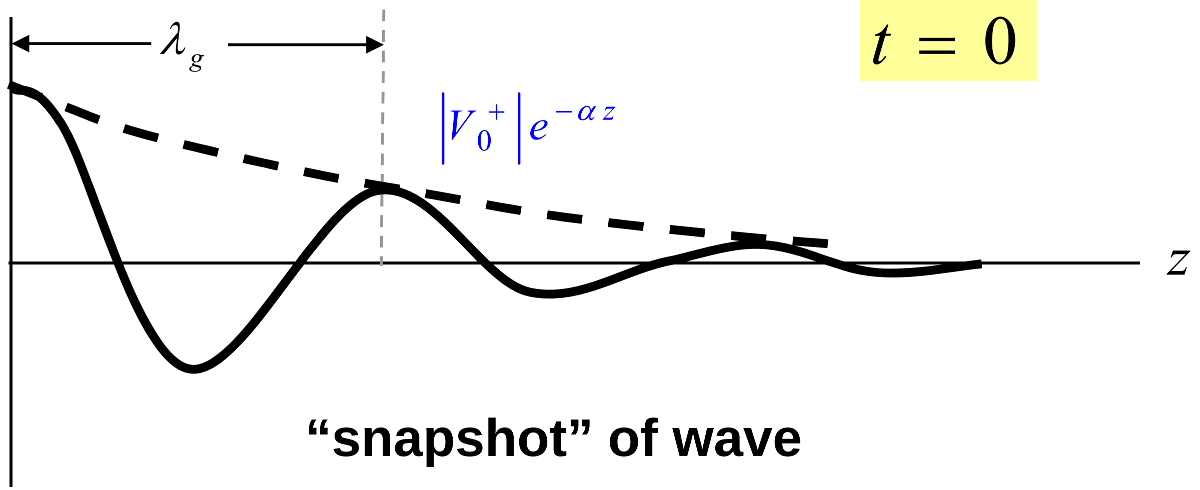
$$k_z = -j\sqrt{(R + j\omega L)(G + j\omega C)}$$ "(complex) wavenumber"

TEM Transmission Line (cont.)

Forward travelling wave (a wave traveling in the positive z direction):

$$V^{+}(z) = V_0^{+} e^{-\gamma z} = V_0^{+} e^{-\alpha z} e^{-j\beta z}$$

$$\begin{aligned} v^{+}(z,t) &= \mathrm{Re}\left\{ \left(V_0^{+} e^{-\alpha z} e^{-j\beta z} \right) e^{j\omega t} \right\} \\ &= \mathrm{Re}\left\{ \left(\left(\left| V_0^{+} \right| e^{j\phi} \right) e^{-\alpha z} e^{-j\beta z} \right) e^{j\omega t} \right\} \\ &= \left| V_0^{+} \right| e^{-\alpha z} \cos\left(\omega t - \beta z + \phi \right) \end{aligned}$$



"snapshot" of wave

The wave "repeats" when:

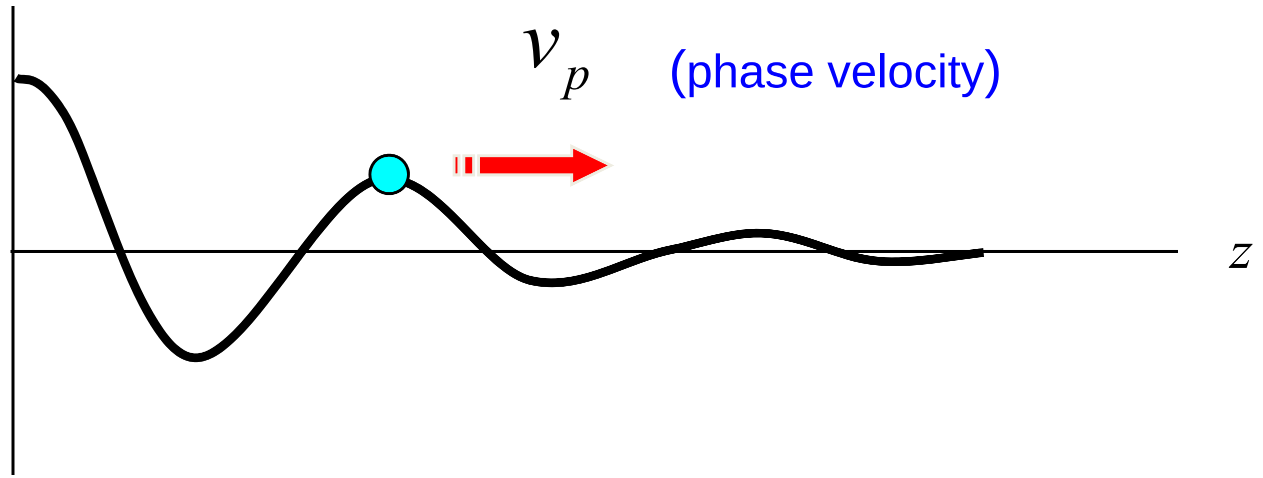
$$\beta \lambda_g = 2\pi$$

Hence:

$$\beta = \frac{2\pi}{\lambda_g}$$

Phase Velocity

Let's track the velocity of a fixed point on the wave (a point of constant phase), e.g., the crest of the wave.



$$v^{+}(z,t) = \left|V_0^{+}\right| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

Phase Velocity (cont.)

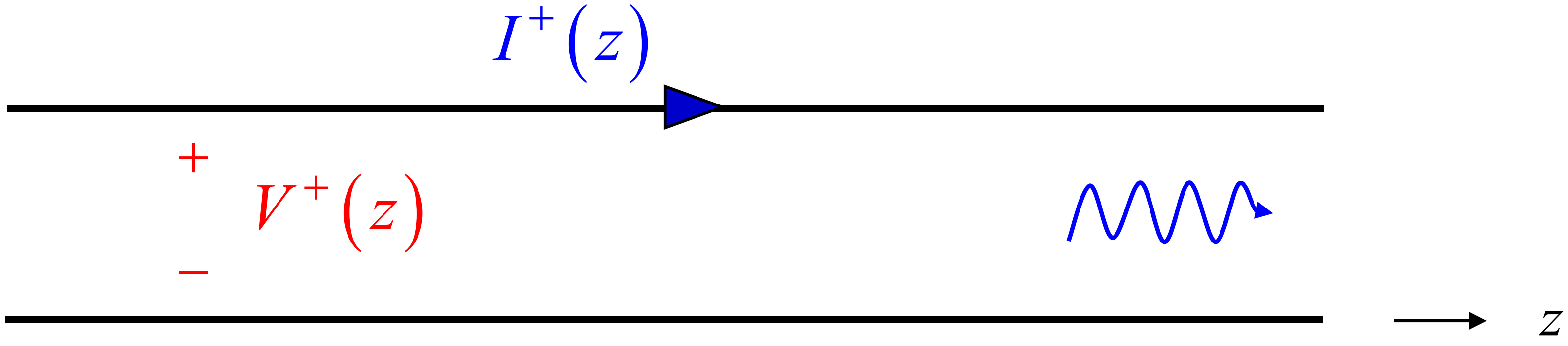
Set $\omega t - \beta z = \text{constant}$

$$\omega - \beta \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\omega}{\beta}$$

Hence $$v_p = \frac{\omega}{\beta}$$

Characteristic Impedance Z_0



Assumption: A wave is traveling in the positive z direction.

$$Z_0 \equiv \frac{V^+(z)}{I^+(z)} \text{ : characteristic impedance}$$

$$V^+(z) = V_0^+ e^{-\gamma z}$$

$$I^+(z) = I_0^+ e^{-\gamma z}$$

so $$Z_0 = \frac{V_0^+}{I_0^+}$$

(Note: Z_0 is a number, not a function of z.)

Characteristic Impedance Z_0 (cont.)

Use first Telegrapher's Equation:

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

so

$$\frac{dV}{dz} = -RI - j\omega LI = -ZI$$

Recall:
$$\begin{cases} V^{+}(z) = V_0^{+}e^{-\gamma z} \\ I^{+}(z) = I_0^{+}e^{-\gamma z} \end{cases}$$

Hence
$$-\gamma V_0^{+}e^{-\gamma z} = -ZI_0^{+}e^{-\gamma z}$$

Characteristic Impedance Z_0 (cont.)

From this we have: $$Z_0 = \frac{V_0^+}{I_0^+} = \frac{Z}{\gamma} = \frac{Z}{\sqrt{ZY}}$$

Use:

$$\left.\begin{aligned} Z &= R + j\omega L \\ Y &= G + j\omega C \end{aligned}\right\}$$ Both are in the first quadrant

$$\Rightarrow \gamma = \sqrt{ZY} \in \text{first quadrant}$$

$$\Rightarrow Z_0 = Z / \sqrt{ZY} \in \text{right - half plane}$$

$$\Rightarrow \operatorname{Re} Z_0 \geq 0$$

$$\Rightarrow \boxed{Z_0 = \sqrt{\frac{Z}{Y}}}$$ (principal square root)

Characteristic Impedance Z_0 (cont.)

Hence, we have

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

RLGC from Z_0 and γ

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \equiv \frac{1}{Y_0}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma Z_0 = R + j\omega L, \ \gamma Y_0 = G + j\omega C$$

$$R = \mathrm{Re}(\gamma Z_0)$$

$$L = \mathrm{Im}(\gamma Z_0) / \omega$$

$$G = \mathrm{Re}(\gamma Y_0)$$

$$C = \mathrm{Im}(\gamma Y_0) / \omega$$

General Case (Waves in Both Directions)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$= \left|V_0^+\right| e^{j\phi^+} e^{-\alpha z} e^{-j\beta z} + \left|V_0^-\right| e^{j\phi^-} e^{+\alpha z} e^{+j\beta z}$$

Wave in $+z$ direction

Wave in $-z$ direction

In the time domain:

$$v(z,t) = \mathrm{Re}\left\{V(z) e^{j\omega t}\right\}$$

$$= \left|V_0^+\right| e^{-\alpha z} \cos\left(\omega t - \beta z + \phi^+\right)$$

$$+ \left|V_0^-\right| e^{+\alpha z} \cos\left(\omega t + \beta z + \phi^-\right)$$

Backward and Forward Waves

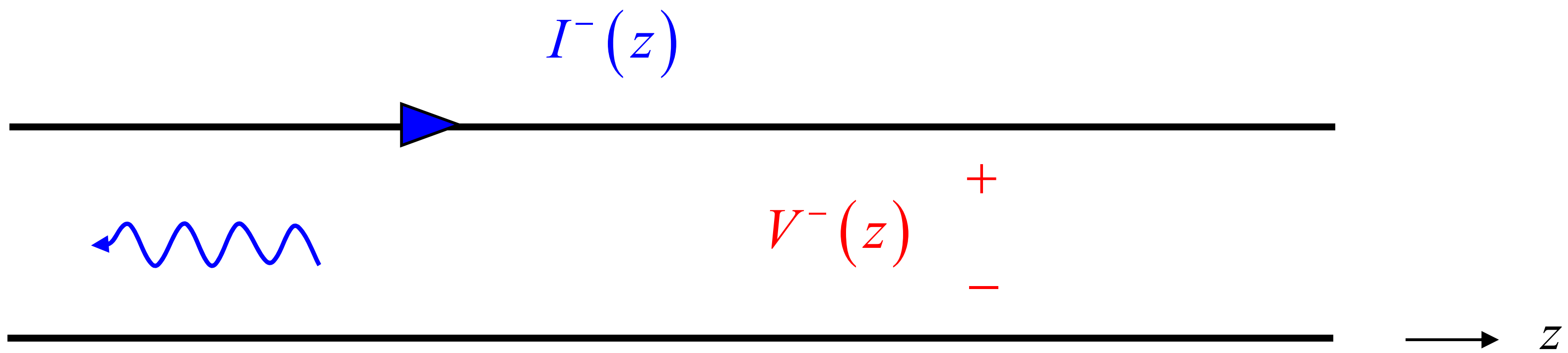
$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \quad \rightarrow \quad V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \quad \rightarrow \quad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

Forward wave

Backward wave

Backward-Traveling Wave

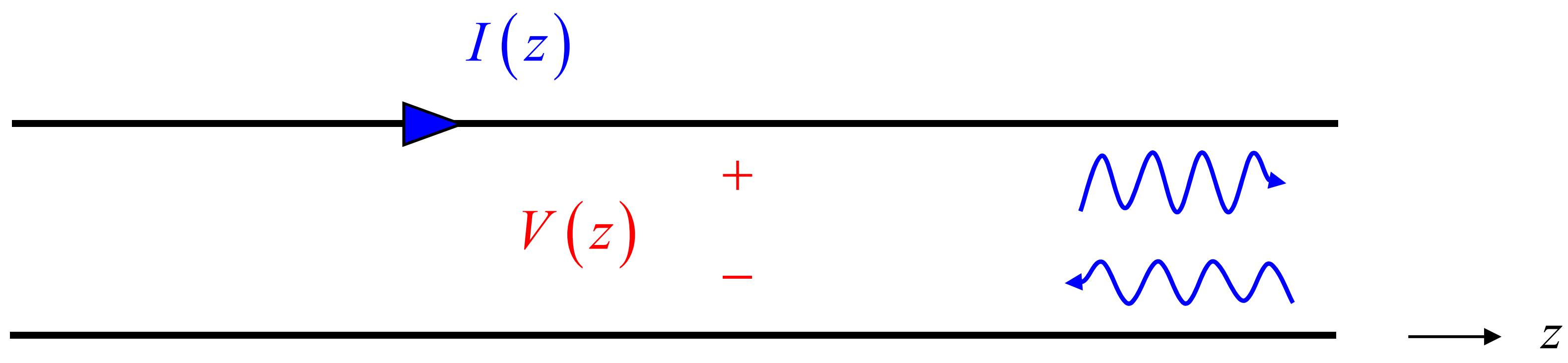


A wave is traveling in the <u>negative</u> z direction.

$$\frac{V^-(z)}{-I^-(z)} = Z_0 \qquad \text{so} \qquad \frac{V^-(z)}{I^-(z)} = -Z_0$$

Note:
The <u>reference directions</u> for voltage and current are chosen the same as for the forward wave.

General Case



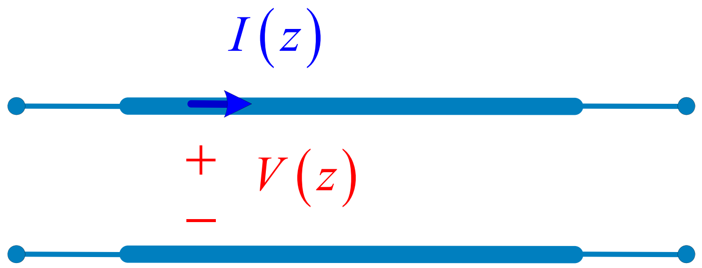
Most general case:

A general superposition of forward and backward traveling waves:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{1}{Z_0}\left[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right]$$

Summary of Basic TL formulas



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Guided wavelength:

$$\lambda_g = \frac{2\pi}{\beta} \; [\mathrm{m}]$$

Phase velocity:

$$v_p = \frac{\omega}{\beta} \; [\mathrm{m/s}]$$

Attenuation in $\mathrm{dB/m} = 8.686\alpha$

Small Loss Case

$$R << \omega L,\ G << \omega C$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(j\omega L)(j\omega C)}\sqrt{\left(1 - j\frac{R}{\omega L}\right)\left(1 - j\frac{R}{\omega C}\right)} \cong j\omega\sqrt{LC}\left(1 - j\frac{R}{2\omega L}\right)\left(1 - j\frac{G}{2\omega C}\right)$$

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + \frac{G}{Y_0}\right)$$

$$\beta = \omega\sqrt{LC} - \frac{RG}{4\omega\sqrt{LC}} < \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{j\omega L\left(1 - j\frac{R}{\omega L}\right)}{j\omega C\left(1 - j\frac{R}{\omega C}\right)}} = \sqrt{\frac{L}{C}}\left(1 - j\frac{R}{2\omega L}\right)\left(1 + j\frac{G}{2\omega C}\right)$$

$$= \sqrt{\frac{L}{C}}\left(1 + \frac{RG}{4\omega^2 LC}\right) - j\sqrt{\frac{L}{C}}\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)$$

Lossless Case

$$R = 0, \;\; G = 0$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= j\omega\sqrt{LC}$$

so

$$\alpha = 0$$
$$\beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta}$$

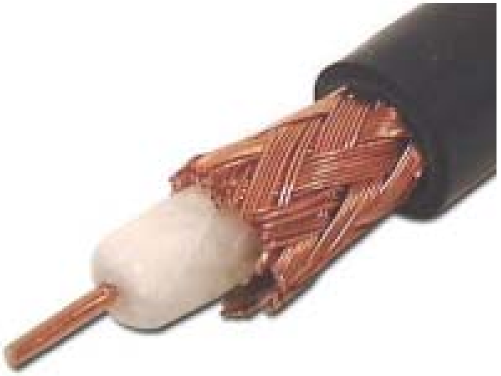
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \Rightarrow Z_0 = \sqrt{\frac{L}{C}}$$

$$v_p = \frac{1}{\sqrt{LC}}$$

(real and independent of freq.) (independent of freq.)

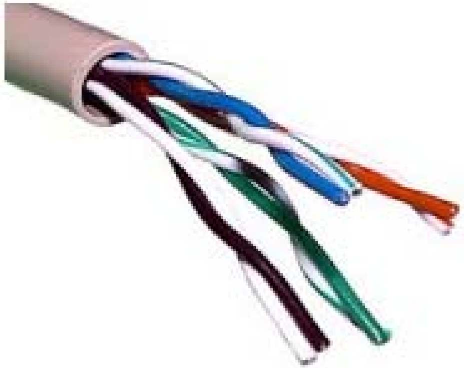
Example RLGC Parameters

RG-59 Coax



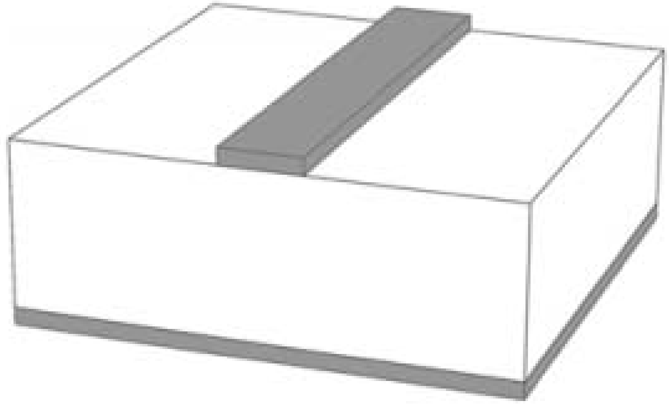
$$R = 36 \ \text{m}\Omega/\text{m}$$
$$L = 430 \ \text{nH/m}$$
$$G = 10 \ \mu\mho/\text{m}$$
$$C = 69 \ \text{pF/m}$$
$$Z_0 = 75 \ \Omega$$

CAT5 Twisted Pair



$$R = 176 \ \text{m}\Omega/\text{m}$$
$$L = 490 \ \text{nH/m}$$
$$G = 2 \ \mu\mho/\text{m}$$
$$C = 49 \ \text{pF/m}$$
$$Z_0 = 100 \ \Omega$$

Microstrip



$$R = 150 \ \text{m}\Omega/\text{m}$$
$$L = 364 \ \text{nH/m}$$
$$G = 3 \ \mu\mho/\text{m}$$
$$C = 107 \ \text{pF/m}$$
$$Z_0 = 50 \ \Omega$$

The Lossless Circular Coax

APPLIED EM

Fundamental Parameters (derived in EE 3321)

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \quad (\text{F/m})$$

$$L = \frac{\mu}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right] \quad (\text{H/m})$$

Attenuation Coefficient, α

$$\alpha = 0$$

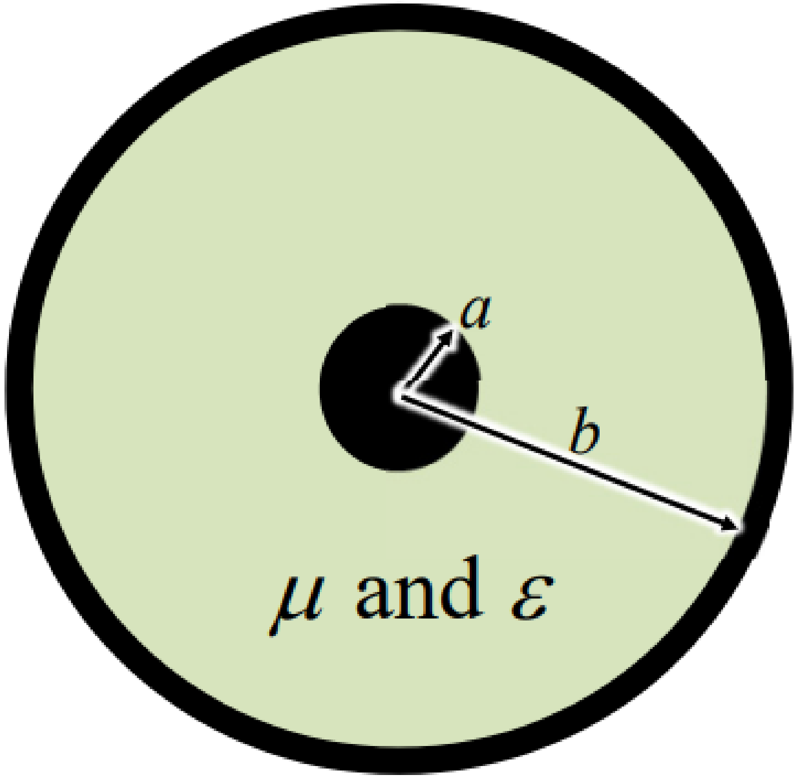
Phase Constant, β

$$\beta = \omega\sqrt{\mu\varepsilon}$$

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right) \qquad a \ll b$$

$$R_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right) \qquad X_0 = 0$$



Typical RLGC for RG-59 Coax at 2 GHz

The typical RG-59 coaxial cable operating at 2.0 GHz has the following RLGC parameters:

$$R = 36 \ \text{m}\Omega/\text{m}$$
$$L = 430 \ \text{nH/m}$$
$$G = 10 \ \mu\mho/\text{m}$$
$$C = 69 \ \text{pF/m}$$

Calculate the transmission line parameters γ, α, β, and Z_0.

Classify the line as lossless, weakly absorbing, distortionless, etc.

Our equations mostly utilize the angular frequency ω instead of the ordinary frequency f.

$$\omega = 2\pi f = 2\pi\left(2.0\times10^{9}\ \text{s}^{-1}\right) = \underline{12.5664\times10^{9}\ \text{rad/s}}$$

The characteristic impedance Z_0 is

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{(36\ \text{m}\Omega/\text{m}) + j\left(12.5664\times10^{9}\ \text{rad/s}\right)(430\ \text{nH/m})}{(10\ \mu\mho/\text{m}) + j\left(12.5664\times10^{9}\ \text{rad/s}\right)(69\ \text{pF/m})}}$$

$$= \boxed{78.94 + j1.92\times10^{-4}\ \Omega}$$

Note the imaginary part of Z_0 is very small indicating that our line is very low loss.

The complex propagation constant γ is

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{\begin{array}{l}\left[(36\ \text{m}\Omega/\text{m}) + j\left(12.5664\times10^{9}\ \text{rad/s}\right)(430\ \text{nH/m})\right] \\ \cdot\left[(10\ \mu\mho/\text{m}) + j\left(12.5664\times10^{9}\ \text{rad/s}\right)(69\ \text{pF/m})\right]\end{array}}$$

$$= \boxed{6.23\times10^{-4} + j68.45\ \text{m}^{-1}}$$

From this result, we read off α and β.

$$\gamma = \alpha + j\beta = 6.23\times10^{-4} + j68.45\ \text{m}^{-1}$$

$$\boxed{\alpha = 6.23\times10^{-4}\ \ \text{Np/m}}$$

Np is Nepers

$$\boxed{\beta = 68.45\ \ \text{rad/m}}$$

rad is radians

01-Python: Z0,gamma from R,L,G,C

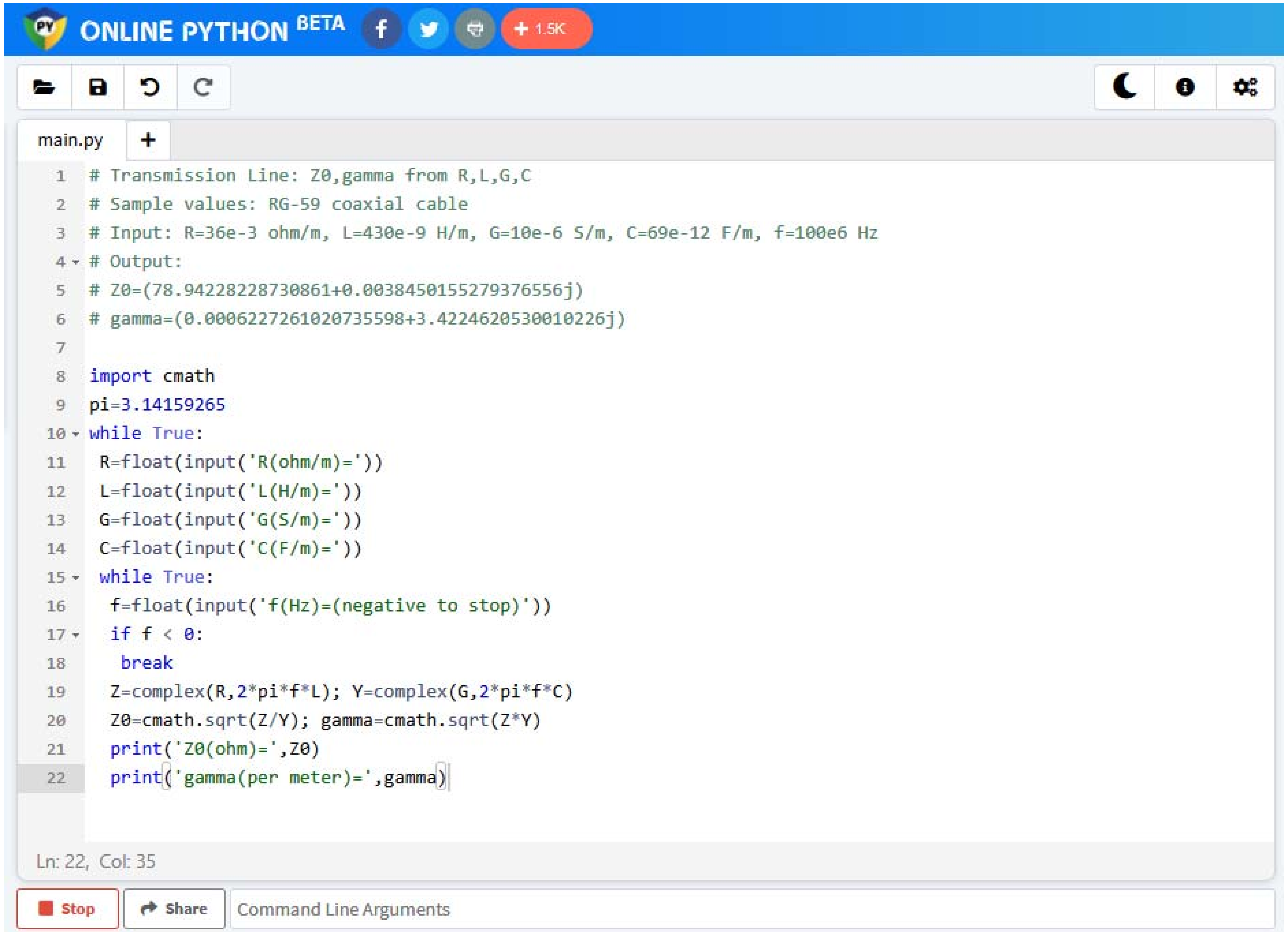
```
# Transmission Line: Z0,gamma from R,L,G,C
# Sample values: RG-59 coaxial cable
# Input: R=36e-3 ohm/m, L=430e-9 H/m, G=10e-6 S/m, C=69e-12 F/m, f=100e6 Hz
# Output:
# Z0=(78.94228228730861+0.0038450155279376556j)
# gamma=(0.0006227261020735598+3.4224620530010226j)

import cmath
pi=3.14159265
while True:
 R=float(input('R(ohm/m)='))
 L=float(input('L(H/m)='))
 G=float(input('G(S/m)='))
 C=float(input('C(F/m)='))
 while True:
  f=float(input('f(Hz)=(negative to stop)'))
  if f < 0:
   break
  Z=complex(R,2*pi*f*L); Y=complex(G,2*pi*f*C)
  Z0=cmath.sqrt(Z/Y); gamma=cmath.sqrt(Z*Y)
  print('Z0(ohm)=',Z0)
  print('gamma(per meter)=',gamma)
```

https://www.online-python.com/ 에 접속하여 소스코드를 위 창에 copy

[Run]아이콘 클릭

아래 창에 데이터 입력 prompt가 표시되면 키보드로 데이터 입력

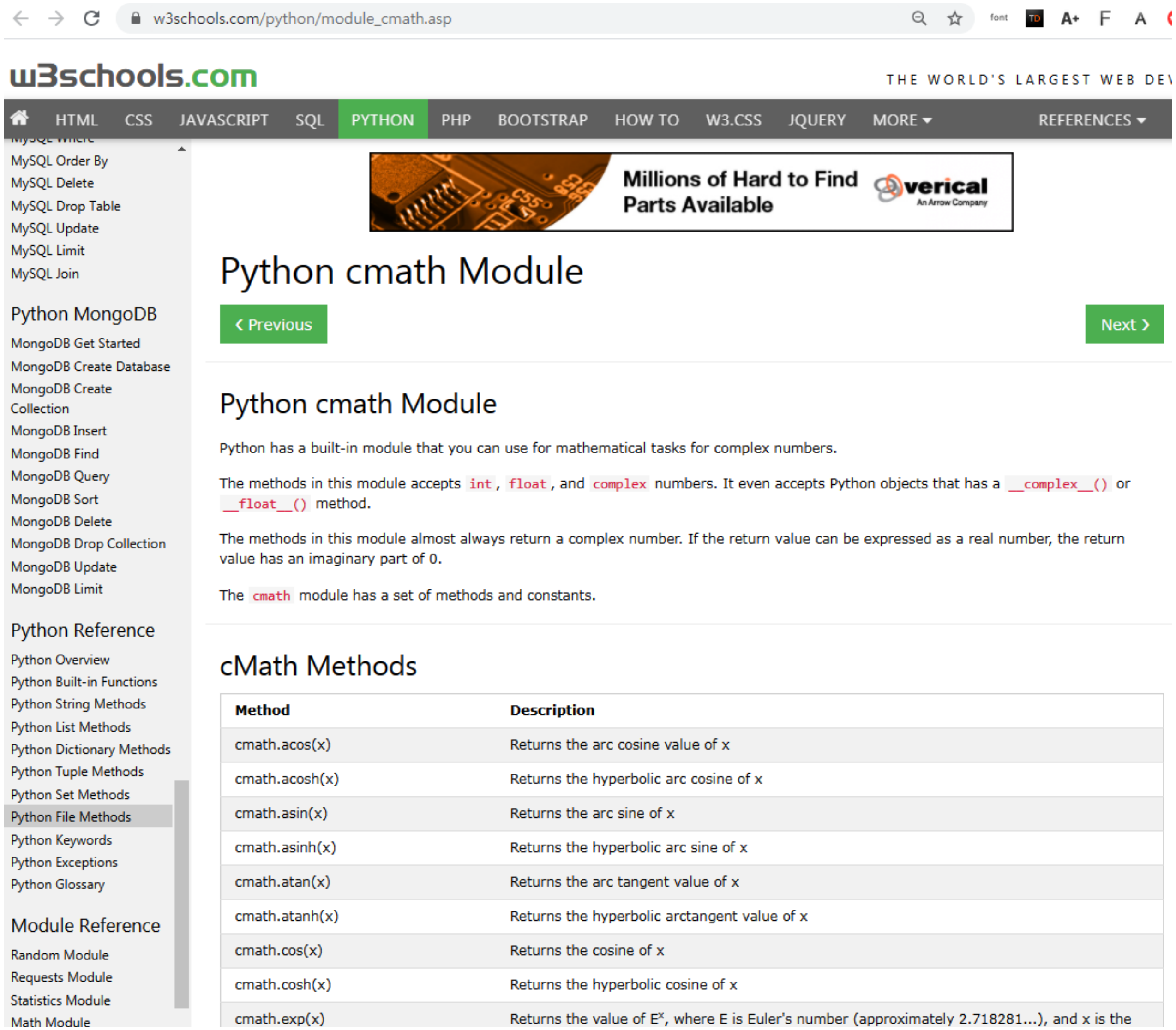
출력은 아래 창에 표시됨.



```
R(ohm/m)=
36e-3
L(H/m)=
430e-9
G(S/m)=
10e-6
C(F/m)=
16e-12
f(Hz)=(negative to stop)
100e6
Z0(ohm)= (163.9359080752083+0.07061310733582303j)
gamma(per meter)= (0.0009294786999524588+1.6480642065475533j)
f(Hz)=(negative to stop)
```

Python cmath 모듈 확인

- 구글에서 python cmath modules 입력
- 결과 중에서 적절한 것을 선택하여 열어 봄.



w3schools.com/python/module_cmath.asp

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Python cmath Module

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Python cmath Module

Python has a built-in module that you can use for mathematical tasks for complex numbers.

The methods in this module accepts `int`, `float`, and `complex` numbers. It even accepts Python objects that has a `__complex__()` or `__float__()` method.

The methods in this module almost always return a complex number. If the return value can be expressed as a real number, the return value has an imaginary part of 0.

The `cmath` module has a set of methods and constants.

cMath Methods

Method	Description
cmath.acos(x)	Returns the arc cosine value of x
cmath.acosh(x)	Returns the hyperbolic arc cosine of x
cmath.asin(x)	Returns the arc sine of x
cmath.asinh(x)	Returns the hyperbolic arc sine of x
cmath.atan(x)	Returns the arc tangent value of x
cmath.atanh(x)	Returns the hyperbolic arctangent value of x
cmath.cos(x)	Returns the cosine of x
cmath.cosh(x)	Returns the hyperbolic cosine of x
cmath.exp(x)	Returns the value of E^x, where E is Euler's number (approximately 2.718281...), and x is the

Fin

(End)