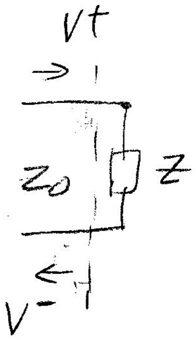
2-10 스미스차트

2-10-1 매개방정식



$\Gamma = \dfrac{V^-}{V^+}$: 반사계수

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\bar{z} - 1}{\bar{z} + 1} = \Gamma_r + j\Gamma_i$$

Γ_r ↑ 실수부, $j\Gamma_i$ ↑ 허수부

$\bar{z} = \dfrac{Z}{Z_0}$: 정규화한 임피던스(normalized impedance)

$$\bar{z} = r + jx$$

r ↑ 실수부, jx ↑ 허수부

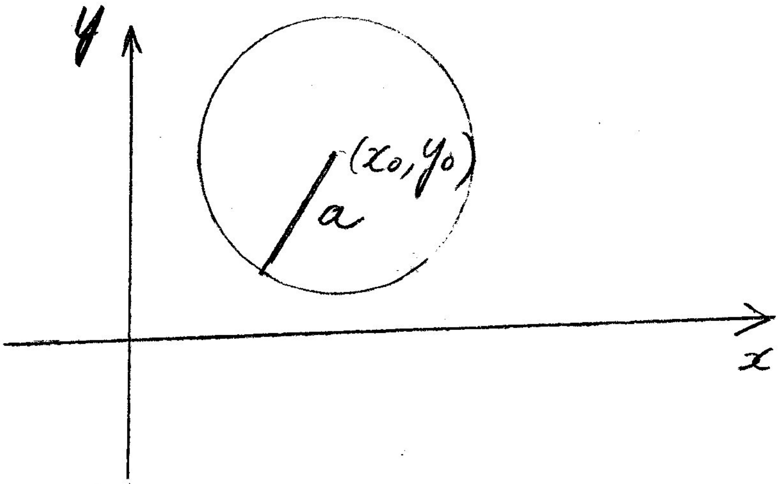
$$\boxed{\Gamma = \frac{\bar{z} - 1}{\bar{z} + 1}, \quad \bar{z} = \frac{1 + \Gamma}{1 - \Gamma}}$$

$$r + jx = \frac{1 + \Gamma_r + j\Gamma_i}{1 - \Gamma_r - j\Gamma_i}$$

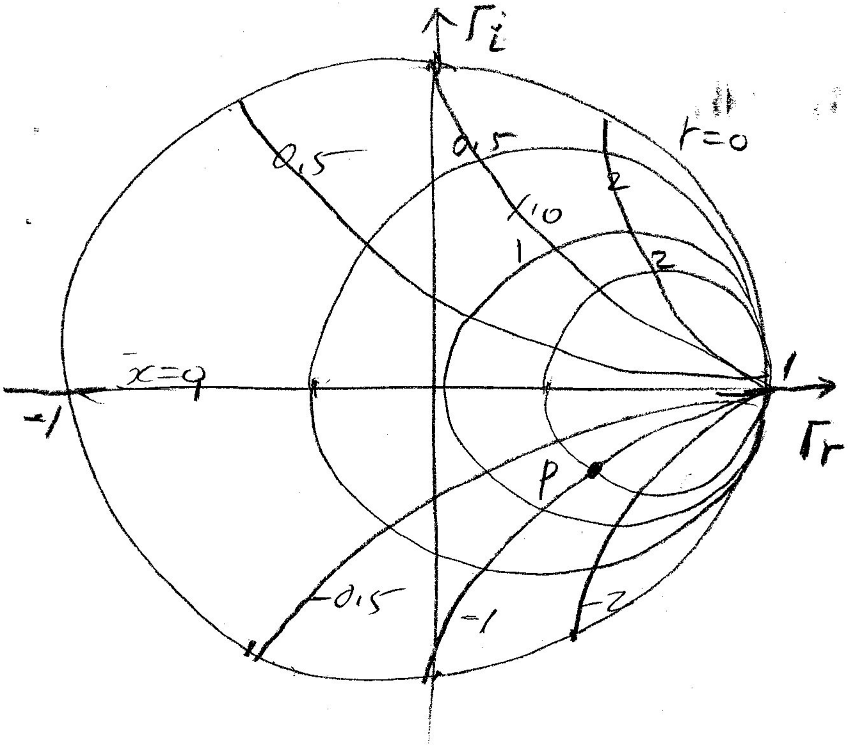
$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \quad : x = const$$

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2 \quad : r = const$$

$(x-x_0)^2+(y-y_0)^2=a^2$: 원의 방정식



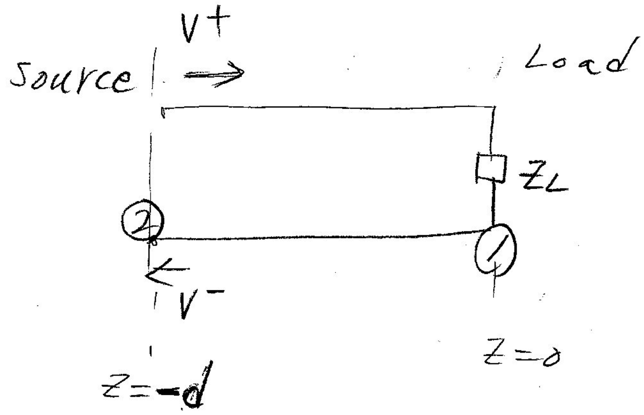
- 스미스 도표
  - 도식적으로 전송선상의 반사계수, 입력 임피던스 계산
  - 임피던스 정합 (Z_{in}을 Z_0가 되도록 회로 추가)에 매우 편리한 툴



$P : \bar{z} = 2 - j1$

$z = 100 - j50\Omega \; (Z_0 = 50\Omega)$

2.10.2 파동임피던스 (임피던스 계산)



$$Z(-d) = \frac{V(-d)}{I(-d)} = Z_0 \frac{1+\Gamma(-d)}{1-\Gamma(-d)}$$

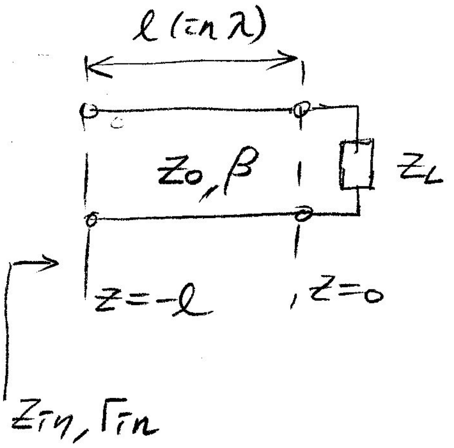
$$\bar{Z}(-d) = \frac{1+\Gamma(-d)}{1-\Gamma(-d)}$$

$$\Gamma(-d) = \Gamma_L e^{-j2\beta d}$$

$\theta = 2\beta d = 2\frac{2\pi}{\lambda}d$; $\underline{d = \frac{\lambda}{2} \rightarrow \theta = 2\pi \; , \; \frac{\lambda}{4} \rightarrow \pi}$

$-2\beta d = -\theta$: 위상감소 → 복소수 평면에서 시계방향회전

실습 (2.10-2.12) : PLab-04, Python programming



$\beta = \frac{2\pi}{\lambda}$, $\beta = \beta_o$ (free space β_o)

Find Z_{in}, Γ_{in} (polar) at $f_1 \le f \le f_2$

입력 :

$Z_L(\Omega)$ (complex) =

$Z_o(\Omega)$, l (m) =

$f_1, f_2, \Delta f$ (Hz) =

출력 : 파일에 출력

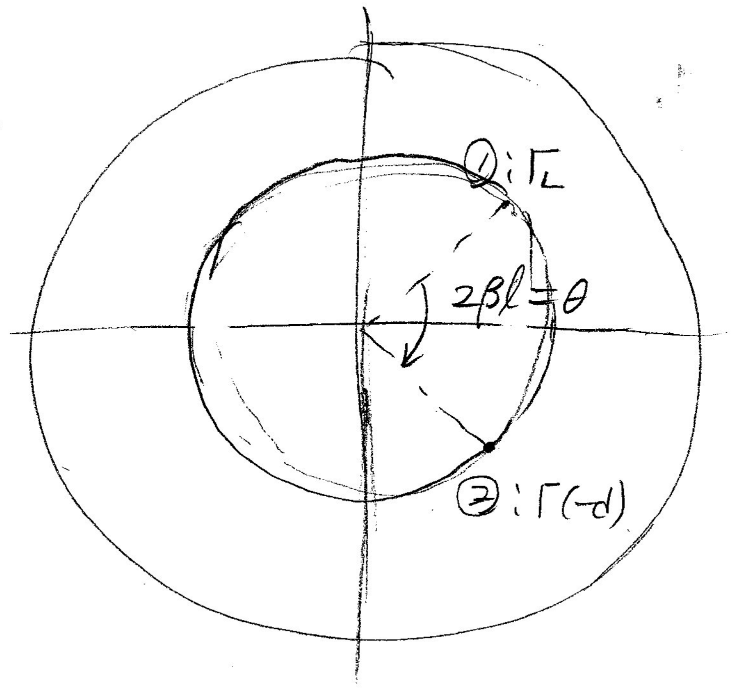
RUN = (실행 번호)

$Z_L(\Omega)$ =

$Z_o(\Omega)$, l (m) =

$f_1, f_2, \Delta f$ (Hz) =

f_1 (Hz)	Z_{in} (Ω)	mag (Γ_{in})	phase (Γ_{in})
10.0000e9	26.4736 − j6.2784	0.1847	26.74
. . . .			
12.000e9	82.4124 − j20.5198	0.3804	72.64



$\Gamma_L : \bar{z}_L$

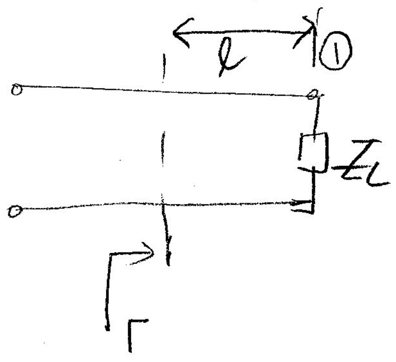
$\Gamma(-d) : \bar{z}(-d)$

WTG(wavelength toward generator)

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$|\Gamma| = const$ (원) $\rightarrow$ SWR = const. circle

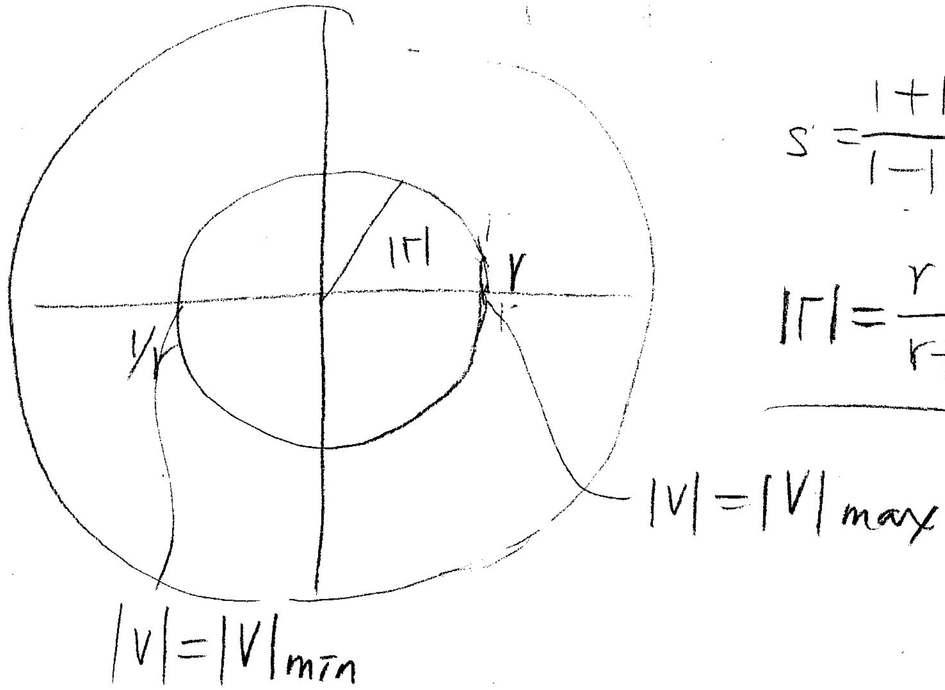
2-10-3 SWR, 최대전압, 최소전압



$V = V^+(1+\Gamma)$

$V = |V|max$ when $\Gamma = |\Gamma| = |\Gamma|e^{j0°}$

$V = |V|min$ when $\Gamma = -|\Gamma| = |\Gamma|e^{j180°}$

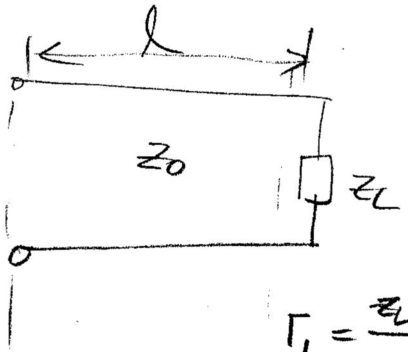


$$s = \frac{1+|\Gamma|}{1-|\Gamma|}, \quad |\Gamma| = \frac{s-1}{s+1}$$

$$|\Gamma| = \frac{r-1}{r+1} \equiv \frac{s-1}{s+1} \quad (x=0)$$

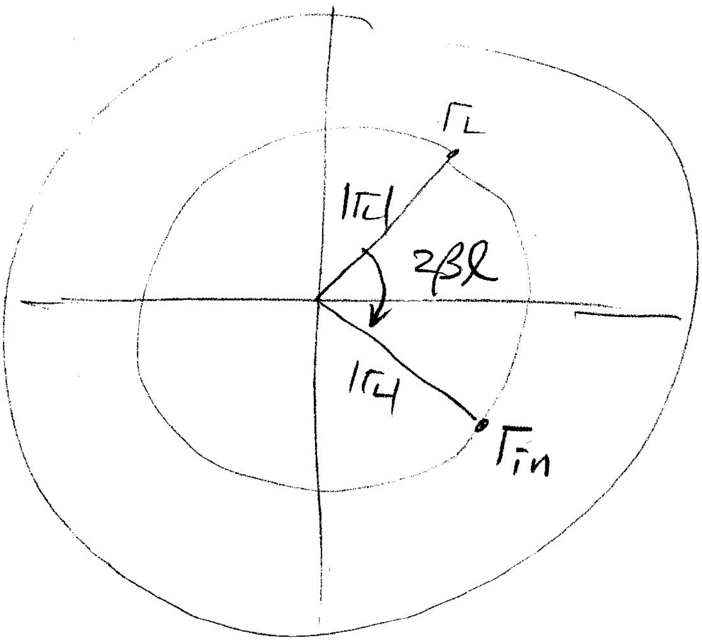
$$\boxed{s = r}$$

2.10.4 임피던스 변환 $(\bar{z} \leftrightarrow \bar{y})$



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_{in} = \Gamma_L e^{-j2\beta l}$$



$$2\beta l = 2\frac{2\pi}{\lambda} = \frac{720^\circ}{\lambda} l$$

$Z = R + jX$: 임피던스

R: 저항, X: 리액턴스

$Y = \frac{1}{Z} = G + jB$: 어드미턴스

G: 컨덕턴스, B: 서셉턴스

$Z = \frac{V}{I}$, V: 전압페이저, I: 전류페이저

$$Y = \frac{1}{R+jX} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

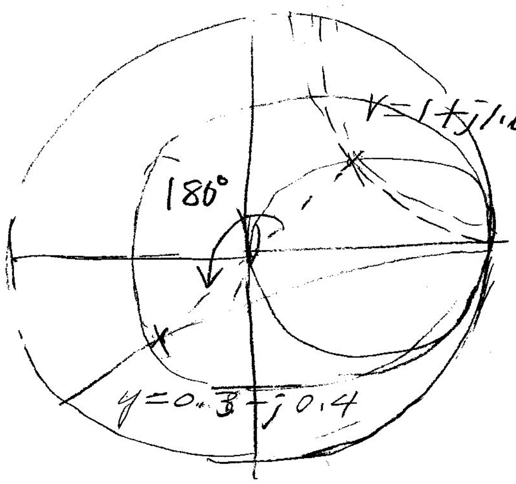
$\bar{Z} = \frac{Z}{Z_0} = r + jx$: 정규화한 임피던스

$\bar{Y} = \frac{Y}{Y_0} = Z_0(B+jX) = g + jb$: 정규화한 어드미턴스

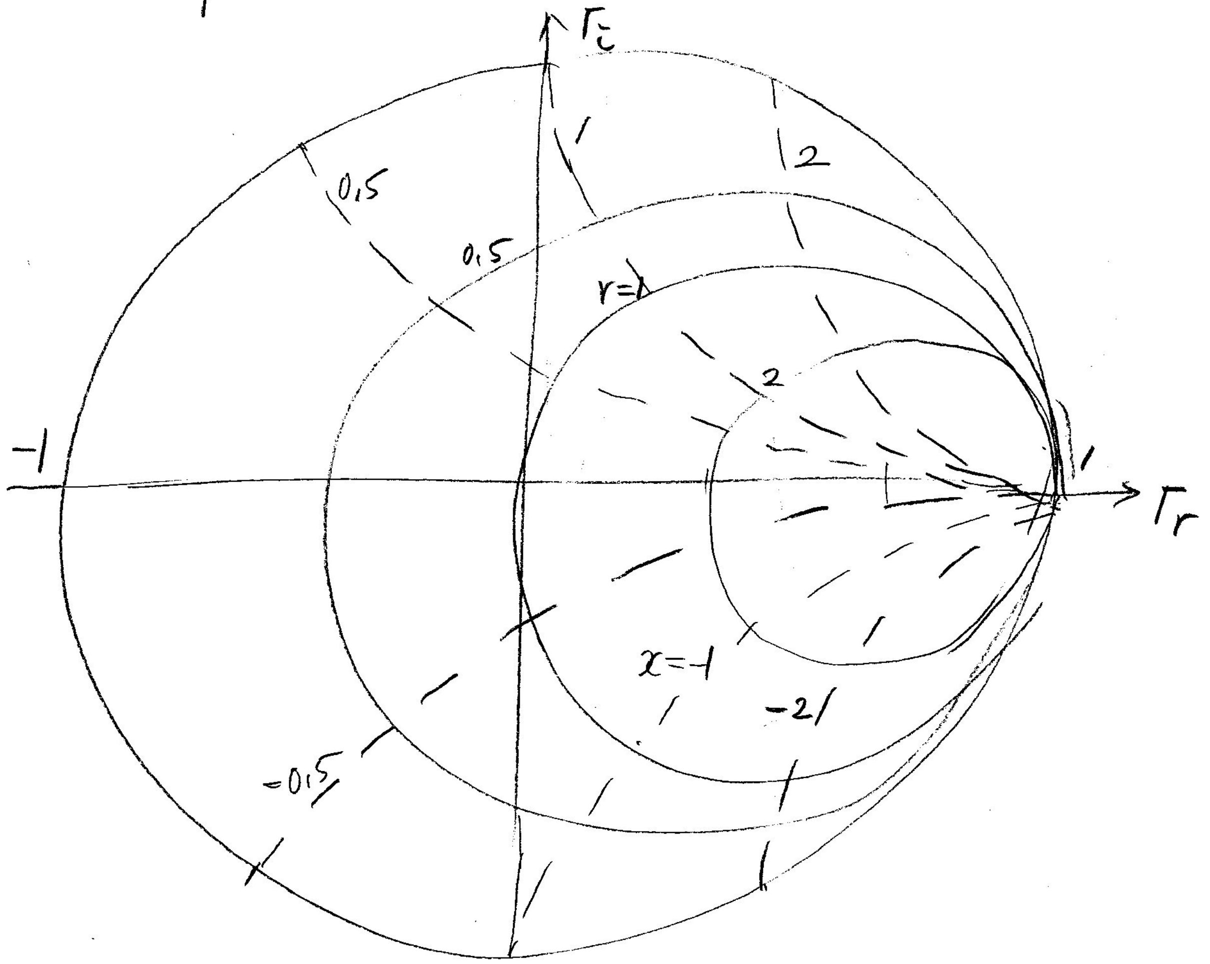
$$\bar{Z} = \frac{1+\Gamma}{1-\Gamma}$$

$\bar{Y} = \frac{1-\Gamma}{1+\Gamma} = \frac{1+(-\Gamma)}{1-(-\Gamma)}$: use Smith chart with $\Gamma \to -\Gamma$

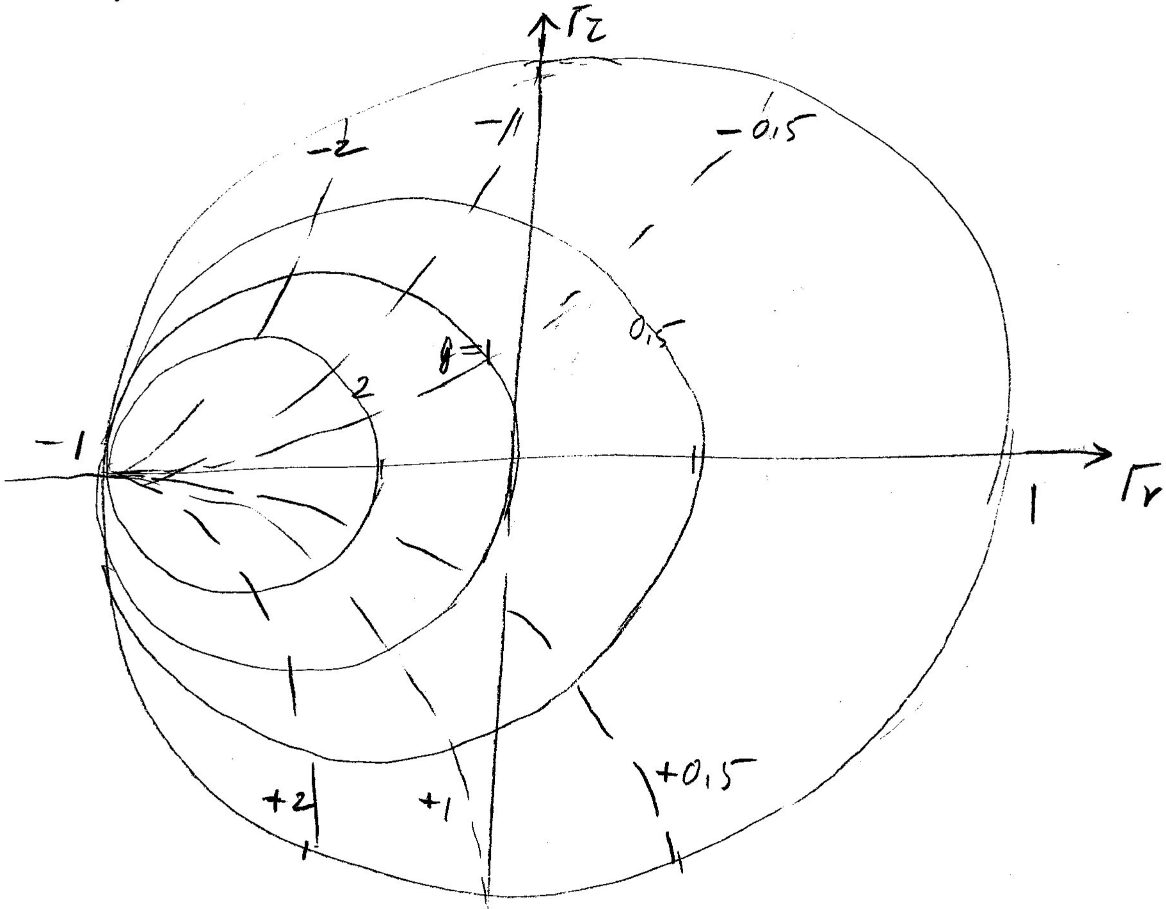
$-\Gamma = \Gamma \underbrace{e^{j\pi}}_{180^\circ \text{ 회전}}$



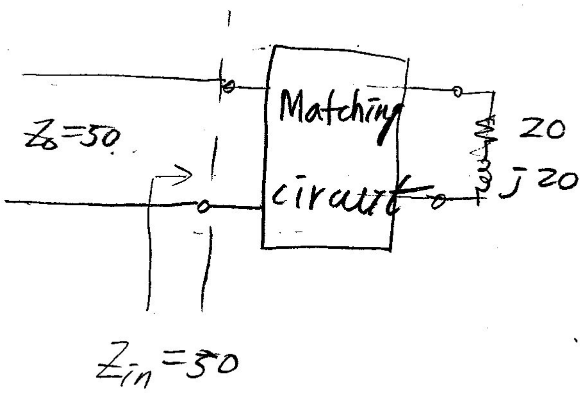
Impedance chart



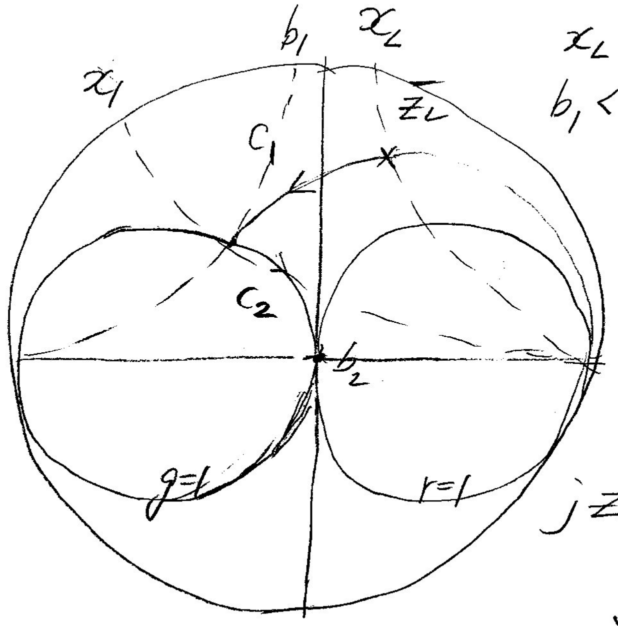
Admittance chart: 임피던스 차트를 180° 회전



2.11 임피던스 정합(matching)

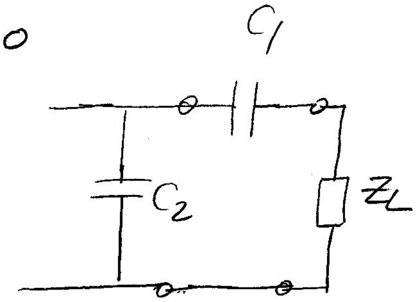


2.11.1 집중소자(lumped element) 정합



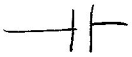
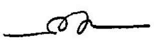
$x_L > x_1$

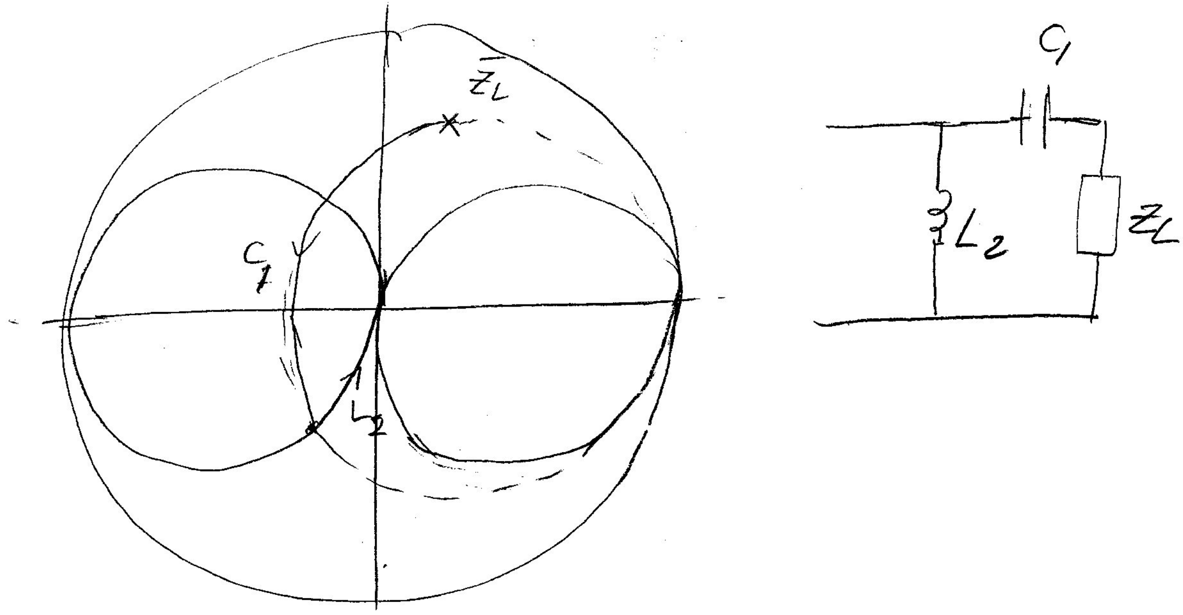
$b_1 < 0, \; b_2 = 0$

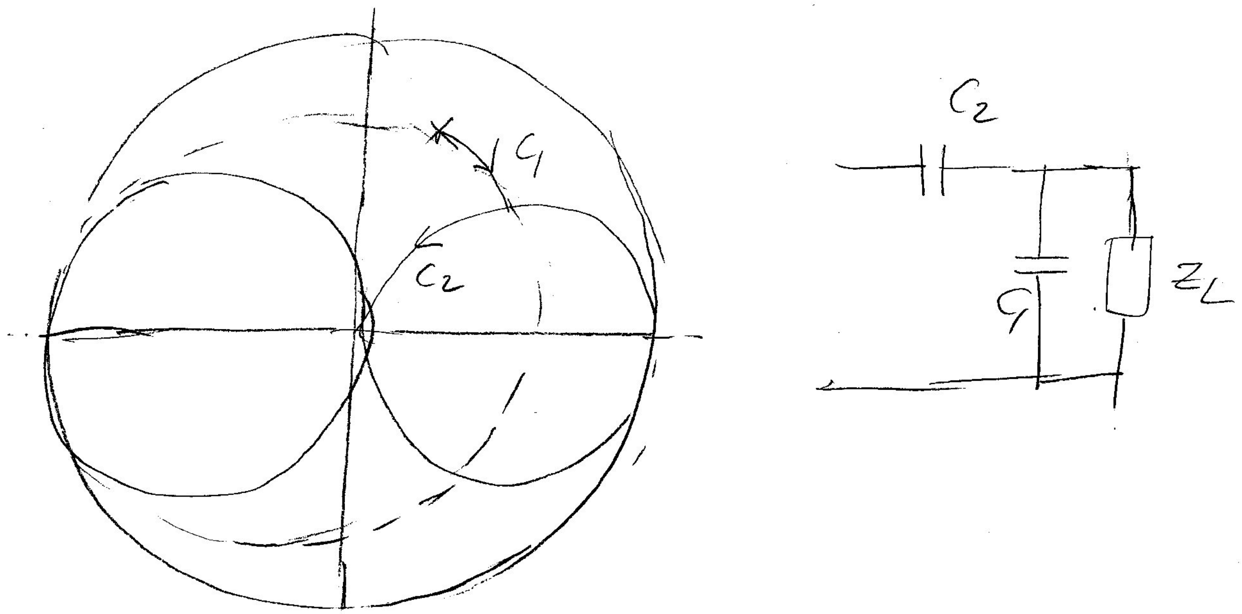


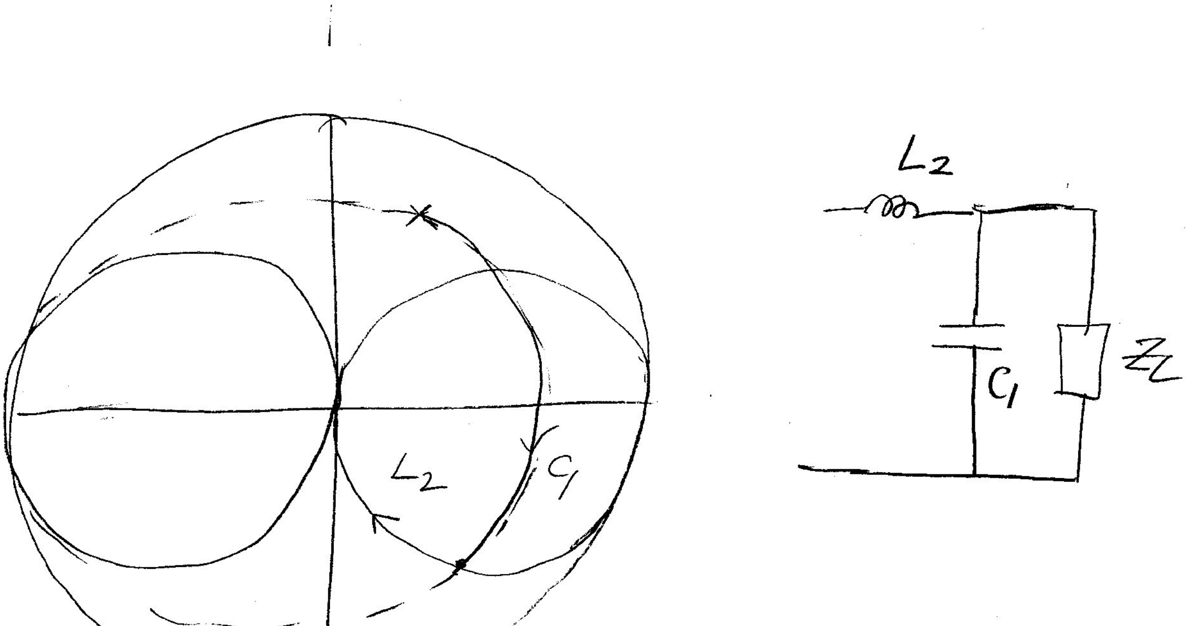
$$jZ_0(x_L) - j\frac{1}{\omega C_1} = j(x_1)Z_0$$

$$jY_0(b_1) + j\omega C_2 = 0 \quad (b_1 < 0)$$

element	Z	Y
	$-j\frac{1}{\omega C}$	$j\omega C$
	$j\omega L$	$-j\frac{1}{\omega L}$


Z_L
C_1
L_2
C_1
L_2
Z_L

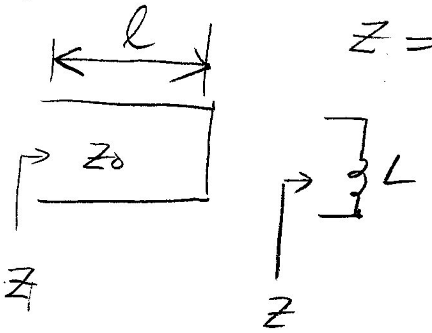

C_1
C_2
C_2
C_1
Z_L


L_2
C_1
L_2
C_1
Z_L

2.11.2 단일 스터브 정합

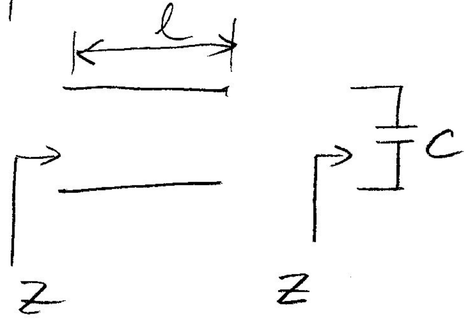
stub : 그루터기, 전송선에 짧은 전송선이 병렬 또는 직렬로 연결된것

short stub



$$Z = jZ_0 \tan\beta l = j\omega L$$

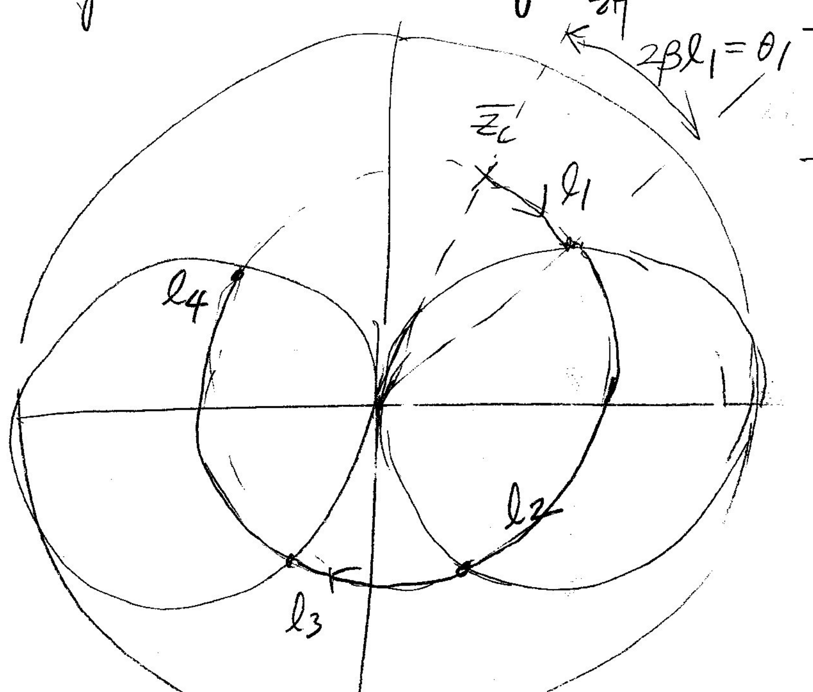
open stub:

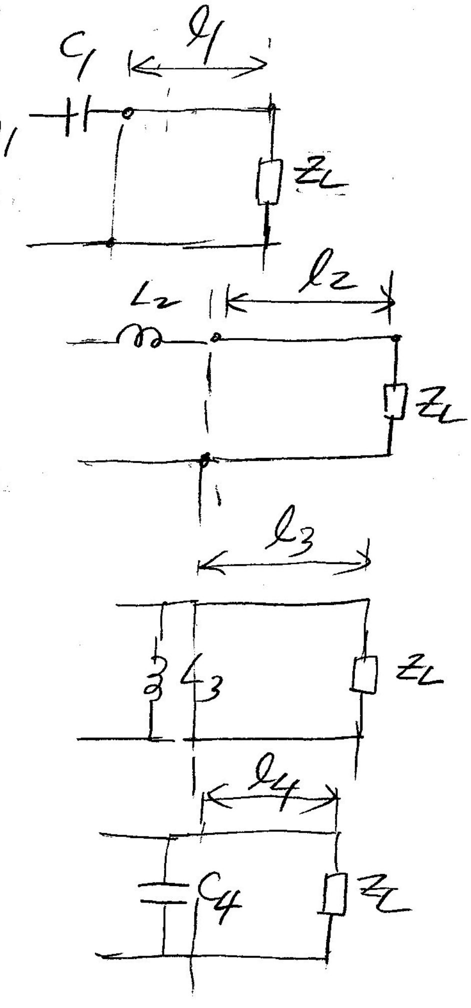


$$Z = -j\frac{Z_0}{\tan\beta l} = -j\frac{1}{\omega C}$$

$$Y = jY_0 \tan\beta l = j\omega C$$

single-stub matching: 4가지 해

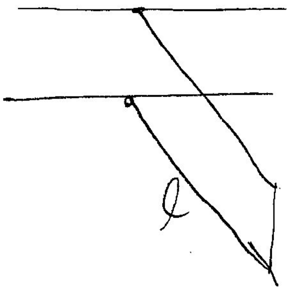




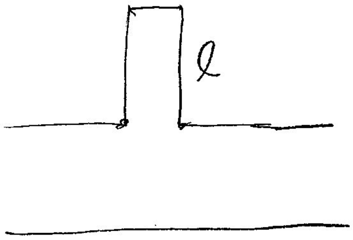
$$\theta_1 = 2\beta l_1 = \frac{4\pi}{\lambda} l_1$$

C → use open stub

L → use short stub

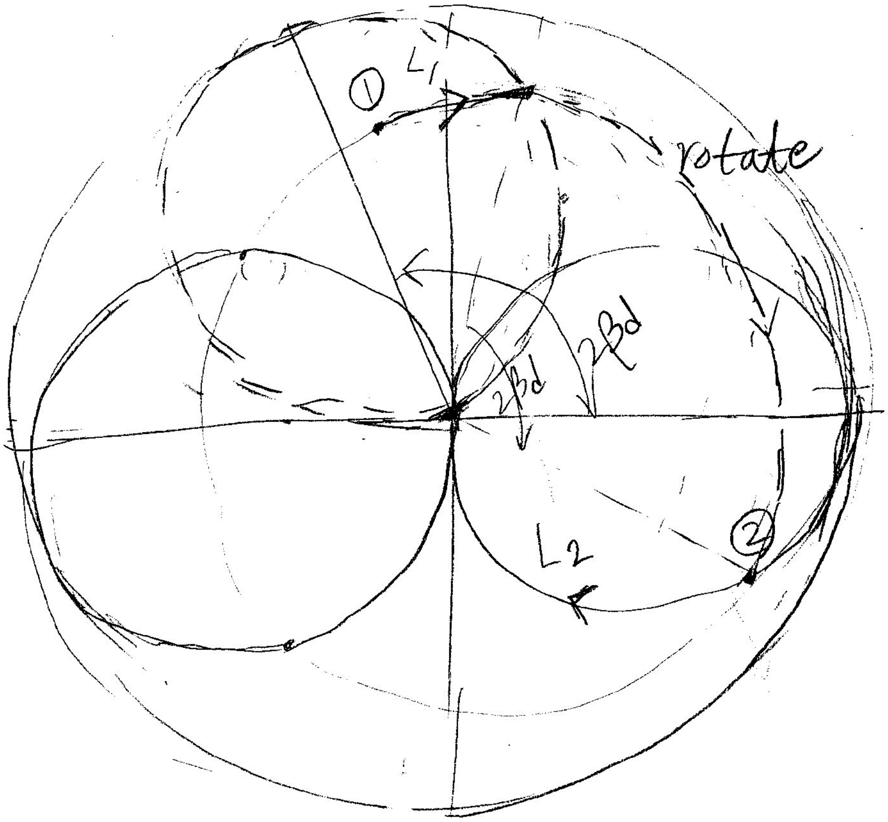


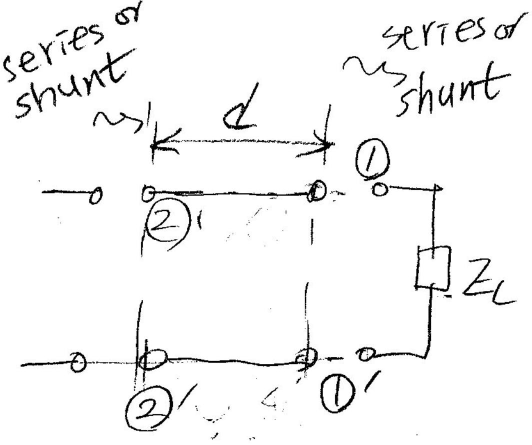
shunt L



serie L

Double-stub tuning

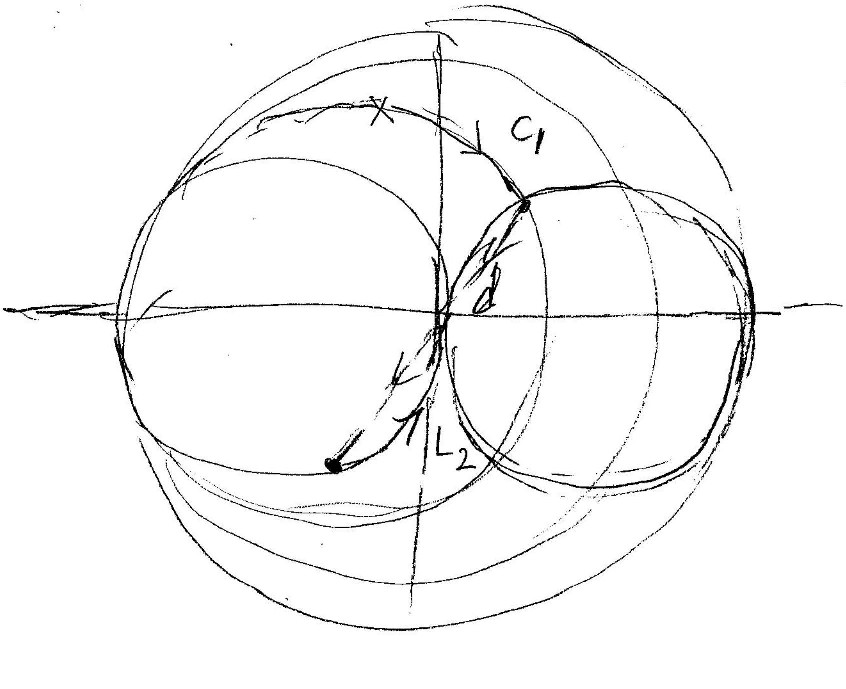




$d = \lambda/8 - 3\lambda/8$ (fixed)

At ① : series element
shunt element) both possible → series element selected

At ② : series element

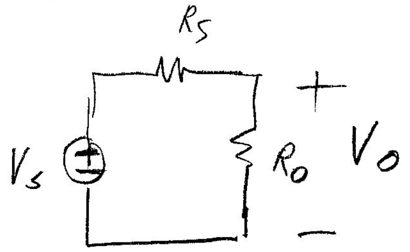


$d = \frac{\lambda}{4}$

C_1 : shunt

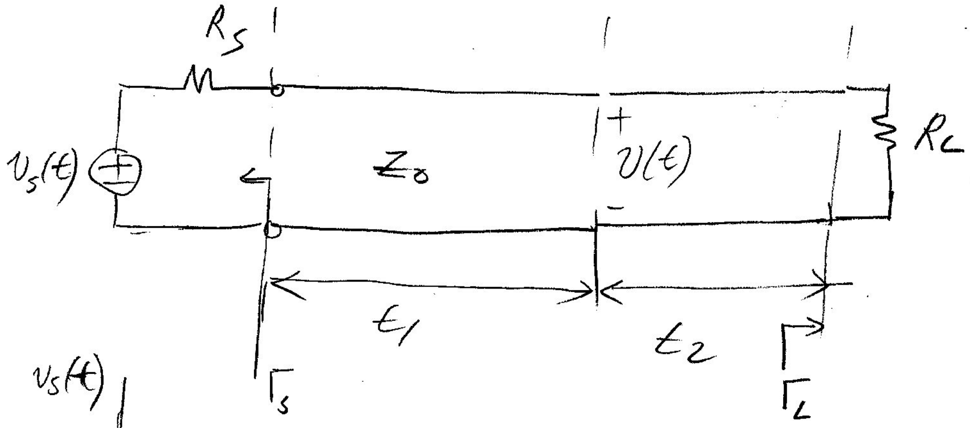
L_2 : shunt

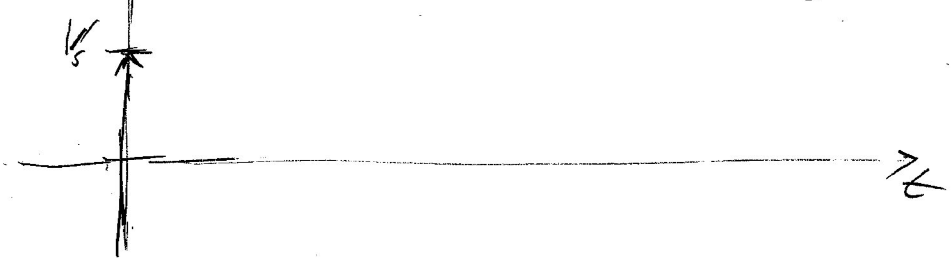
2.12 전송선의 과도현상

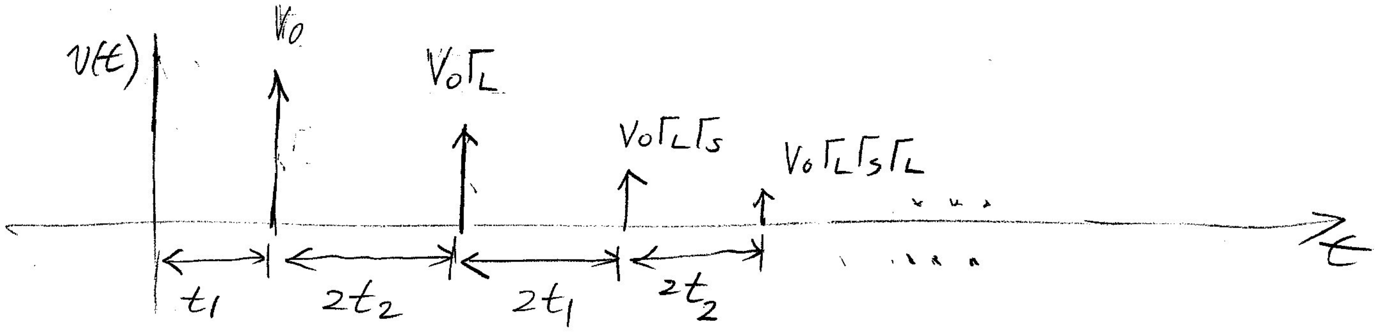


$$V_0 = \frac{R_0}{R_s + R_0} V_s$$

i) Impulse 입력





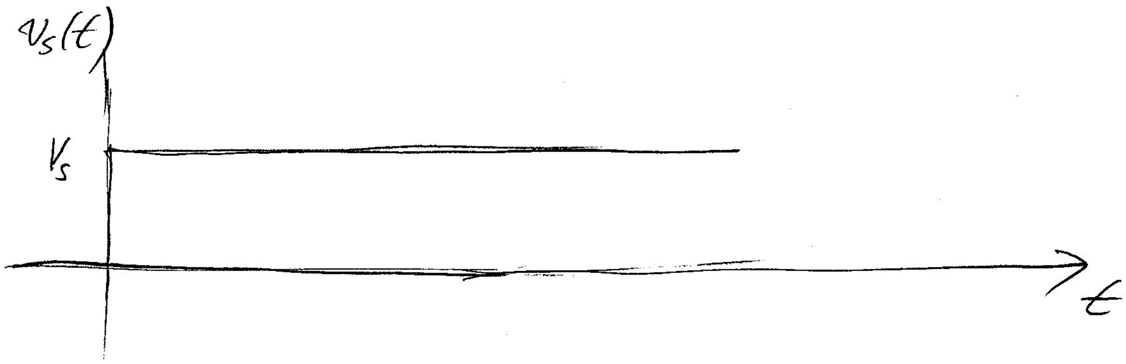


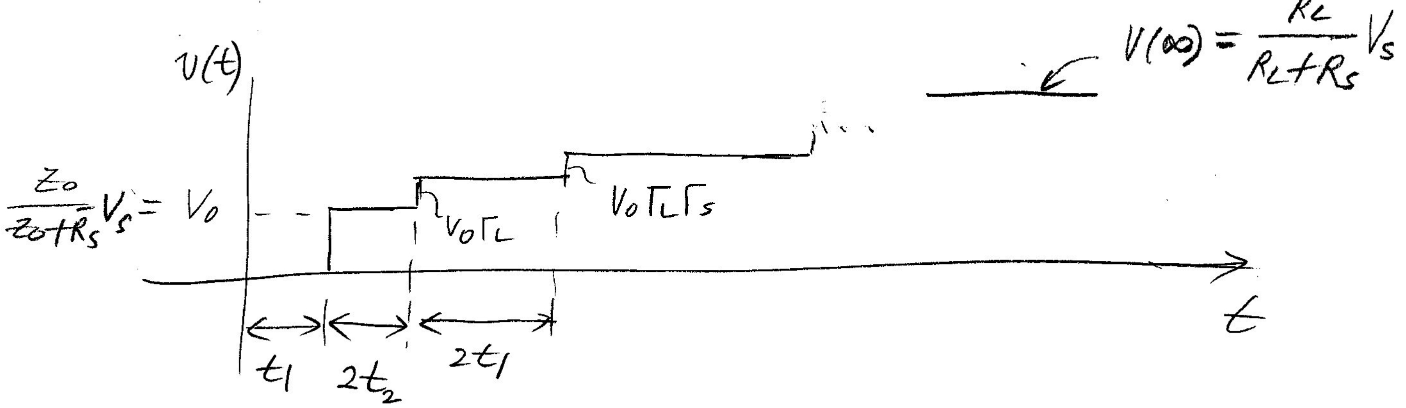
$$V_0 = \frac{Z_0}{Z_0 + R_S} V_s \;;\; V_s = 1$$

$t=0$ 인순간에 $v_s(t)$는 전송선의 Z_0 만 감지하고 전송선 종단에 무엇이 연결되었는지 감지하지 못한다.

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}, \quad \Gamma_s = \frac{R_s - Z_0}{R_s + Z_0}$$

2) step 응답





$t = t_1$:

$$v = V_0 = \frac{Z_0}{Z_0+R_s}V_s$$

$t = \infty$ (DC state)

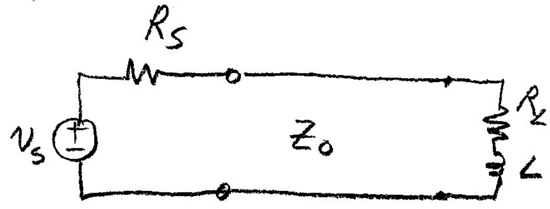
$$v = \frac{R_L}{R_L+R_s}V_s$$

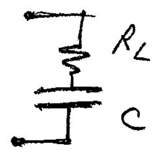
$$v = V_0 + V_0\Gamma_L + V_0\Gamma_L\Gamma_s + V_0\Gamma_L\Gamma_s\Gamma_L + V_0\Gamma_L^2\Gamma_s^2 + V_0\Gamma_L^3\Gamma_s^2 + \cdots$$

$$= V_0(1+\Gamma_L) + V_0(1+\Gamma_L)\Gamma_L\Gamma_s + V_0(1+\Gamma_L)(\Gamma_L\Gamma_s)^2 + \cdots$$

$$= V_0(1+\Gamma_L)\frac{1}{1-\Gamma_L\Gamma_s} = \frac{R_L}{R_L+R_s}V_s$$

(참고. $S = a + ax + ax^2 + ax^3 + \cdots = \frac{a}{1-x}$, $|x|<1$)





$$V(\infty) = V_S \frac{R_L}{R_S + R_L}$$

$$V(\infty) = V_S$$

연습문제 (2.10 - 2.12)

스미스 도표상에서 다음을 도시하라.

1. $r=0,\ -\infty < x < \infty$

2. $r=1,\ -\infty < x < \infty$

3. $g=0,\ -\infty < b < \infty$

4. $g=1,\ -\infty < b < \infty$

5. $r=1,\ x=0$

6. $r=\infty,\ x=0$

7. $r=0,\ x=0$

8. $g=1,\ b=0$

9. $g=\infty,\ b=0$

10. $g=0,\ b=0$

다음의 경우를 스미스 도표에 표시하라.

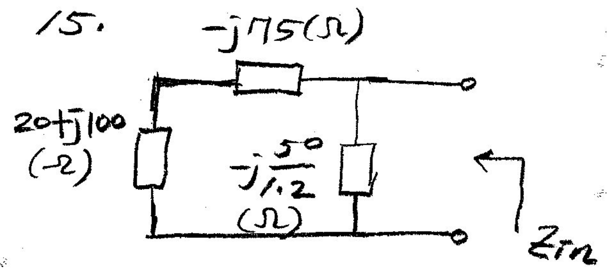
11. $Z_{in} = 50 + j50\,(\Omega),\ Z_0 = 50\,\Omega$

12. $Y_{in} = \dfrac{1}{50+j50}\,(S),\ Y_0 = \dfrac{1}{50}\,(S)$

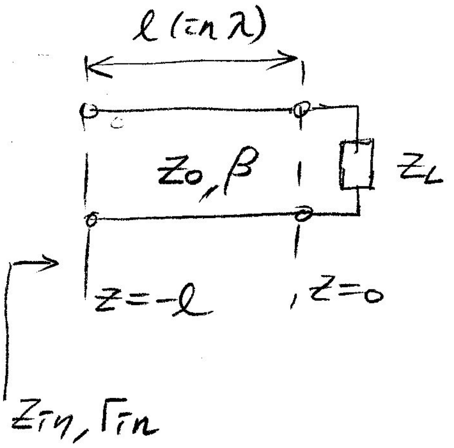
13. $Z_{in} = 25 - j25\,(\Omega),\ Z_0 = 50\,\Omega$

14. $Y_{in} = \dfrac{1}{25-j25}\,(S),\ Y_0 = \dfrac{1}{25}\,(S)$

15.



실습 (2.10-2.12) : PLab-04, Python programming



$\beta = \frac{2\pi}{\lambda}$, $\beta = \beta_0$ (free space β_0)

Find Z_{in}, Γ_{in}(polar) at $f_1 \le f \le f_2$

입력 :

$Z_L(\Omega)$ (complex) =

$Z_0(\Omega)$, l (m) =

f_1, f_2, Δf (Hz) =

출력 : 화일에 출력

RUN = (실행 번호)

$Z_L(\Omega)$ =

$Z_0(\Omega)$, l (m) =

f_1, f_2, Δf (Hz) =

f (Hz)	$Z_{in}(\Omega)$	mag(Γ_{in})	phase(Γ_{in})
10.0000e9	26.4736-j6.2784	0.1847	26.74
. . . .			
12.000e9	82.4124-j20.5198	0.3804	72.64

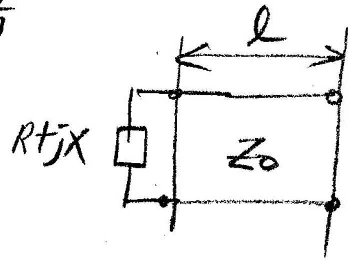
실습 (2.10−2.12) : GLab-04, Online Smith chart

1.

1) Quicksmith online Smith chart에 접속

2) 설정에서

Length Unit : Wave Lengths로 변경



3) 종단부하 : $R=100$, $X=100$으로 설정 (마우스 우클릭)

4) 종단부하 다음에 전송선 연결 : $Z_0=50$, $l=0.25\lambda$

5) 설정에서 Sweep Variable을 Frequency로 설정

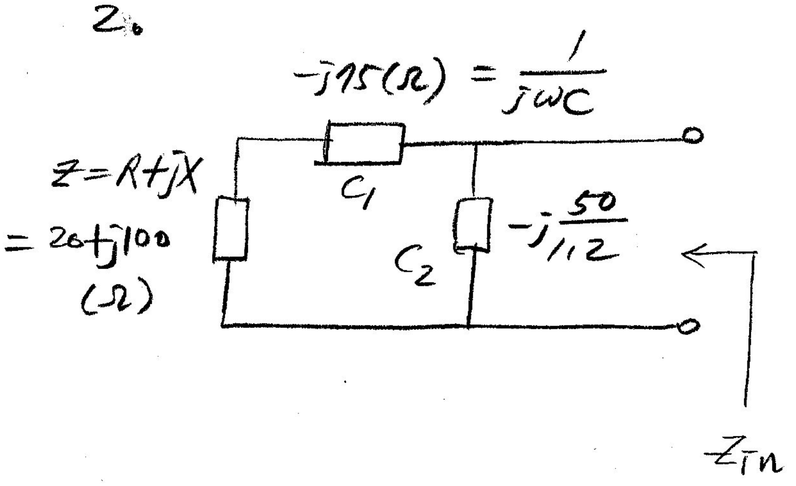
6) 설정에서 Sweep start : 1, sweep stop : 100, Sweep Step : 1

7) Sweep에서 Generate

8) $1 \le R \le 100$ sweep

9) $0.01 \le l \le 0.25$ Sweep (step 0.01)

2.



$f=100\,\text{MHz}$

$C : -jX = \frac{1}{j\omega C} \rightarrow C = \frac{1}{\omega X}$

$C_1 = 21.2\,\text{pF}$

$C_2 = 38.2\,\text{pF}$

QuickSmith를 이용하여 Z_{in}을 구하라.