Fundamentals of OFDM Communication Technology

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Outline

- Fundamentals of OFDM – An Introduction
- OFDM System Design Considerations
- Key OFDM Receiver Functional Blocks
- Example: LTE OFDM PHY Layer

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Fundamentals of OFDM – An Introduction

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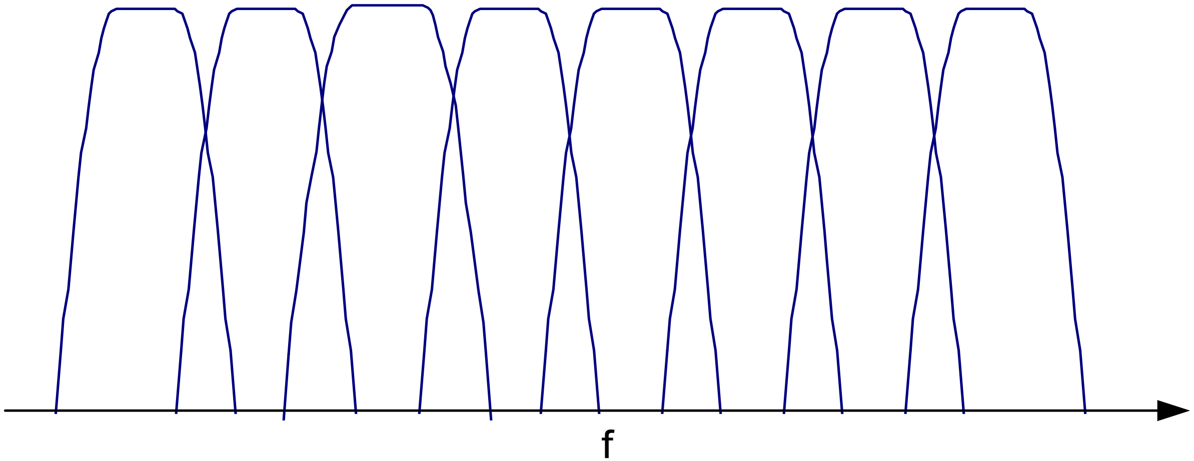
OFDM Signal Spectrum

- **OFDM vs. FDM**
- **To improve Spectrum efficiency, the multiple subcarrier spectrum should and can overlap as long as they are still orthogonal**
  - **Definition of Orthogonality:**

$$\int_{t_0}^{t_0+T} s_1(t)s_2^*(t)dt = 0$$



Signal spectrum of an early OFDM System

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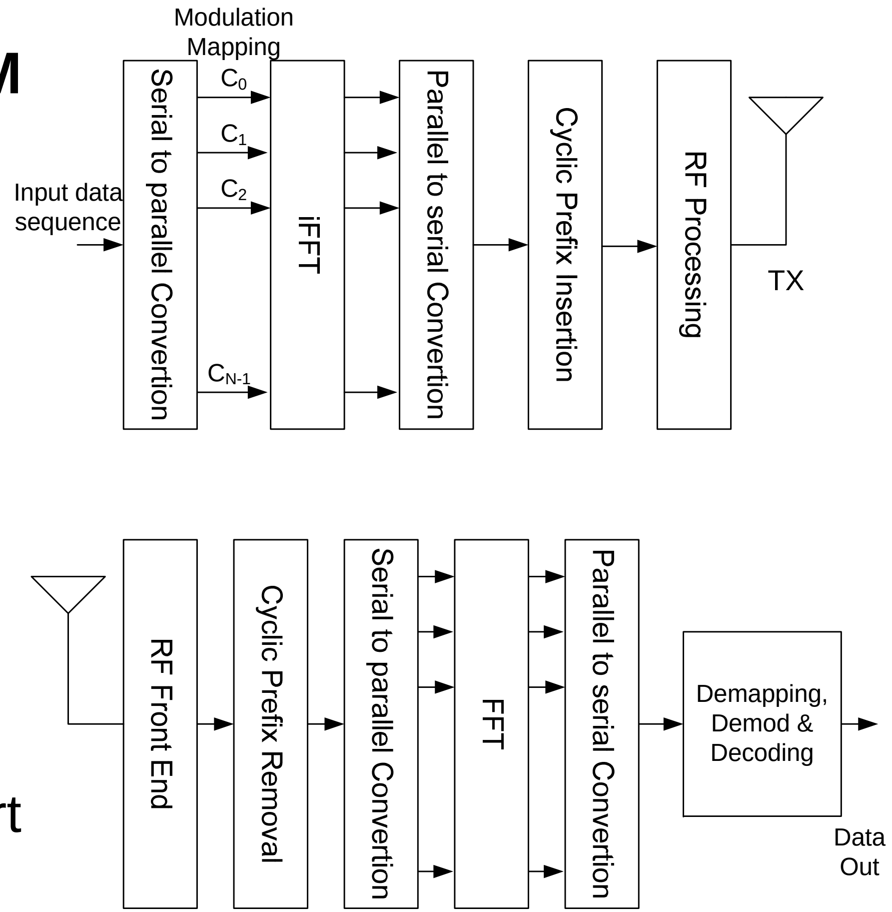
FFT/IFFT Based OFDM Systems

- **To reduce implementation complexity, the modern OFDM systems are FFT/IFFT based.**
  - A multicarrier data signal is effectively the Fourier Transform of the original serial data train and the banks of coherent demodulator is effectively the Inverse Fourier Transform (Salz and Weinstein, 1969)
  - A system realization was proposed by Weinstein and Ebert in 1971



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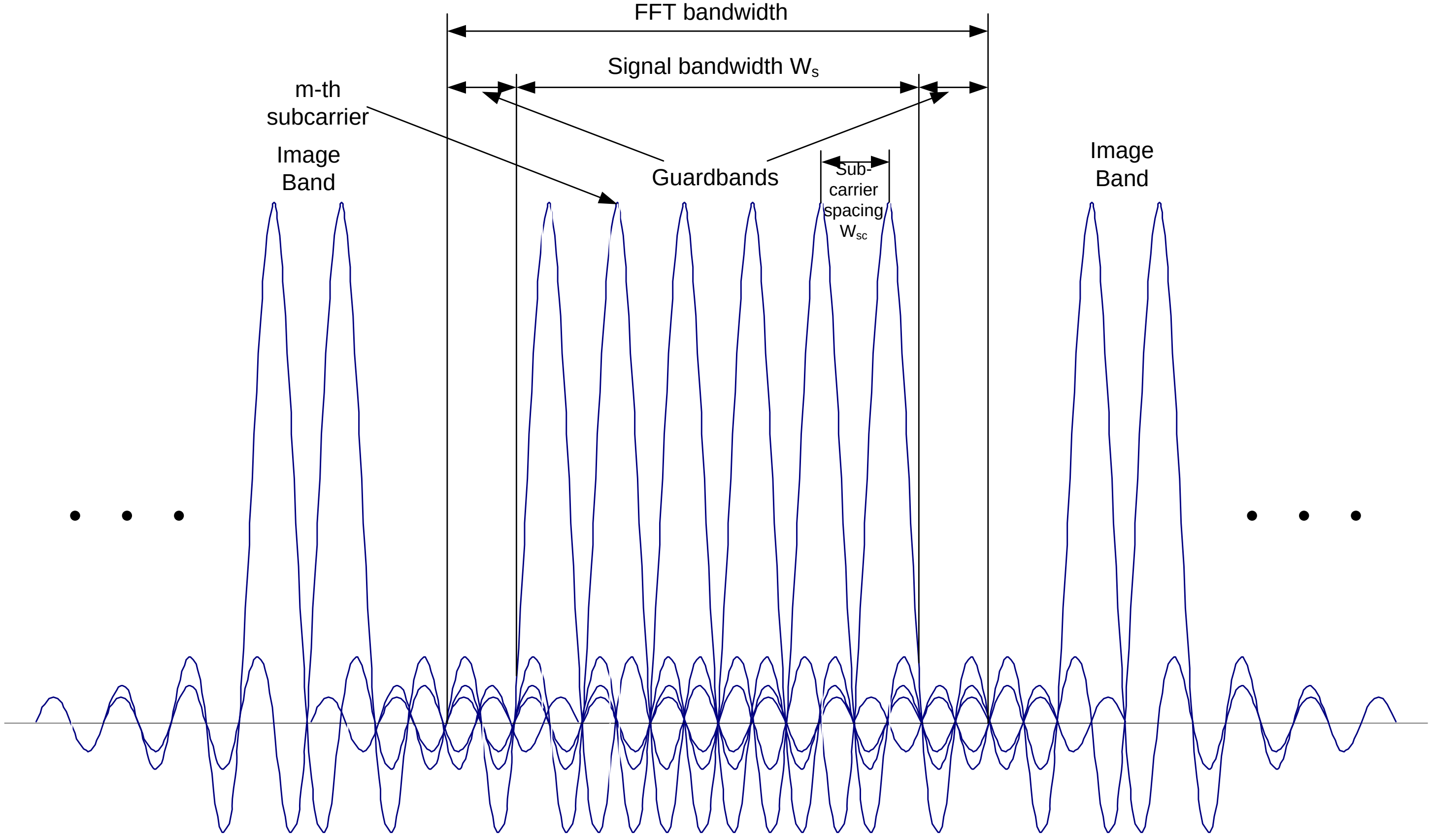
Spectrum of FFT based OFDM Signal (Digital)



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Characteristics of OFDM

- **OFDM is mainly useful for communication over multipath channels with long delay spread and at high SNR**
  - For such channels ISI becomes the major impairments
  - For such channels OFDM can be capacity achieving
- **Single carrier design cannot achieve capacity with known equalizer implementation for broadcasting channels**
  - OFDM and DFE without feedback error, or pre-coding, are equivalent
  - DFE without feedback error is practically impossible
  - Pre-coding is not applicable to broadcasting and communications over fast fading channels
- **For such channels OFDM receiver is easier to implement than a single carrier receiver that require a complicated equalizer or special design (frequency domain equalizer)**

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Advantages of OFDM (cont.)

- **In large cell single frequency (broadcasting) networks (SFN), the signals from different transmitters appear as multiple delayed copies of the transmitted signal to the receiver.**
  - Can have delay spreads up to, or greater than, 100 microseconds (as opposed to 5-6 microseconds in cellular networks).
  - OFDM is especially suitable for Broadcasting/Multicasting applications
- OFDM is specially suitable for MIMO implementation.
- Overhead in an OFDM System
  - Overhead due to cyclic Prefix (CP) – More overhead than that in a single carrier (SC) system.
  - Overhead due to pilot for channel estimation (also in SC)

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OFDM System Design Considerations

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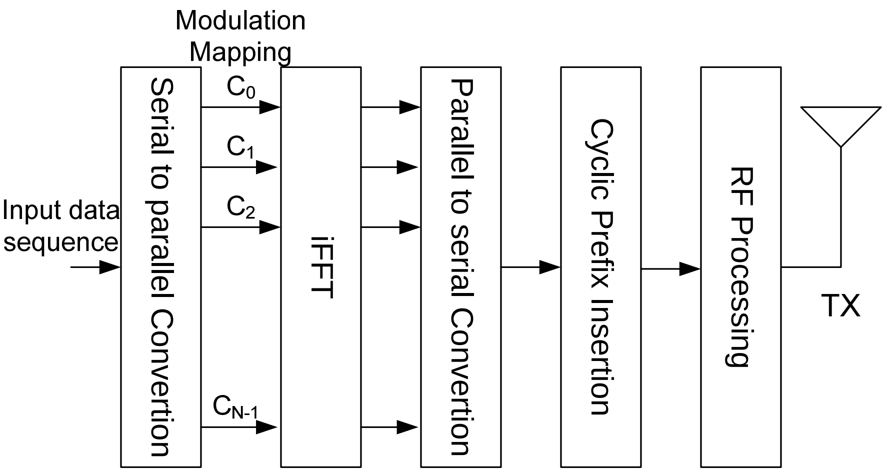
OFDM Transmitter Operation

- The input bit stream is divided into N substreams, mapped to N parallel modulation symbol streams and modulated onto N subcarriers of OFDM symbols, respectively.



- For each OFDM symbol, the modulation is performed by a 2^M point iFFT of the N modulation symbols after appending guard (zero) symbols on each sides and maybe also at DC.
- 2^M iFFT output samples are pre-extended by Cyclic Prefix (CP) samples
- CP extended iFFT outputs are converted to serial and to analog form, filtered and transmitted

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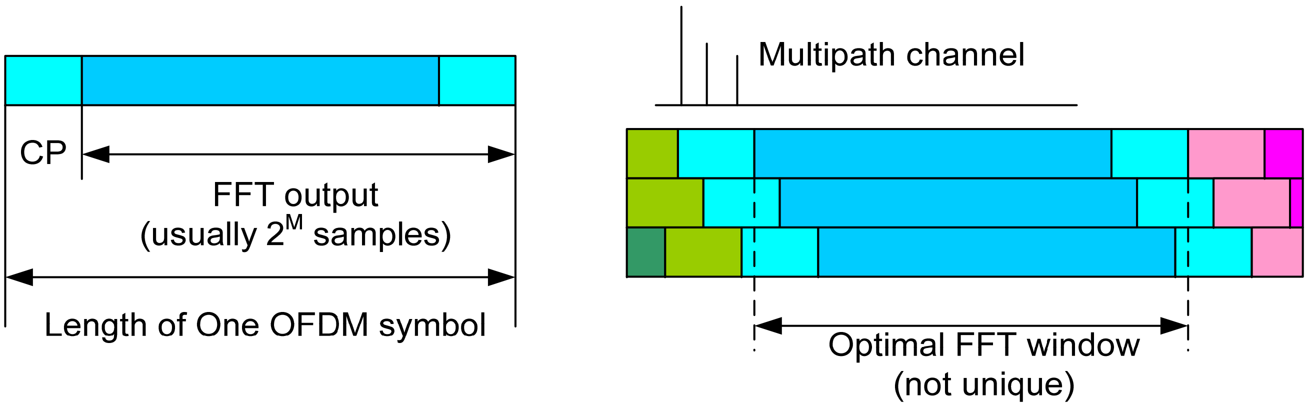
Cyclic Prefix (CP) in OFDM

- In a single path channel, each FFT output is equal to the transmitted symbol weighted by the path gain (no CP needed)
- In a multipath channel cyclic prefix is added into transmitted signal to mitigate the interference of the signal passing through different paths



Transmitted signal

Received signal

- The received signal samples in the FFT window contains one cycle of the (cyclically shifted) TX signal passing though each path

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Cyclic Prefix (CP) in OFDM – The Optimality of OFDM

- The FFT output is the sum of the copies of the corresponding TX symbol passing through all of the paths weighted by the path magnitudes and phases
- The magnitude of each FFT output is proportional to the channel frequency response of the corresponding subcarrier
- With powerful coding and proper power allocation of each subcarrier an OFDM receiver can approach the channel capacity minus CP and pilot overheads

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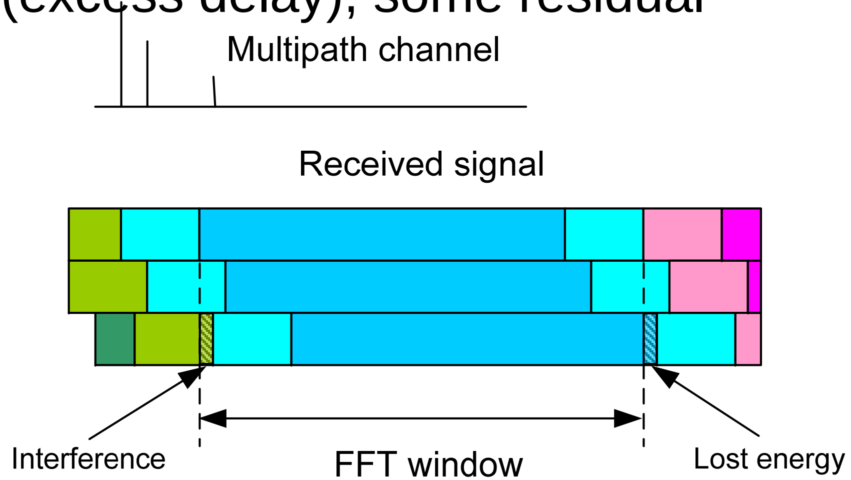
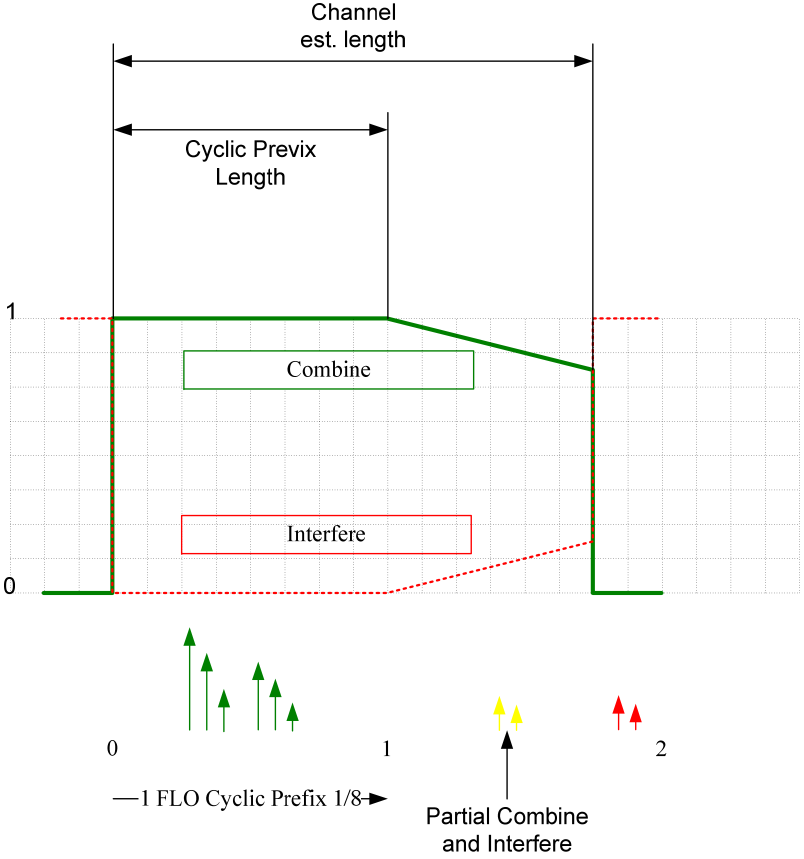
Cyclic Prefix (CP) in OFDM –the effect of excess delay

- As long as the CP is greater than the path delay spread, ICI/ISI due to multipath are totally eliminated
- If the delay spread is larger than CP (excess delay), some residual ICI/ISI will present, however
  - Only part of the excess delay will cause interference
  - Some performance degradation due to loss of energy
  - The loss is proportional to ratio of the excess delay energy to the total symbol energy (unlike the single carrier case where all ISI are interfering)
  - This is under the assumption the complete channel is estimated



Cyclic Prefix (CP) in OFDM –the effect of excess delay (2)

- **CP constitutes overhead**
- **Trade offs need to be made between longer CP and tolerance of interference**
- **CP is not the only factor that determines the forbearance of channel delay spread, channel estimation length is more important than CP length**



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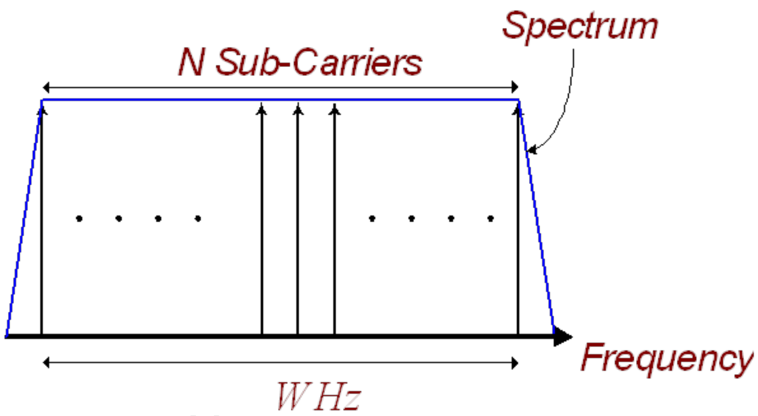
OFDM Signal Design Parameters

- Total number of subcarriers = $N_{data} + N_{guard}$ = 2^M (FFT size)
- (i)FFT sampling rate, aka chip rate = FFT Bandwidth



- Subcarrier spacing ΔF_{sc} = chip rate/ 2^M
- Signal bandwidth = ΔF_{sc} x N_{data} < FFT Bandwidth
- Purpose of guardband:
  - Facilitate the Transmitter implementation in meeting the spectrum mask
  - More importantly - avoiding alias after receiver sampling
  - In addition, simplify adjacent channel interference (ACI) rejection

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Impact of carrier frequency error in OFDM and single carrier systems

- **It is often stated that OFDM is more sensitive to carrier frequency error than single carrier**
- **This is true in the sense that the ideal receiver assumes the phase does not change during one OFDM symbol duration (coherent time = $1/W_{sc}$)**
- **The ideal single carrier receiver only need coherent time of $1/W_s$, which is much smaller than $1/W_{sc}$**
- **However, for a realistic pilot assisted coherent receiver, the required coherent time is determined by channel estimation (usually averaged over multiple samples)**
- **Conclusion – the difference is not as large as it appears**

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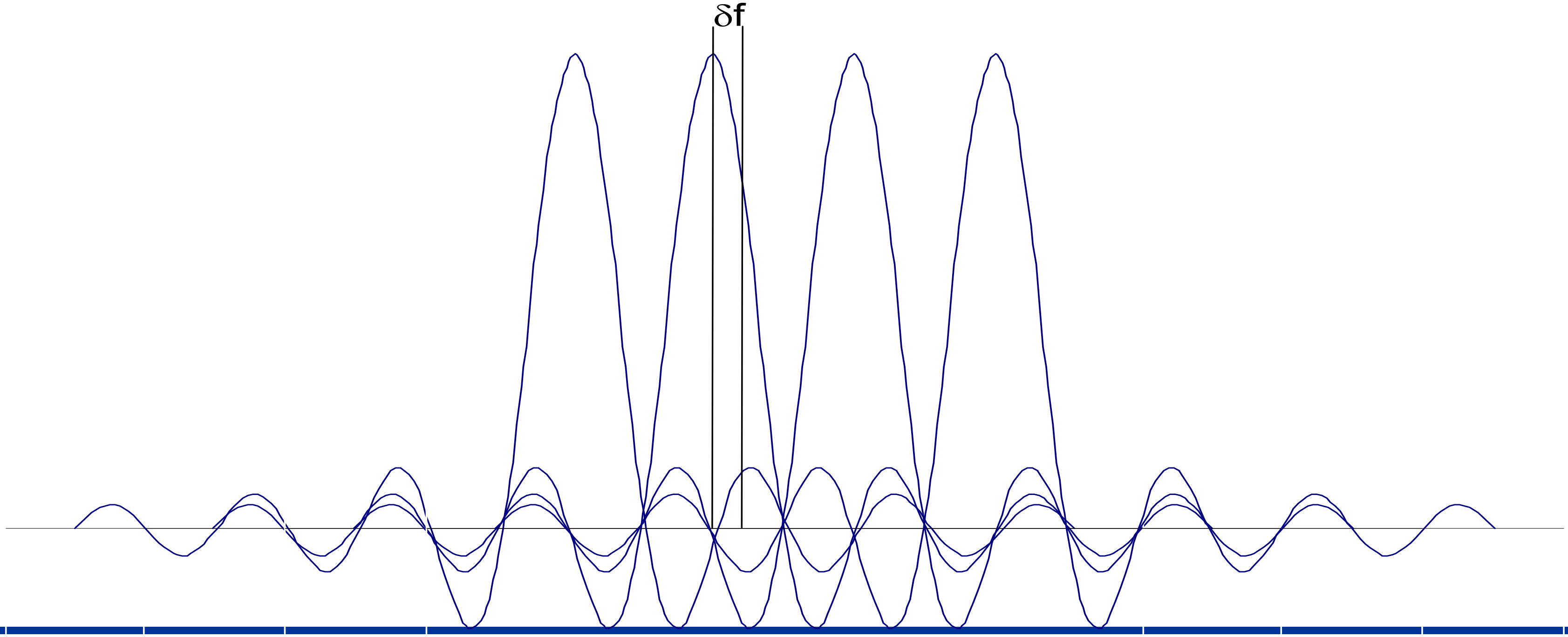
OFDM demodulation with carrier frequency error

- **No frequency error – demodulated desired subcarrier at maximum, no ICI**
- **With frequency error – lower demodulated desired subcarrier, ICI from other subcarriers**



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OFDM demodulation with carrier frequency error (Cont.)

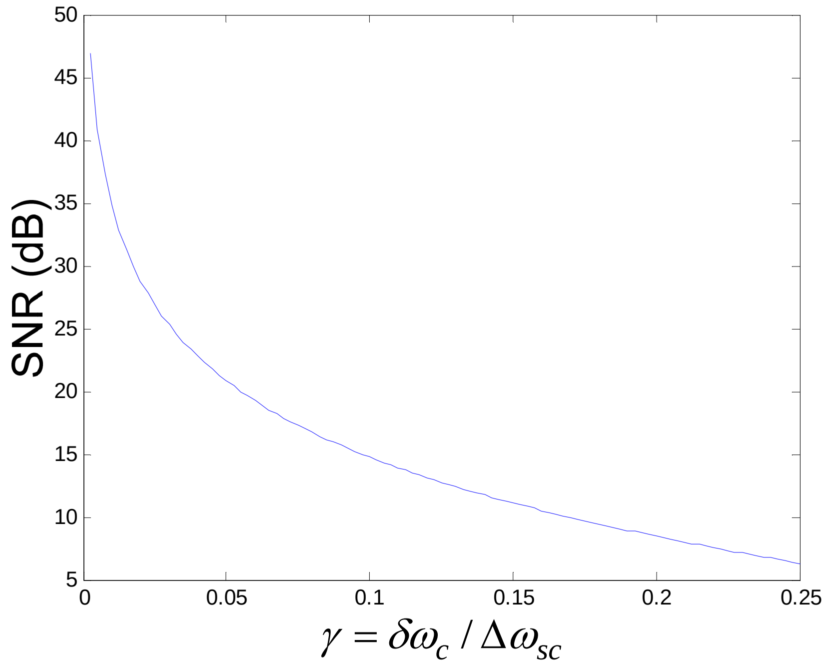
- **Signal to ICI ratio at subcarrier *l*:**

$$\frac{S}{ICI_{\delta\omega_c}} = \frac{\mathrm{sinc}^2(-\gamma)}{\sum_{\substack{k=0 \\ k\neq l+n}}^{N-1} \mathrm{sinc}^2(k-l-\gamma)}$$

where N is the number active subcarriers and

$$\gamma = \delta\omega_c / \Delta\omega_{sc}$$

- This result can also be used to evaluate the impact of Doppler



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OFDM demodulation with time-varying channels

- For time-varying channel response, the effect of ICI can be quantified as follows (as shown below for a single path channel):

$$\hat{s}_m = \frac{1}{T_s}\int_{t=0}^{T_s}\left(h(t)\sum_{k=1}^{N} s[k]e^{j2\pi f_k t}\right)e^{-j2\pi f_m t}dt$$

$$= \sum_{k=1}^{N}\left(\frac{1}{T_s}\int_{t=0}^{T_s} h(t)e^{j2\pi(f_k - f_m)t}dt\right)s[k]$$

$$= a_0 s[m] + \sum_{k=1,k\neq m}^{N} a_{k-m}s[k]$$

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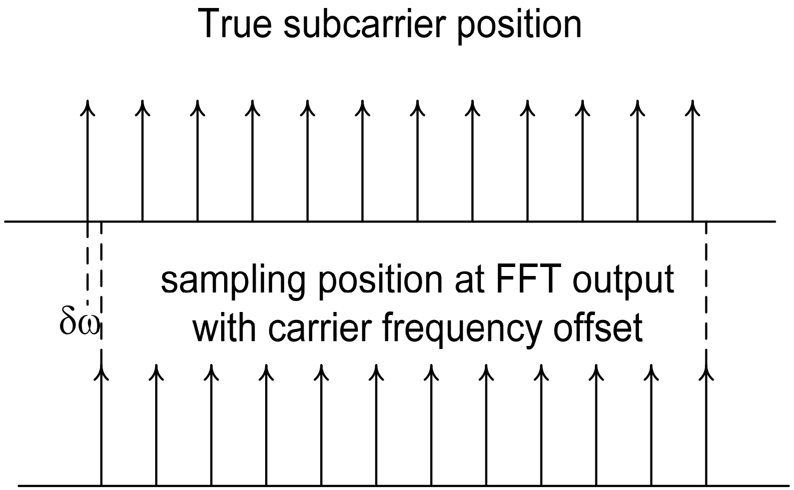
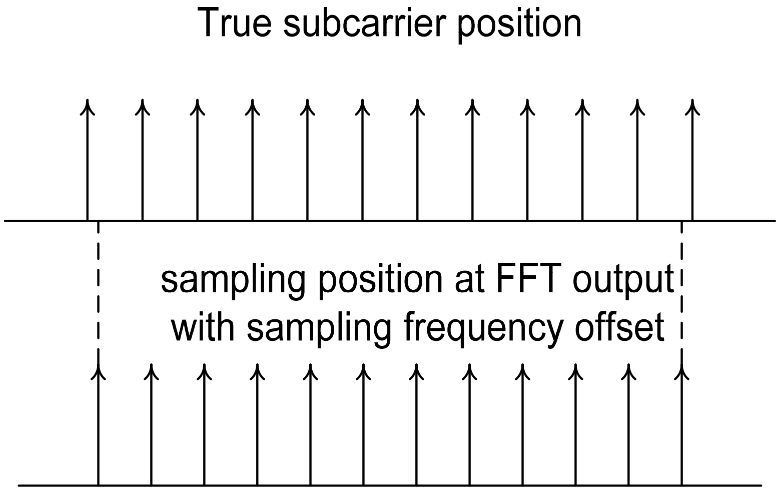
OFDM demodulation with sampling frequency error

- **Local oscillator frequency error will also cause sampling frequency error.**
- **It's impact to receiver is similar to carrier frequency error but in a somewhat different way – see the figures below:**





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OFDM demodulation with sampling frequency error (Cont.)

- **Signal to ICI ratio at subcarrier *l*:**

$$\frac{S}{ICI_{\delta\omega_s}} = \frac{\mathrm{sinc}^2[l(\zeta - 1)]}{\sum_{\substack{k=-N/2 \\ k \neq l+n}}^{N/2-1} \mathrm{sinc}^2(\zeta k - 1)}$$

where N is the number active subcarriers and

$$\zeta = \delta\omega_s / \omega_s$$

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Other aspects of OFDM system design

- **Peak to average Ratio**
  - OFDM signal can be viewed as a large number of sinusoidal summed together – has a Gaussian-like distribution
  - The peak to average (PA) ratio of its complex envelop is larger than 12 dB which demands highly linear power amplifier
  - Clipping (saturation) can reduce PA ratio but would cause out-band interference (“spectrum regrowth”)
    - For high power broadcasting system, clipping is not desired due to tight outband emission mask requirement
  - Research efforts have been devoted for reduction of the PA ratio

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Other aspects of OFDM system design (cont.)

- Coding
  - **The subcarrier magnitudes at the demodulator (FFT) output can vary widely when signal passes through a multipath channel**
  - **It is very important to make sure the decoding decision is made over the average of the decoding metrics taking from multiple FFT outputs across the entire band**
    - Averaging in decoding process is more important than that in a single carrier system
    - Strong code and effective interleaving are essential to ensure good performance in an OFDM system
    - Turbo code, other concatenated code schemes (e.g., Convolutional code + Reed Solomon code) and LDPC are effective ways to achieve such averaging

Key OFDM Receiver Functional Blocks

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Key Receiver Functional Blocks

- Most receiver resources are devoted to FFT/iFFT and (Turbo) decoder.
  - These blocks are straightforward to implement, even though resource consuming
- Synchronization related blocks are most difficult to design and optimize, but with greatest impact to receiver performance
  - Channel estimation
  - Timing control
  - Frequency offset compensation

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Channel Estimation in OFDM Receiver

- In an OFDM system, a coherent receiver usually provides better performance than a non-coherent receiver (e.g. using differential coding/decoding)
- To perform coherent demodulation, the complex frequency domain channel of each data subcarrier need to be estimated
- The estimates of frequency domain channels can be expressed as the Fourier transform of the channel impulse response in time domain
- Channel estimation can be performed using TDM or FDM pilots
- The channel estimate based on FDM pilots could be more effective (less interference between data and pilots) and efficient (less complex)

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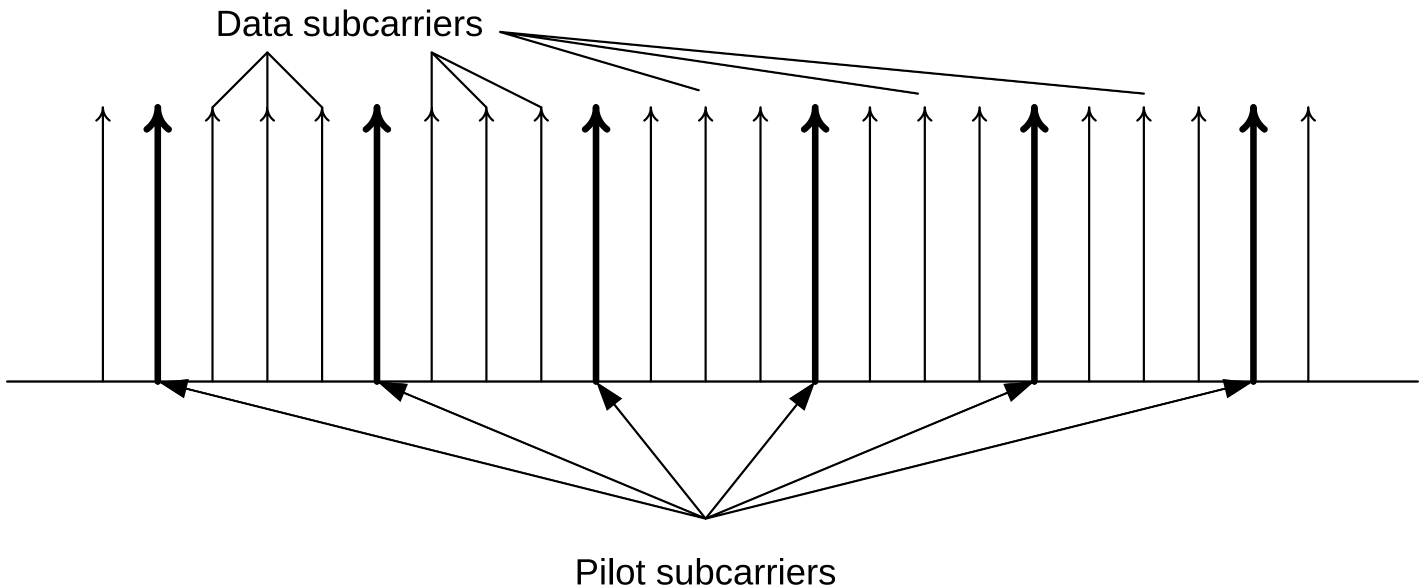
Channel Estimation (cont.)

- FDM pilots – subcarriers with known modulation symbols
- Frequency response (FR) of pilot subcarriers can be easily estimated by descrambling corresponding FFT output
- The FRs of data subcarriers are computed by interpolation



Channel Estimation (cont.)

- How many pilot subcarriers are needed in each OFDM symbol?
  - We can view this from different angles:

    (1) If the time domain channel has N_p taps, N_p pilots are needed

    (2) It can be viewed as a frequency domain Nyquist sampling problem for equally spaced pilots, which yield lowest estimation error

    - Assuming the spacing is M subcarries, alias occurs every N_{FFT}/M time domain samples, i.e., channel time span should be $< N_{FFT}/M$
    - This result is approximate due to finite number of pilots in each OFDM symbol.

    (3) Pilot spacing should be less than channel coherent bandwidth which is approximately 1/channel time span
  - These view points are equivalent under given conditions

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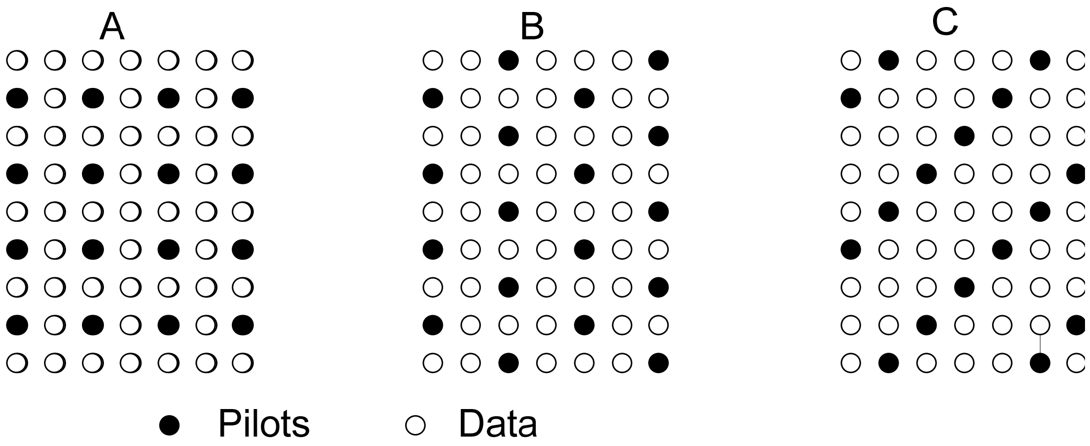
Channel Estimation (cont.)

- How frequent pilots should be inserted?
- Based on Nyquist sampling theorem, its frequency should be AT LEAST twice of the maximum Doppler frequency
- FDM pilot symbol patterns



- A and B would have same performance with infinitely long filter. Practically, B should have better performance.
- C can handle channels twice as long at a half of Doppler frequency

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Channel Estimation – further considerations

- Pilot interpolation can be efficiently implemented using an iFFT/FFT pair *if the FFT size is divisible by the pilot spacing P*
- The result iFFT yields an estimate of time domain channel impulse response (CIR)
- For sparse CIR, the variance in frequency domain channel estimate can be reduced by eliminating taps due to noise/interference in estimated CIR

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Timing control in OFDM

- With guardbands, the receiver can be viewed as a fractional spaced system
  - No inter-chip/sample timing adjustment is needed
- The optimal receiver timing selection is equivalent to optimal FFT window placement
- If the cyclic prefix is longer than the channel span, the optimal timing is not unique. In such a case:
  - Optimal timing does not need to be precise
  - Emphases should be put on robustness
- In a mobile communication environment, receiver timing control is still the most challenging task (in some sense, as the finger control in CDMA receivers)

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Timing control in OFDM (cont.)

- Selection of the optimal FFT window
  - Definition:
    - CP of the received signal (RX): N_{CP} samples prior to the FFT window
    - Position of a channel path: the sample corresponding to the beginning of the channel path
  - Selection criterion:

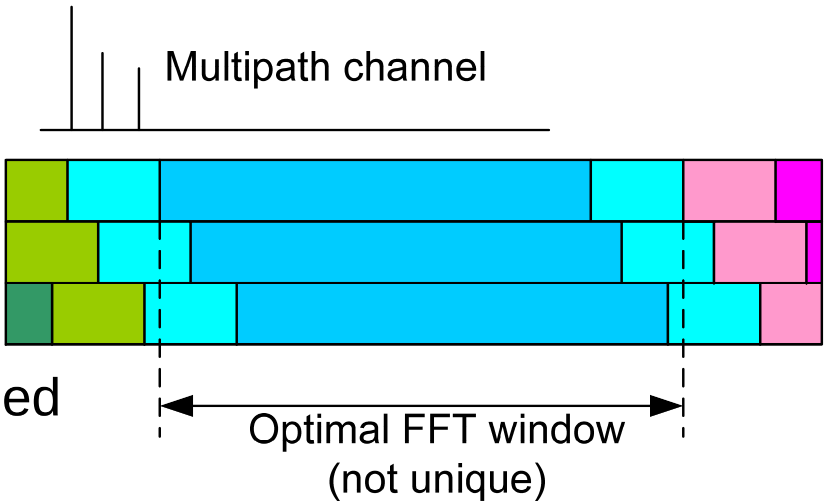
    (1) The RX CP shall cover the first arriving path (FAP)

    (2) The Rx CP should cover the last arriving path (LAP)

    (3) If both (1) and (2) cannot be satisfied simultaneously:

    > Satisfy (1) first

    > and/or Rx CP should cover the paths with most of the energy



  - Path positions are determined from time domain channel estimates

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Carrier frequency Synchronization

- Carrier frequency offset compensation is needed to reduce the degradation due to such offset
- The compensation can be done by either adjusting the local oscillator frequency or using a digital phase rotator
- Compensation is usually controlled by an AFC loop that is driven by a circuit, which detects phase error of every Δt
- Phase error detection can be done on signal Pre-FFT (time domain) or Post-FFT (frequency domain)
- In both cases, the offset frequency up to about half of the OFDM symbol rate can be detected and compensated
- Other more sophisticated schemes are possible to compensate even larger frequency offsets

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Carrier frequency Synchronization (cont.)

- Pre-FFT: compare the phases of CP samples and their corresponding portion at the end of the OFDM symbol
  - Frequency offset = $\Delta\phi_{Pre}/(N_{FFT}*Tc)$
  - Maximum detectable frequency offset = $1/(N_{FFT}*Tc)$
- Post-FFT compare the phases of the corresponding (pilot) signals of an OFDM symbol and the subsequent OFDM symbol
  - Frequency offset = $\Delta\phi_{Post}/[(N_{FFT}+N_{CP})*Tc]$
  - Maximum detectable frequency offset = $1/[(N_{FFT}+N_{CP})*Tc]$

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Example:

LTE OFDM PHY Layer

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Overview of LTE PHY Layer

- Forward Link – OFDM Parameters
  - Frequency organization:
    - Bandwidth: 1.4, 3, 5, 10, 20 MHz
    - Subcarrier Spacing: 15 kHz (also 7.5 kHz for MBSFN)
  - Time domain organization:
    - Frame: 10 ms
    - Sub frame: 1 ms
    - Slot: 0.5 ms
    - OFDM Symbol duration: 0.5/7 ms and 0.5/6 ms (also 0.5/3 ms for MBSFN)
- Reverse Link – SC-FDMA (lower P/A ratio)
- Modulation – QPSK, 16QAM and 64QAM

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LTE Forward Link Organization

- Frequency domain OFDM parameters:

Bandwidth (MHz)	1.4	3	5	10	15	20
Active Subcarriers + zero subcarrier	73	181	301	601	901	1201
FFT Size	128	256	512	1024	1536	2304
Sampling Rate	1.92	3.84	7.68	15.36	23.04	30.72

  - For 15 kHz subcarrier spacing
  - OFDM symbol length without CP is 1/15000 = 66.67 μs
  - Obtain desired signal bandwidth by select the number of guard carriers
  - Modulation and demodulation can be done by using larger FFT sizes

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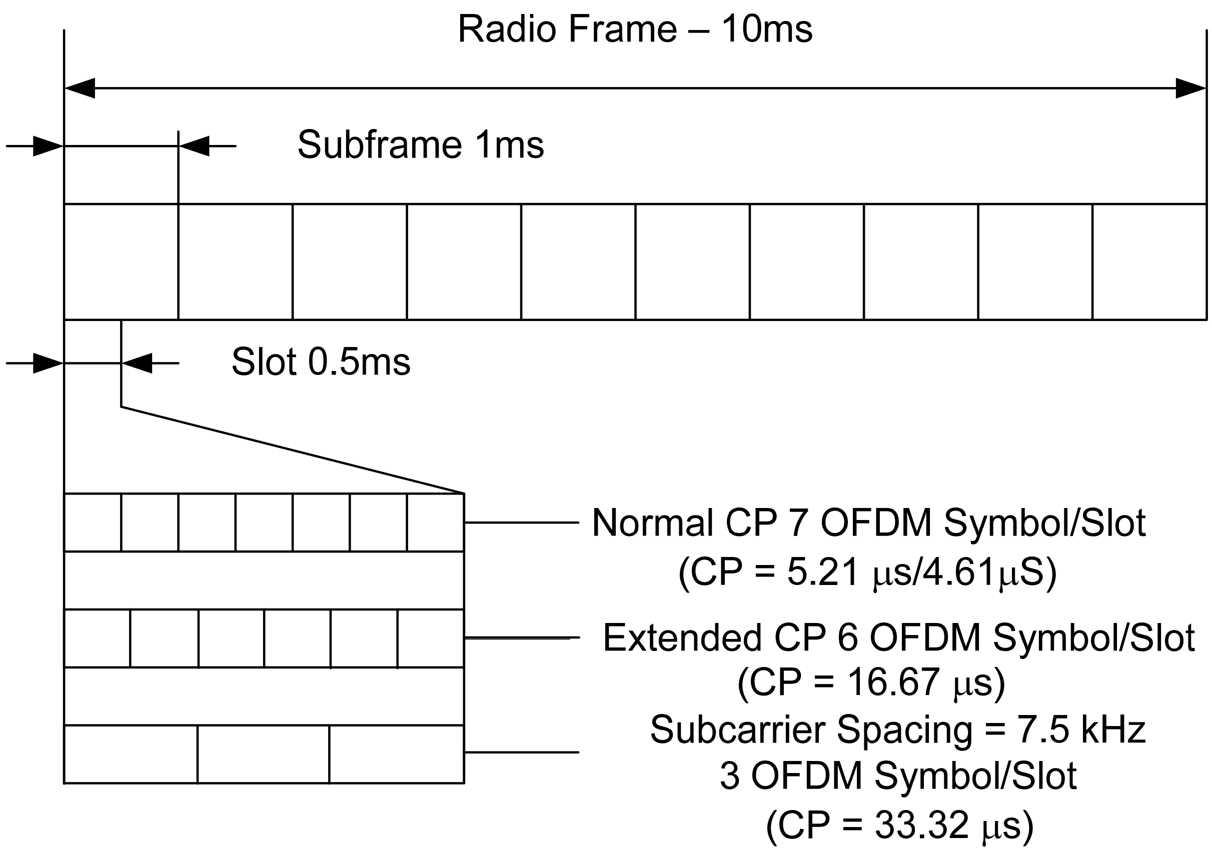
LTE Forward Link Organization (cont.)

- Time Domain Frame Structure



LTE Resources Organization

- Resource Element: One subcarrier in one OFDM symbol
- Resource Block: 12/24 subcarriers per OFDM symbols in one Slot, for Δf = 15kHz/7.5kHz

Bandwidth (MHz)	1.4	3	5	10	15	20
Resources blocks per Slot	6	15	25	50	75	100

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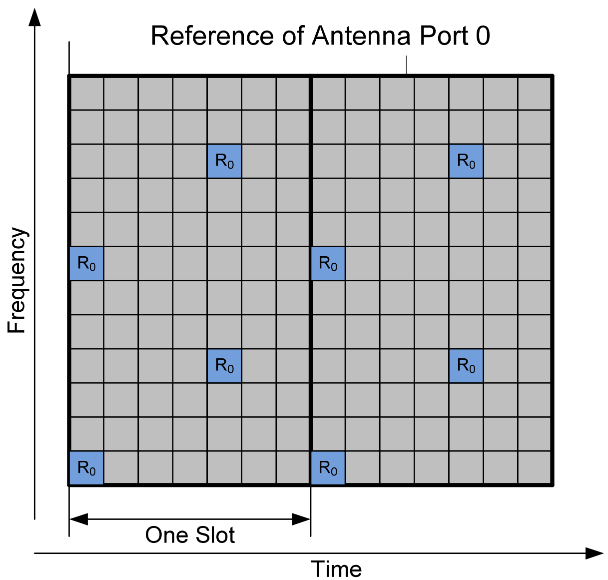
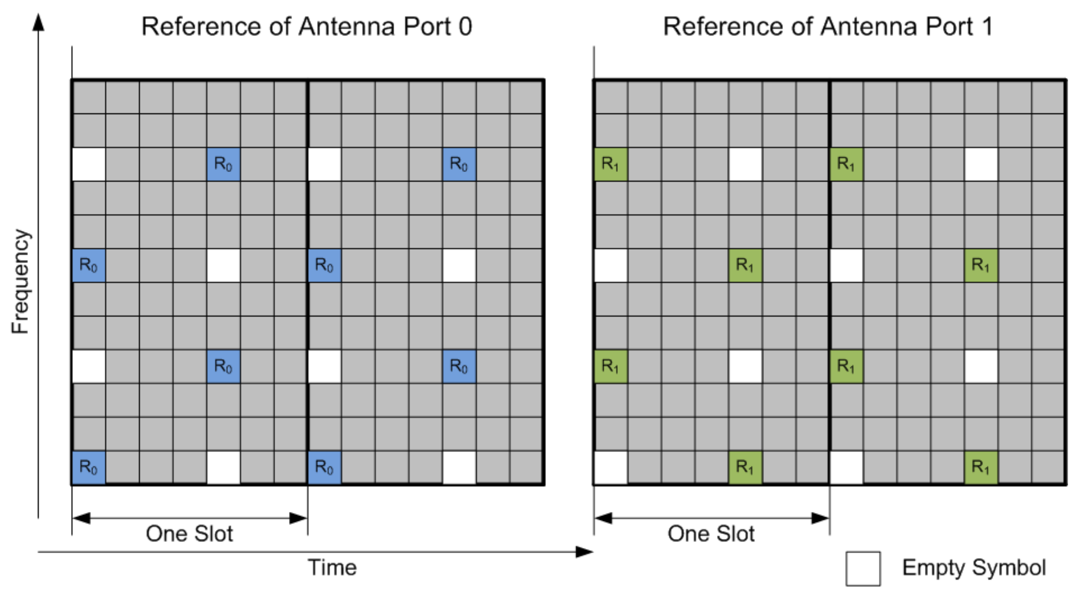
Forward Link Reference Symbols

- Reference symbols organization depends on the number of antenna ports and CP types
- Cell-Specific Reference symbols of regular length CP



Single Antenna Tx



Two Antenna Tx

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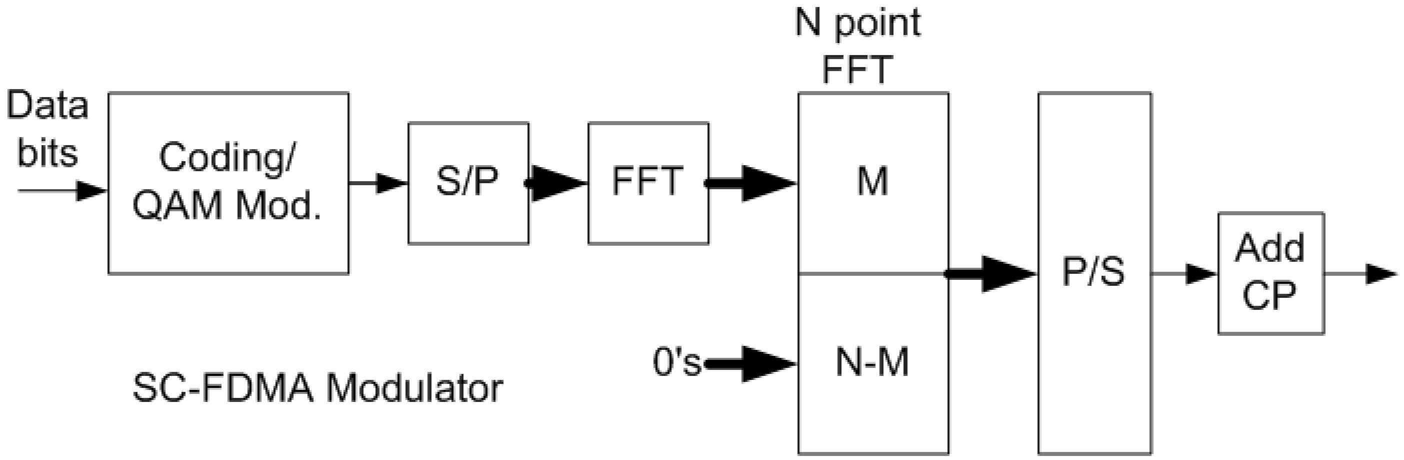
Forward Link Reference Symbols (cont.)

- Above show *cell specific* reference symbol patterns
  - Reference symbol arrangement of one and two antenna depicted
  - Reference symbol support up to four antenna Tx
    - Mainly for dense cell sites areas, low vehicle speeds
    - One column of reference symbols per slot
  - Mainly for Spatial Multiplexing and Transmitter Diversity
- User specific reference symbol can also be deployed
  - One set per user
  - Mainly for beam forming

LTE Reverse Link

- Reverse Link employs Single Carrier FDMA (SC-FDMA)



- Why SC-FDMA:
  - Lower peak to average ratio (a single carrier system)
  - FDMA between users
  - Easy to perform frequency domain equalization (FDE)

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