2-6. 무손실 전송선로

신호가 전송선을 감쇠 없이 진행

$\boxed{R=0}$: 도체 손실=0 ← $\sigma_c = \infty$ (초전도체, 저항이 없는 도체)

$\boxed{G=0}$: 유전체 손실 =0 ← $\tan\delta_e = 0$

자성체 손실 =0 ← $\tan\delta_m = 0$

$$\gamma = \alpha + j\beta$$

$$\boxed{\alpha = 0}$$

$$j\beta = j\omega\sqrt{LC}, \quad \boxed{\beta = \omega\sqrt{LC}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

- TEM 선로

$\boxed{LC = \mu\varepsilon}$ (모든 TEM 선로에 적용됨)

$$\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon}, \quad \boxed{\beta = \beta_0\sqrt{\mu_r\varepsilon_r}}$$

$\boxed{\beta_0 = \omega\sqrt{\mu_0\varepsilon_0}}$: 진공의 전파상수

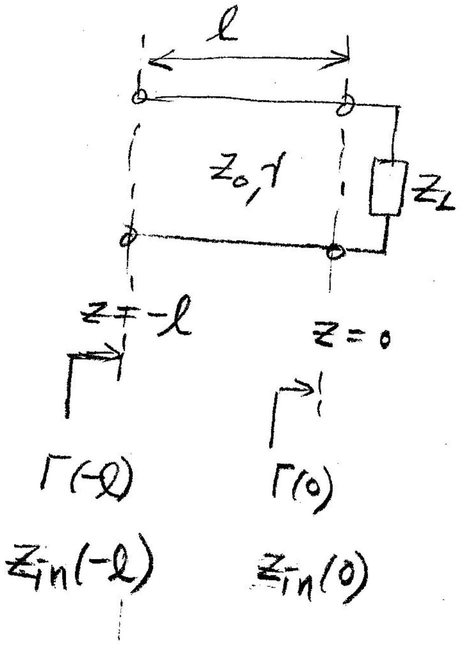
$\boxed{\lambda = \frac{2\pi}{\beta}} = \frac{2\pi}{\beta_0}\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\mu_r\varepsilon_r}}$: 파장

$\boxed{\lambda_0 = \frac{c}{f}}$: 진공에서의 파장, $c = 3\times10^8\,m/s$: 진공에서의 광속

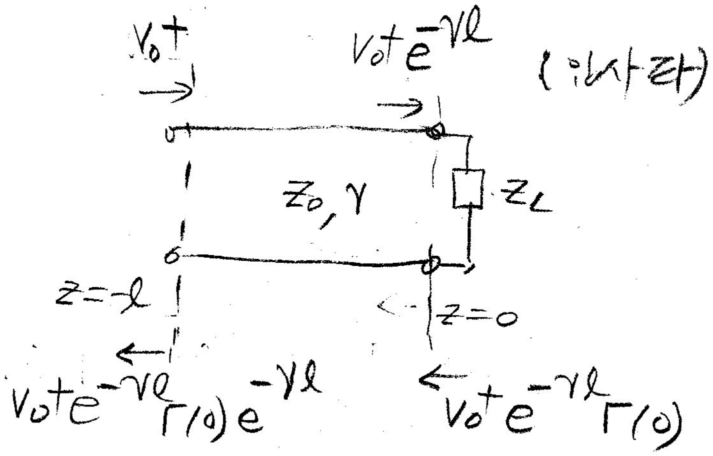
$\boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}}$: 위상속도

• 반사계수 (reflection coefficient) : 전송선에서 반사파와 입사파의 비

$$\Gamma \equiv \frac{V^-(z)}{V^+(z)} = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \Gamma(0)\, e^{2\gamma z}$$: (전압) 반사계수



$$\boxed{\Gamma(-l) = \Gamma(0)\, e^{-2\gamma l}}$$



$$\boxed{Z_{in} \equiv \frac{V(z)}{I(z)}}$$: 입력 임피던스 (impedance)

$z=0$:

$$V(0) = V_0^+ + V_0^-$$

$$I(0) = \frac{1}{Z_0}(V_0^+ - V_0^-)$$

$$\frac{V(0)}{I(0)} = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} = Z_0 \frac{1+\Gamma(0)}{1-\Gamma(0)} = Z_L$$

$$Z_{in} = Z_0 \frac{1+\Gamma}{1-\Gamma} \quad \text{at any } z$$

$$\boxed{\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{\bar{z}_{in} - 1}{\bar{z}_{in} + 1}}$$ at any z, $|\Gamma| \le 1$ if $Re(z_{in}) \ge 0$

$$\boxed{\bar{z}_{in} \equiv \frac{Z_{in}}{Z_0}}$$: 정규화된 입력 임피던스

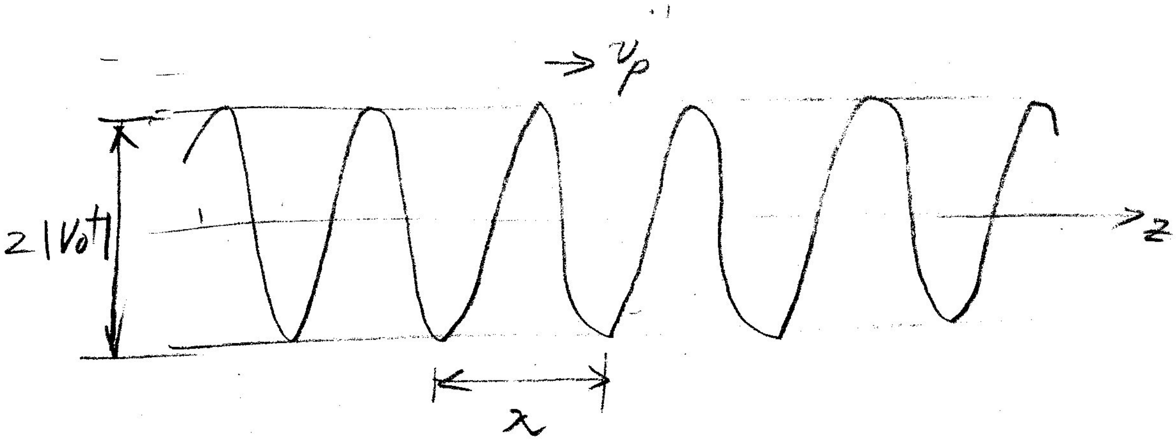
• 정재파(standing wave): 전송선에서 입사파와 반사파가 간섭을 일으켜 총파동의 크기가 주기적으로 최대와 최소가 반복되는 현상

$\alpha \doteq 0$ (무손실 선로)

$\Gamma = 0$ (무반사) $\rightarrow V_0^- = 0$

$$V(z) = V_0^+ e^{-j\beta z}$$

$$|V(z)| = |V_0^+| = \text{const}$$



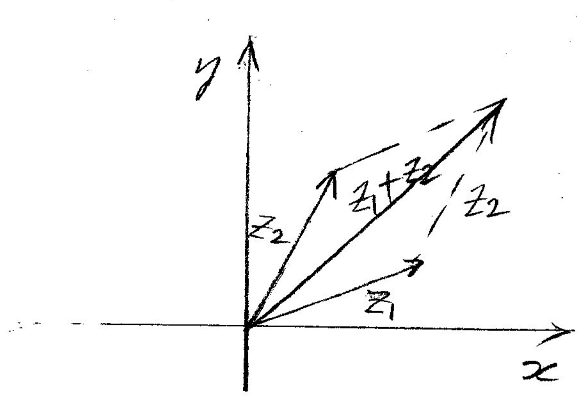
$\Gamma \neq 0$

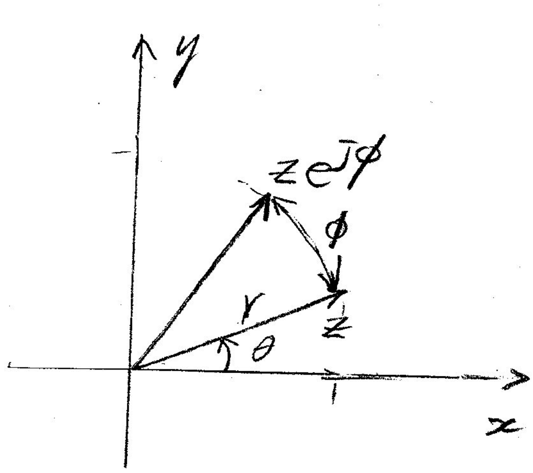
$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} = V_0^+ e^{-j\beta z}\left[1+\Gamma(0) e^{j2\beta z}\right]$$

$$|V(z)| = |V_0^+|\,\left|1+\Gamma(0) e^{j2\beta z}\right|$$

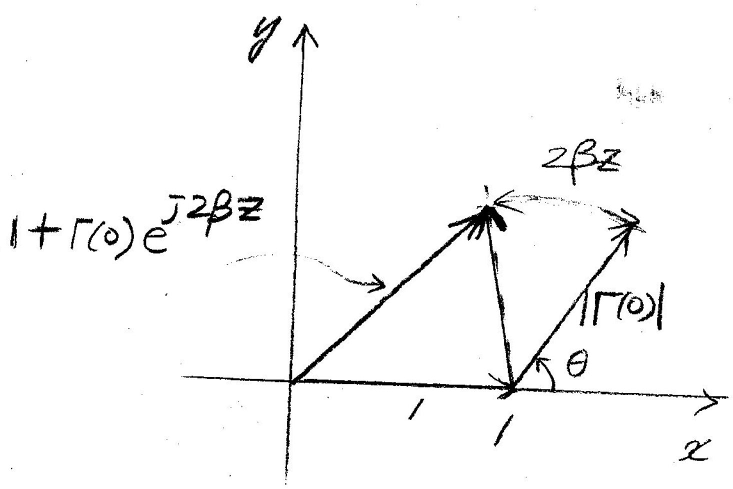
- 벡터의 합과 회전: 도식적 해법

$z_1 = x_1 + jy_1,\ z_2 = x_2 + jy_2\ ;\ z = re^{j\theta}$





$\Gamma(0) = |\Gamma(0)| e^{j\theta}$



$$\max\left|1+\Gamma(0)e^{j2\beta z}\right| = 1+|\Gamma(0)| \quad \text{at} \quad \measuredangle\Gamma(0)+2\beta z = 2n\pi$$

$$\min\left|1+\Gamma(0)e^{j2\beta z}\right| = |1-\Gamma(0)| \quad \text{at} \quad \measuredangle\Gamma(0)+2\beta z = 2n\pi+\pi$$

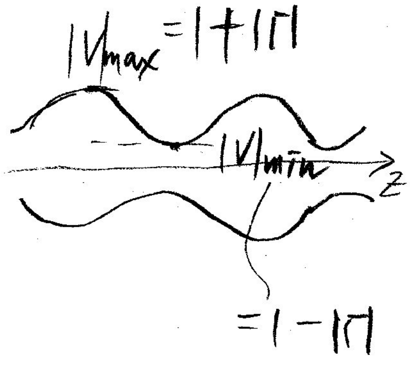
$$V(z) \equiv V_0^+ e^{-j\beta z}|S(z)|e^{j\measuredangle S(z)}, \quad S(z) = 1+\Gamma(0)e^{j2\beta z}$$

$$v(z,t) = \mathrm{Re}\left[V(z)e^{j\omega t}\right]$$

$$= |V_0^+|\,|S(z)|\cos\left[\omega t-\beta z+\measuredangle S(z)\right] \equiv A(z)\cos\left[\omega t-\beta z+\measuredangle S(z)\right]$$

z 에서의 정재파 진폭

$$A(z) \equiv |V_0^+|\,|S(z)| = |V_0^+|\,\left|1+\Gamma(0)e^{j2\beta z}\right|$$

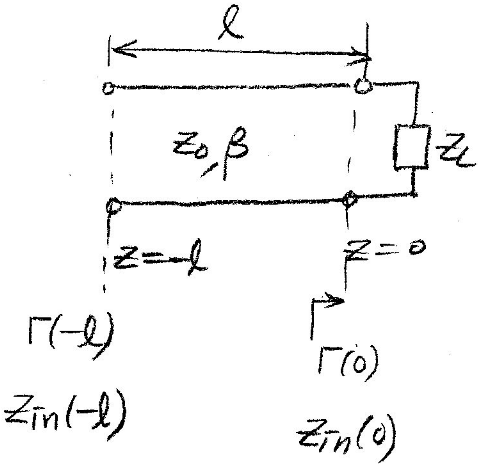


$$\boxed{S = \frac{|V|_{max}}{|V|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} = VSWR = SWR}$$

VSWR (voltage standing wave ratio) : 전압 정재파비

SWR (standing wave ratio) : 정재파비

2.7 입력 임피던스 (Input Impedance): 전송선의 입력단에서 전압을 전류로 나눈 값

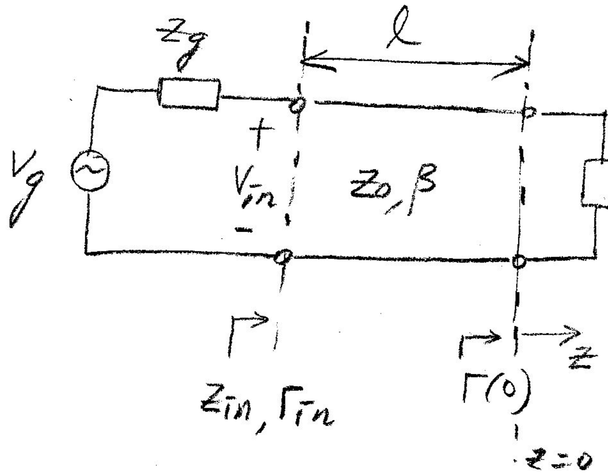


$Z_{in}(0) = Z_L$

$$\boxed{\Gamma(0) = \frac{Z_{in}(0) - Z_0}{Z_{in}(0) + Z_0}}$$

$\boxed{\Gamma(-l) = \Gamma(0)\, e^{-j2\beta l}}$: derivation omitted

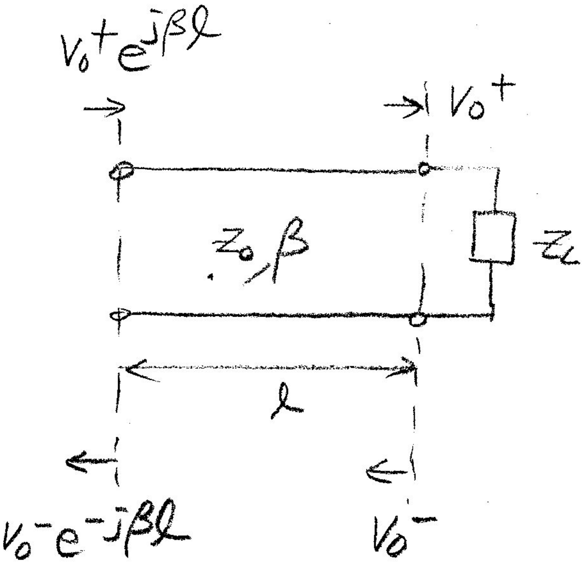
$$\boxed{Z_{in}(-l) = Z_0 \frac{1+\Gamma(l)}{1-\Gamma(-l)} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}}$$ derivation omitted



$$\boxed{\begin{aligned} V(z) &= V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \\ I(z) &= \frac{1}{Z_0}\left(V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right) \\ \frac{V_0^-}{V_0^+} &= \Gamma(0)\, e^{-j2\beta l} \\ \Gamma(0) &= \frac{Z_L - Z_0}{Z_L + Z_0} \end{aligned}}$$

$\Gamma(0) = \dfrac{Z_L - Z_0}{Z_L + Z_0}$, $\Gamma_{in} = \Gamma(0)\, e^{-j2\beta l}$, $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

$$V_{in} = V_g \frac{Z_{in}}{Z_g + Z_{in}} = V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l} = V_0^+\left[e^{j\beta l} + \Gamma(0) e^{-j\beta l}\right]$$



$$\boxed{V_0^+ = V_g \frac{Z_{in}}{Z_g + Z_{in}} \frac{1}{e^{j\beta l} + \Gamma(0) e^{-j\beta l}}}$$

2-7b. 유손실 선로

$$\boxed{\gamma = \alpha + j\beta z}$$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} = V_0^+ [e^{-\gamma z} + \Gamma(0) e^{\gamma z}]$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = \frac{V_0^+}{Z_0} [e^{-\gamma z} - \Gamma(0) e^{\gamma z}]$$

$$\boxed{\Gamma(-l) = \Gamma(0) e^{-2\gamma l}}$$

$V^+(z) = V_0^+ e^{-\gamma z}$: $+z$ 방향으로 진행하는 파동

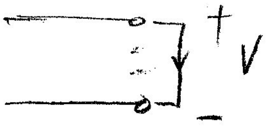
$V^-(z) = V_0^+ e^{\gamma z}$: $-z$ 방향으로 진행하는 파동

$$\Gamma(-l) = \frac{V_0^-(-l)}{V_0^+(-l)} = \frac{V_0^-}{V_0^+} e^{-2\gamma l}, \quad \Gamma(0) = \frac{V_0^-}{V_0^+}$$

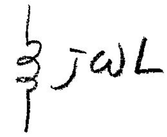
$$\boxed{Z_{in}(-l) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}}$$

2·8 단락/개방 전송선, 1/4 파장 변환기

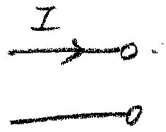
단락 : $V=0$, $Z_L=0$ (short)



→



개방 : $I=0$, $Y_L=\frac{1}{Z_L}=0$ (open)



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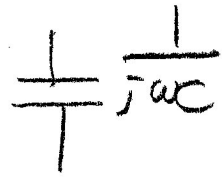
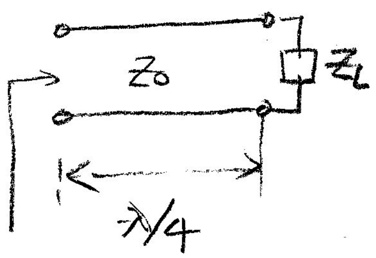


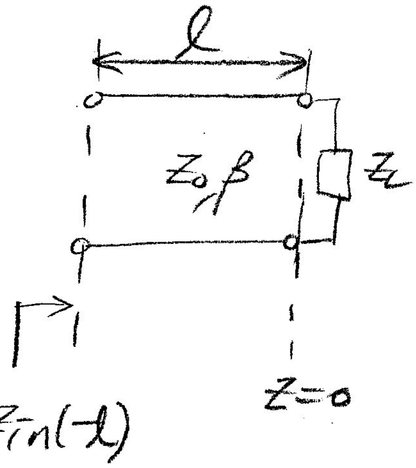
1/4 파장 변환기(quarter-wave transformer)



$Z_{in}=Z_0$ if $Z_0=\sqrt{Z_{in}Z_L}$

o. 전송선상의 임피던스공식

$$\boxed{Z_{in}(-l)=Z_0\frac{Z_L+jZ_0\tan\beta l}{Z_0+jZ_L\tan\beta l}}$$



o. 단락 전송선

$Z_L=0$

$$\boxed{Z_{in}=jZ_0\tan\beta l\equiv j\omega L}\quad \left(\beta l<\frac{\pi}{2}\right)$$

o. 개방전송선

$Z_L = \infty$

$$\boxed{Z_{in} = \frac{Z_0}{j\tan\beta l} = -jZ_0\cot\beta l} \quad (\beta l < \tfrac{\pi}{2})$$

$$Y_0 = \frac{1}{Z_0}$$

o. 1/4 파장 변환기

$$\beta l = \frac{\pi}{2} \rightarrow l = \frac{1}{\beta}\frac{\pi}{2} = \frac{\lambda}{2\pi}\frac{\pi}{2} = \frac{\lambda}{4}$$

$$\tan\beta l = \tan\frac{\pi}{2} = \infty$$

$$Z_{in} = \frac{Z_0^2}{Z_L} \rightarrow Z_0 = \sqrt{Z_{in}Z_L}$$

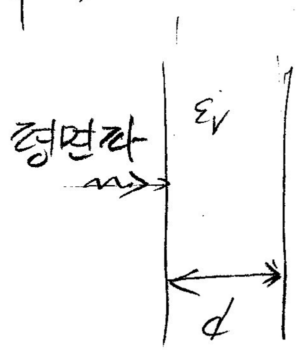
Z_{in}, Z_L을 주어지고 Z_0를 구한다.

o. 1/2 파장 선로

$$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi, \quad \tan\beta l = \tan\pi = 0$$

$$\boxed{Z_{in} = Z_L}$$

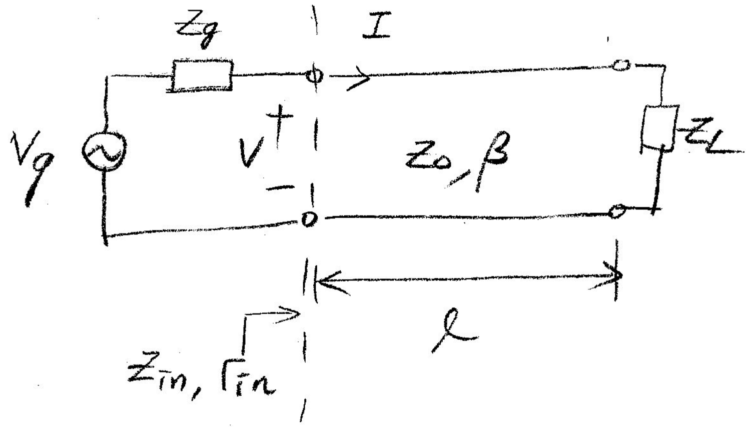
예: 1/2 파장 레이돔



$$d = \frac{\lambda}{2} = \frac{\lambda_0}{2\sqrt{\varepsilon_r}}$$: 반파장 레이돔

→ 평면파가 반사없이 레이돔 투과

2.9 전송선의 전력흐름



$\boxed{P_{av} = \frac{1}{2} Re(V^* I)}$: 시평균 소모전력 , $*$: 공액(켤레) 복소수

$\boxed{V = Z_{in} I \ , \quad I = Y_{in} V \ , \quad Y_{in} = \frac{1}{Z_{in}}}$

$$\boxed{P_{av} = \frac{1}{2} Re(Z_{in}) |I|^2 = \frac{1}{2} Re(Y_{in}) |V|^2}$$

$$V = V_0^+ + V_0^- = V_0^+ (1 + \Gamma)$$

$$I = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} (1 - \Gamma)$$

$$V^* I = (V_0^+)^* (1 + \Gamma^*) \frac{V_0^+}{Z_0} (1 - \Gamma)$$

$$= \frac{|V_0^+|^2}{Z_0} (1 - |\Gamma|^2 + \Gamma^* - \Gamma)$$

$$Re(V^* I) = \frac{|V_0^+|^2}{Z_0} (1 - |\Gamma|^2)$$

$$P_{av} = \frac{1}{2} \frac{|V_0^+|^2}{Z_0} - \frac{1}{2} \frac{|V_0^+|^2}{Z_0} |\Gamma|^2 = P^+ - P^-$$

$$\boxed{P_{av} = P^+ - P^-}$$

$\boxed{P^+ = \frac{|V_0^+|^2}{2 Z_0}}$: 입사전력 , $\boxed{P^- = P^+ |\Gamma|^2}$: 반사전력

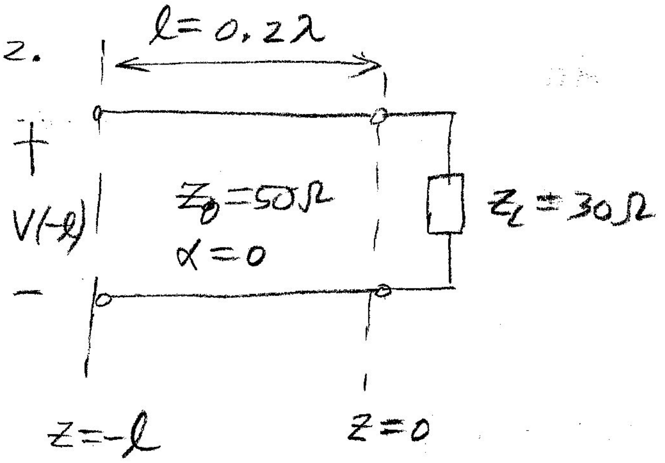
연습문제 (2.6–2.9)

1. $R=0,\ G=0,\ L=200\,\mathrm{nH/m},\ C=10\,\mathrm{PF/m},\ f=1\,\mathrm{GHz}$

(a) $Z_0(\Omega)=$

(b) $\lambda(m)=$

2.



(a) $\Gamma(0)=$

(b) $\Gamma(-l)=$

(c) $Z_{in}(-l)=$

(d) $V(-l)=10$ 일경우 $z=-l$에서 $P_{av}=$

(e) $z=-l$에서 입사전력 $P_i=$

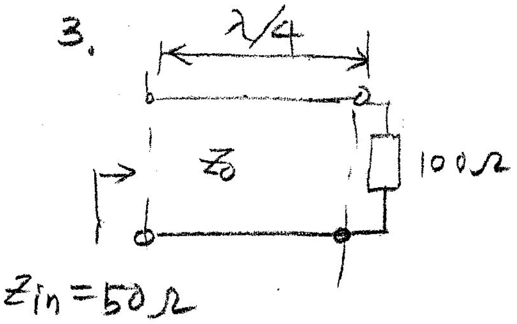
(f) $z=-l$에서 반사전력 $P_r=$

(g) VSWR =

(h) $Z_L=0$ 일 경우 $Z_{in}(-l)=$

(i) $Z_L=\infty$ 일 경우 $Z_{in}(-l)=$

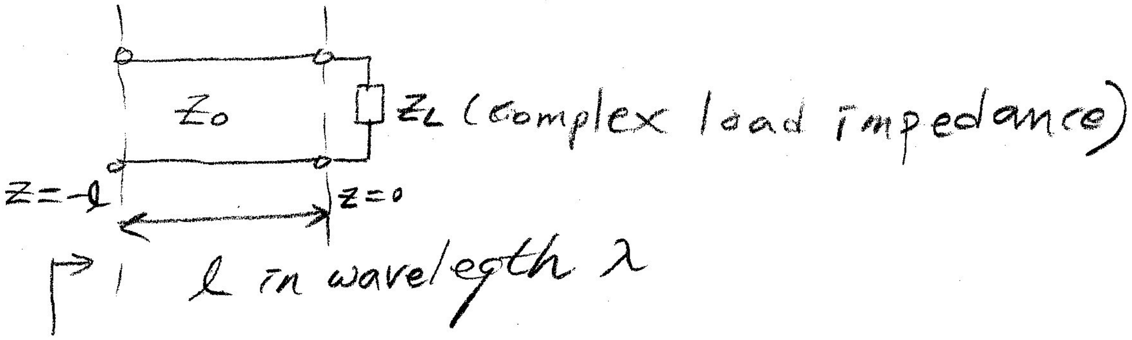
3.



$Z_0=$

실습 (2.6–2.9)

PLab-03



$Z_{in}(-\ell)$

$\Gamma_{in}(-\ell)$

입력 :

$Z_L(\Omega) =$

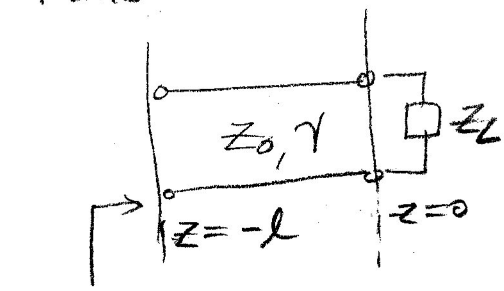
$Z_0(\Omega), \ell(\lambda) =$

출력 :

$|\Gamma_{in}(-\ell)|, \measuredangle \Gamma_{in}(-\ell) =$

$Z_{in}(-\ell) =$

PLab-03b



$\Gamma_{in}(-\ell)$

$Z_{in}(-\ell)$

입력 :

$\alpha(Np/m) =$

$\beta/\beta_0 =$

$Z_0(\Omega), \ell(m) =$

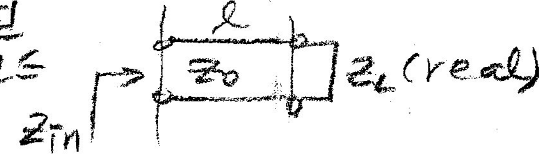
$f(Hz) =$

출력 :

$\Gamma_{in}(-l) =$

$Z_{in}(-l) =$

종단에 부하가 연결된
전송선의 입력 임피던스
Glab-03



$$\frac{Z_{in}}{Z_0} = \frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l} = \frac{(Z_L/Z_0) + j\tan\beta l}{1 + j(Z_L/Z_0)\tan\beta l}$$

$$= \frac{1}{1+[(Z_L/Z_0)t]^2}\Big[Z_L/Z_0 + (Z_L/Z_0)t^2$$

$$+ j\left[-(Z_L/Z_0)^2 t + t\right]\Big], \quad t = \tan\beta l, \quad \beta l = 2\pi x, \quad x = \frac{l}{\lambda}$$

$$y = \frac{(Z_L/Z_0)(1+\tan^2(2\pi x))}{1+(Z_L/Z_0)^2\tan^2(2\pi x)} = Re(Z_{in}/Z_0) = r, \text{ red}$$

$$y = \frac{[1-(Z_L/Z_0)^2]\tan(2\pi x)}{1+(Z_L/Z_0)^2\tan^2(2\pi x)} = Im(Z_{in}/Z_0) = x, \text{ blue}$$

$Z_L = 0 - 1000$ (Z_L으로 표현)

$Z_0 = 50$

$x = 0.15$, black (그래프 색깔이 커서를

그래프 범위 : $-1 \le x \le 1$, $-10 \le y \le 10$

1. $Z_L = 5$, $Z_0 = 50$일때 graph plot

2. 다음표 작성 : $Z_0 = 50\Omega$

$x = l/\lambda$	$Z_L(\Omega)$	$Re(Z_{in}/Z_0)$	$Im(Z_L/Z_0)$
0.15	0		
0.15	5		
0.15	50		
0.15	500		
0.15	5000		

GLab-03b : Standing wave I

1. Graph Plotter 접속

2. 다음 그래프 입력

$$y = \cos\left(\frac{2\pi}{10}x - \frac{2\pi}{2}t\right)$$: 입사파 , red

$$y = R\cos\left(\frac{2\pi}{10}t + \frac{2\pi}{2}t + a\right)$$: 반사파 , blue

반사계수 $\Gamma = Re^{ja}$

$$y = \cos\left(\frac{2\pi}{10}t - \frac{2\pi}{t}\right) + R\cos\left(\frac{2\pi}{10}t + \frac{2\pi}{2}t + a\right)$$: 입사파 + 반사파

green

$R = 0 \quad (0 \le R \le 1)$

$a = 0 \quad (0 \le a \le 2\pi$

$t = 0 \quad (0 \le t \le 2,\ \Delta t = 0.01,\ 0.2X)$

축범위 : $-15 \le x \le 15$, $-3 \le y \le 3$

3.
1) $R = 0.5,\ a = 0,\ t = 0$ 인 경우 그래프 도시
2) $R = 0.5,\ a = 0,\ t = 0.4$ 인 경우 그래프 도시

3) 다음 표 완성 ; $0 \le t \le 2$ 로 animation

R	a	x_{max}	$\lvert V \rvert_{max}$	x_{min}	$\lvert V \rvert_{min}$
0.5	0				
0.5	π				

4) $R = 1,\ a = 0,\ 0 \le t \le 2$ 일 경우 입사파 + 반사파가 x 방향으로 진행하지 않고 정지하며 진동하는 것을 확인하라.

x_{max} : 입사파 + 반사파의 진폭이 x 축상에서 최대가 되는 x의 위치. $\lvert V \rvert_{max}$는 그 때의 진폭

x_{min} : 〃 최소

GLab-03c: Standing wave II

GLab-03b의 그래프에 다음 식으로 주어지는 standing wave amplitude envelope 추가. 아래 두 식은 정재파의 envelope를 표현한다.

$$y = \sqrt{\left[1+R\cos\left(2\frac{2\pi}{10}x+\alpha\right)\right]^2+\left[R\sin\left(2\frac{2\pi}{10}x+\alpha\right)\right]^2}\ ,\ \text{black}$$

$$y = -\sqrt{\left[1+R\cos\left(2\frac{2\pi}{10}x+\alpha\right)\right]^2+\left[R\sin\left(2\frac{2\pi}{10}x+\alpha\right)\right]^2}\ ,\ \text{black}$$

1. $R=0.5$, $a=0$, $t=0.58$일 때 그래프 도시
2. $R=0.5$, $a=-0.4\pi$, $t=0.58$일 때 그래프 도시
3. $a=0$에서 $a=-0.4\pi$로 변할 때 접근선에는 어떤 변화가 생기는가?