

Scattering Coefficients and Circuit Analysis

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INTRODUCTION

Consider a simple two-conductor transmission line, such as a coaxial cable or parallel pair. These types of transmission lines support transverse electromagnetic (TEM) waves and therefore the wave transmission can be expressed purely in terms of the voltage between the conductors and the current flowing through the conductors. If the line is terminated by a load (Fig. 1), that is not perfectly matched to the transmission line, then some of the incident signal will be reflected back from the load. In terms of the voltage and current along the line we have, at a point z ,

$$V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z} \quad (1)$$

and

$$I(z) = \frac{1}{Z_0} (V^+ e^{-j\beta z} - V^- e^{+j\beta z}) \quad (2)$$

where β is the phase constant (rads m^{-1}), V^+ is the voltage amplitude of the forward propagating signal and V^- is that of the reverse. The $e^{-j\beta z}$ terms denote forward propagation (towards the load), the $e^{+j\beta z}$ terms denote reverse propagation (away from the load). Z_0 is the characteristic impedance of the transmission line and is dependent on the geometry and material of the structure. For simplicity, the time dependence has been omitted from equations (1) and (2). The actual voltage and current is given by $\text{Re}\{V e^{j\omega t}\}$ and $\text{Re}\{I e^{j\omega t}\}$, respectively.

Suppose we choose a reference plane at $z = 0$. We define the reflection coefficient at this point by

$$\Gamma = \frac{V^-}{V^+} \quad (3)$$

We further define the impedance of the termination by

$$Z = \frac{V}{I} \quad (4)$$

where the z dependence has been omitted. Note that we are free to set the reference plane anywhere along the line, this might be at the connector interface, at the load element or some distance along the line.

Substituting equations (1) and (2) into equation (4) gives the well-known relationship between the reflection coefficient of the termination and its impedance at the reference plane:

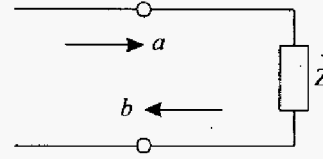


Fig. 1. Transmission line terminated in a mismatched load.

$$Z = Z_0 \frac{V^+ + V^-}{V^+ - V^-} = Z_0 \frac{1 + \Gamma}{1 - \Gamma} \quad (5)$$

Similarly, the relationship between the admittance of the termination and its reflection coefficient is given by

$$Y = Z^{-1} = Y_0 \frac{V^+ - V^-}{V^+ + V^-} = Y_0 \frac{1 - \Gamma}{1 + \Gamma} \quad (6)$$

where Y_0 is the admittance of the transmission line, $Y_0 = Z_0^{-1}$. Another useful expression is given by solving for Γ in equation (5):

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{Y_0 - Y}{Y_0 + Y} \quad (7)$$

Clearly Γ is dependent on the impedance of the termination but we note that it is also dependent on the characteristic impedance of the line. A knowledge of Z_0 is therefore required to define Γ .

Relationships for the power flow can also be defined. It is well known that, using phasor notation, the RMS power is given by

$$P = \frac{1}{2} \text{Re}\{VI^*\}$$

where $*$ denotes the complex conjugate. Substituting for V and I from equation (1) and (2) yields, for $z = 0$,

$$P = \frac{1}{2Z_0} \text{Re}\{(V^+ + V^-)(V^+ - V^-)^*\}$$

which gives

$$P = \frac{1}{2Z_0} (|V^+|^2 - |V^-|^2),$$

where we have made use of the fact that $V^+ * V^- - V^+ V^- *$ is purely imaginary and $V^+ V^{+*} = |V^+|^2$. We also assume that Z_0 is purely real. We notice that the power is dependent on the characteristic impedance. We can however define the network in terms of another set of amplitude constants such that the impedance is not required in power calculations.

Let

$$V = \sqrt{Z_0} (ae^{-j\beta z} + be^{+j\beta z}) \quad (8)$$

and

$$I = \frac{1}{\sqrt{Z_0}} (ae^{-j\beta z} - be^{+j\beta z}) \quad (9)$$

where a and b are defined as the wave amplitudes of the forward and backward propagating signal. With reference to equation (1) it is easy to show that $a = V^+ / \sqrt{Z_0}$ and $b = V^- / \sqrt{Z_0}$. We find now, that if Z_0 is purely real, that the power is simply given by

$$P = \frac{1}{2} (|a|^2 - |b|^2). \quad (10)$$

$|a|^2 / 2$ is the power in the forward propagating wave and $|b|^2 / 2$ is the power in the backward propagating wave. Equation (10) is a very satisfying result since it allows propagation to be defined in terms of wave amplitudes that are directly related to the power in the wave. This is particularly useful for measurement purposes since power is more easy to measure than voltage or current. In fact we shall see that for many microwave networks, voltage and current cannot be measured or even defined.

The analysis so far has dealt with one-port devices. These are completely specified by their impedance Z or reflection coefficient Γ (with respect to Z_0). The more important case of the two-port, or multi-port device, requires a more complicated model.

SCATTERING PARAMETERS

Consider the two-port network shown in Fig. 2. There will, in general, be waves propagating into and out of each of the ports. If the device is linear, the output signals can be defined in terms of the input signals. Thus,

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad (11)$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \quad (12)$$

where b_1 and b_2 are the wave amplitudes of the signals flowing out of ports 1 and 2, respectively. Similarly, a_1 and a_2 are the wave amplitudes of the signals flowing into ports 1 and 2, respectively. S_{11} , S_{21} , S_{12} and S_{22} are the scattering coefficients or scattering parameters. Using the definition of wave amplitude the voltages at port 1 and port 2 are given by

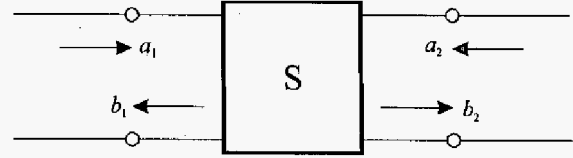


Fig. 2. Two-port device represented by S-parameter matrix.

$$V_1 = \sqrt{Z_{01}} (a_1 + b_1)$$

and

$$V_2 = \sqrt{Z_{02}} (a_2 + b_2),$$

respectively. Where it is assumed that the characteristic impedance is different at each port: Z_{01} at port 1 and Z_{02} at port 2. Similarly the currents entering port 1 and port 2 are

$$I_1 = \frac{1}{\sqrt{Z_{01}}} (a_1 - b_1)$$

and

$$I_2 = \frac{1}{\sqrt{Z_{02}}} (a_2 - b_2),$$

respectively. Equations (11) and (12) can be more neatly written in matrix notation:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (13)$$

or $\mathbf{b} = \mathbf{S}\mathbf{a}$ where

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad \text{and} \quad \mathbf{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$\mathbf{S}$ is the scattering matrix or S-parameter matrix of the two-port network.

If port 2 is terminated by a perfect match of impedance Z_{02} , i.e. all of the incident energy is absorbed in the termination, we have the following properties

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad \text{and} \quad S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

Similarly, if port 1 is terminated by a perfect match of impedance Z_{01} then

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} \quad \text{and} \quad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

With the definitions above we can obtain some insight into the meaning of the individual S-parameters. S_{11} is the reflection coefficient at port 1 with port 2 terminated in a matched load. It therefore gives a measure of the mismatch due to the network and not any other devices that may be connected to port 2. S_{21} is the transmission coefficient from port 1 to port 2 with port 2 terminated in a perfect match. It gives a measure of the amount of signal that is transmitted from port 1 to port 2. S_{22} and S_{12} are similarly defined with S_{22} giving the reflection from port 2 and S_{12} the transmission from port 2 to port 1.

The S-parameter representation equally applies to multi-port devices. For an n -port device, the S-parameter matrix is given by

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

where b_k is the amplitude of the wave travelling away from the junction at port k . Similarly, a_k is the wave amplitude travelling into the junction at port k . The S-parameters are defined as

$$S_{ij} = \left. \frac{b_i}{a_j} \right|_{a_k=0 \text{ for } k \neq j}$$

IMPEDANCE AND ADMITTANCE PARAMETERS

Similar expressions to equations (5) and (7) can be obtained for n -port devices. If we have an n -port device then the voltage and current at the reference plane of port k is given by

$$V_k = \sqrt{Z_{0k}}(a_k + b_k) \quad (14)$$

and

$$I_k = \frac{1}{\sqrt{Z_{0k}}}(a_k - b_k) \quad (15)$$

where Z_{0k} is the characteristic impedance of the transmission line connected to port k . Equations (14) and (15) can be written in matrix notation as

$$\mathbf{V} = \mathbf{Z}_0^{1/2}(\mathbf{a} + \mathbf{b}) \quad (16)$$

and

$$\mathbf{I} = \mathbf{Z}_0^{-1/2}(\mathbf{a} - \mathbf{b}), \quad (17)$$

respectively. Where $\mathbf{a}$ and $\mathbf{b}$ are column vectors containing the wave amplitudes and $\mathbf{V}$ and $\mathbf{I}$ are column vectors containing the port voltages and currents:

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \quad \text{and} \quad \mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

$\mathbf{Z}_0$ is a diagonal matrix with Z_{0k} as its diagonal elements:

$$\mathbf{Z}_0 = \begin{bmatrix} Z_{01} & 0 & \cdots & 0 \\ 0 & Z_{02} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z_{0n} \end{bmatrix} \quad (18)$$

$\mathbf{Z}_0^{1/2}$ denotes a diagonal matrix with $\sqrt{Z_{0k}}$ as its diagonal elements. Often, the characteristic impedance of each of the ports is identical, in which case each of the diagonal elements are equal. From (8) and (9) we have

$$\mathbf{a} = \frac{1}{2} \mathbf{Z}_0^{-1/2}(\mathbf{V} + \mathbf{Z}_0 \mathbf{I}) \quad (19)$$

and

$$\mathbf{b} = \frac{1}{2} \mathbf{Z}_0^{-1/2}(\mathbf{V} - \mathbf{Z}_0 \mathbf{I}). \quad (20)$$

Let $\mathbf{V} = \mathbf{Z}\mathbf{I}$, where $\mathbf{Z}$ is the impedance matrix; extensively used in electrical circuit theory:

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix}$$

Also let $\mathbf{I} = \mathbf{Y}\mathbf{V}$ where $\mathbf{Y}$ is the admittance matrix:

$$\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix}$$

We note that $Z = Y^{-1}$. The individual elements of the impedance and admittance matrices are defined as

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k=0 \text{ for } k \neq j} \quad \text{and} \quad Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k=0 \text{ for } k \neq j}$$

That is Z_{ij} is the ratio of voltage at port i to that of the current at port j with all other port currents set to zero, i.e. short circuit. Y_{ij} is defined as the ratio of current at port i to the voltage at port j with all other port voltages set to zero, i.e. open circuit. Substituting $V = ZI$ and $b = Sa$ into equations (19) and (20) yields

$$Z = Z_0^{1/2} (U - S)^{-1} (U + S) Z_0^{1/2} = Y^{-1} \quad (21)$$

or solving for S

$$S = Z_0^{-1/2} (Z - Z_0) (Z + Z_0)^{-1} Z_0^{1/2} \quad (22)$$

where U is the unit matrix:

$$U = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Examples of equations (21) and (22) for two-port networks are given in Appendix C.

Z and Y parameters can be very useful in the analysis of microwave networks since they can be related directly to simple π or T networks (refer to Table 1). These circuits are fundamental in lumped element circuits such as attenuators and are also important in equivalent circuits for waveguide junctions and discontinuities.

Examples of S-parameter Matrices

Table 1 shows some common examples of microwave networks and their network parameters. Parameters are only shown for the simplest form. The associated S , Z or Y parameters can be determined using equations (21) and (22). In each case it is assumed that the characteristic impedance is identical at each port and equal to $Z_0 = Y_0^{-1}$.

We notice from the table that the S -parameter matrices are symmetrical, i.e., $S_{mn} = S_{nm}$. This is a demonstration of reciprocity in microwave networks and applies to most networks (see Appendix A). A property of lossless scattering matrices is also seen for the line

section. Here, $S^T S^* = U$, which applies to all lossless networks, refer to Appendix B.

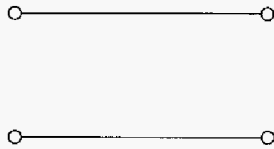
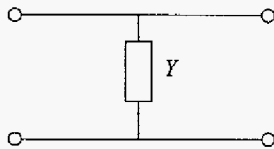
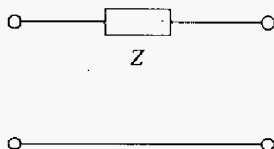
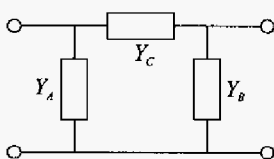
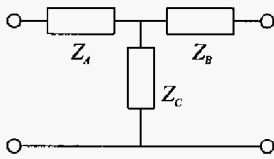
Circuit	Network parameters
Loss less transmission line of length L , phase constant β and characteristic impedance Z_0 	$S = \begin{bmatrix} 0 & e^{-j\beta L} \\ e^{-j\beta L} & 0 \end{bmatrix}$
Shunt admittance Y 	$S = \frac{1}{Y + 2Y_0} \begin{bmatrix} -Y & 2Y_0 \\ 2Y_0 & -Y \end{bmatrix}$
Series impedance Z 	$S = \frac{1}{2Z_0 + Z} \begin{bmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{bmatrix}$
π network 	$Y = \begin{bmatrix} Y_A + Y_C & -Y_C \\ -Y_C & Y_B + Y_C \end{bmatrix}$
T network 	$Z = \begin{bmatrix} Z_A + Z_C & Z_C \\ Z_C & Z_B + Z_C \end{bmatrix}$

Table 1. Network parameters for common microwave networks.

CASCADE PARAMETERS

Another useful transformation of the S-parameter matrix is the cascade matrix. The two-port cascade matrix is given by

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix} \quad (23)$$

where we notice that now the wave amplitudes on port 1 are given in terms of the wave amplitudes on port 2. Note that some textbooks interchange a_1 with b_1 and b_2 with a_2 . Comparing equation (23) with equation (13) gives the following relationships between the cascade matrix elements and the scattering coefficients:

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} 1 & -S_{22} \\ S_{11} & S_{12}S_{21} - S_{11}S_{22} \end{bmatrix} \quad (24)$$

Similarly, the reverse transform is given by

$$S = \frac{1}{T_{11}} \begin{bmatrix} T_{21} & T_{22}T_{11} - T_{12}T_{21} \\ 1 & -T_{12} \end{bmatrix}. \quad (25)$$

Suppose we have two two-port devices cascaded together (refer to Fig. 3). The first network is given by

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$

and the second network,

$$\begin{bmatrix} a'_1 \\ b'_1 \end{bmatrix} = \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{bmatrix} b'_2 \\ a'_2 \end{bmatrix}$$

where, by inspection of Fig. 3 we see that

$$\begin{bmatrix} b_2 \\ a_2 \end{bmatrix} = \begin{bmatrix} a'_1 \\ b'_1 \end{bmatrix}.$$

Therefore,

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{bmatrix} b'_2 \\ a'_2 \end{bmatrix}.$$

We see that in order to calculate the input wave amplitudes in terms of the output amplitudes we simply multiply the cascade matrices together. Often the cascaded two-port is converted back to an S-parameter matrix using equation (25). Any number of cascaded

two-port networks can then be replaced by a single equivalent two-port network.

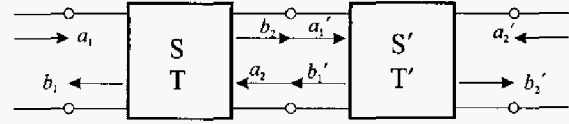


Fig. 3. Two two-port networks cascaded together.

CHARACTERISTIC IMPEDANCE

We have seen that a microwave network can be characterised in terms of its S-parameters and that the S-parameters are defined with respect to the characteristic impedance at the ports of the network. A fundamental understanding of the nature of Z_0 is therefore essential in microwave circuit analysis and measurement. Unfortunately, the true nature of characteristic impedance is often overlooked by microwave engineers and Z_0 is usually considered to be a real valued constant such as 50 Ω . In many cases this is a very good assumption. However, the careful metrologist does not make assumptions and the true nature of the characteristic impedance is imperative in precision microwave measurements. In fact without a knowledge of the characteristic impedance, S-parameter measurements have little meaning and this lack of knowledge is so often the cause of poor measurements. This is particularly important in measured S-parameters from network analysers. S-parameters measured on a network analyser are with respect to the Z_0 of the calibration items used to calibrate the analyser. If this value is ill-defined then so are the measured S-parameters.

Characteristic Impedance in Real Transmission Lines

If a TEM or quasi-TEM line contains dielectric and conductive losses then equations (1) and (2) become

$$V(z) = V^+ e^{-\gamma z} + V^- e^{+\gamma z} \quad (26)$$

and

$$I(z) = \frac{1}{Z_0} (V^+ e^{-\gamma z} - V^- e^{+\gamma z}) \quad (27)$$

where the complex propagation constant is defined as $\gamma = \alpha + j\beta$. Equations (26) and (27) are very similar to equations (1) and (2) however the attenuation constant α adds an exponential decay to the wave's amplitude

as it propagates along the line. It is easy to show that in terms of the transmission line's per length series impedance Z and shunt admittance Y , that the propagation constant γ and characteristic impedance Z_0 are given by [ii]

$$\gamma = \sqrt{ZY} \quad \text{and} \quad Z_0 = \sqrt{\frac{Z}{Y}}.$$

In the loss less case $Z = j\omega L$ and $Y = j\omega C$, representing the series inductance of the conductors and the shunt capacitance between them. The complex propagation constant then reduces to the familiar phase constant $j\beta$ and Z_0 degenerates to a real valued constant dependent only on L and C :

$$\gamma = j\beta = j\omega\sqrt{LC} \quad \text{and} \quad Z_0 = \sqrt{\frac{L}{C}}.$$

In lossy lines there is a series resistive component due to conduction losses and a shunt conductance due to dielectric losses. Hence, $Z = R + j\omega L$ and $Y = G + j\omega C$ and therefore,

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (28)$$

and

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \frac{R + j\omega L}{\gamma} \quad (29)$$

where R , L , G and C are often functions of ω . Two very important facts about Z_0 are immediately evident from equation (29): Z_0 is complex and a function of frequency. Therefore, the assumption that Z_0 is a real valued constant that is independent of frequency is only an approximation. Fortunately, for many transmission lines the loss is small. In this case $R \ll \omega L$ and $G \ll \omega C$ and an approximate expression for the propagation constant is obtained by using a first order binomial expansion. Thus,

$$\alpha \approx \frac{1}{2}\sqrt{LC}\left(\frac{R}{L} + \frac{G}{C}\right), \quad \beta \approx \omega\sqrt{LC} \quad \text{and} \quad Z_0 \approx \sqrt{\frac{L}{C}}.$$

To first order Z_0 is identical to the loss less expression and this is why the assumption that Z_0 is a real valued constant is so often used. There are many cases however when this approximation is far from valid. For example transmission lines and waveguides operating at millimetre-wave frequencies often have very large losses due to the increase of conduction and dielectric loss with frequency. In these cases, precision measurements must consider the complex nature of the transmission line. Furthermore, at low frequencies where ω is small, we find that $R \gg \omega L$ and $G \gg \omega C$.

The complex nature of both γ and Z_0 then plays a very important role.

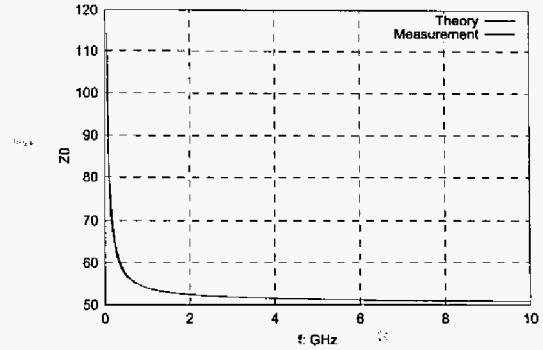


Fig. 4. Real part of characteristic impedance of CPW transmission line on GaAs.

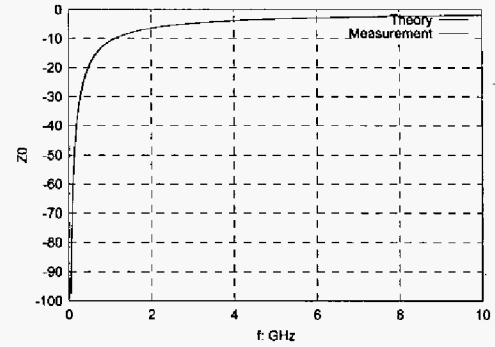


Fig. 5. Imaginary part of characteristic impedance of CPW transmission line on GaAs.

By way of example Figures 4 and 5 show how the real and imaginary parts of Z_0 vary with frequency for a coplanar waveguide (CPW). The parameters of the line are typical for a microwave monolithic integrated circuit (MMIC) with a 400 μm thick Gallium Arsenide (GaAs) substrate and gold conductors of 1.2 μm thickness. We see that above a few GHz the real component of Z_0 approaches the nominal 50 Ω of the design but with a small imaginary part of a few ohms. Below 2 GHz the picture is very different with a rapid increase in the real component of Z_0 and a large decrease in the imaginary component. Although the results are shown for CPW, similar results would be seen for microstrip, stripline and even coaxial cable.

Characteristic Impedance in Non-TEM Waveguides

The usual definition of characteristic impedance is the ratio of the forward voltage to forward current. These are easily determined for simple TEM transmission lines such as coaxial cable where the voltage between the two conductors and the current flowing through

them is uniquely defined. However, it can be more difficult to define voltages and currents in quasi-TEM transmission lines such as microstrip and coplanar waveguide due to their hybrid nature. In fact, many waveguides used in microwave systems may only have a single conductor such as rectangular waveguide or no conductors at all as in a dielectric waveguide. In these cases it becomes impossible to define a unique voltage or current and guides of this type are better explained in terms of their electric and magnetic fields:

$$\mathbf{E}(x, y, z) = C^+ \mathbf{e}_t(x, y) e^{-\gamma z} + C^- \mathbf{e}_t(x, y) e^{+\gamma z} \quad (30)$$

$$\mathbf{H}(x, y, z) = C^+ \mathbf{h}_t(x, y) e^{-\gamma z} - C^- \mathbf{h}_t(x, y) e^{+\gamma z} \quad (31)$$

where $\mathbf{e}_t$ and $\mathbf{h}_t$ are the transverse electric and magnetic fields, respectively. C^- and C^+ are complex-valued constants. In general, all transmission lines are described by equations (30) and (31) and not equations (1) and (2). Equations (30) and (31) can be expressed as [i]

$$\mathbf{E} = \frac{V(z)}{v_0} \mathbf{e}_t \quad (32)$$

and

$$\mathbf{H} = \frac{I(z)}{i_0} \mathbf{h}_t \quad (33)$$

where the equivalent waveguide voltage and current is given by

$$V(z) = v_0 (C^+ e^{-\gamma z} + C^- e^{+\gamma z}) = \sqrt{Z_0} (a e^{-\gamma z} + b e^{+\gamma z}) \quad (34)$$

and

$$I(z) = i_0 (C^+ e^{-\gamma z} - C^- e^{+\gamma z}) = \frac{1}{\sqrt{Z_0}} (a e^{-\gamma z} - b e^{+\gamma z}), \quad (35)$$

respectively. v_0 and i_0 are normalisation constants such that

$$Z_0 = \frac{v_0}{i_0}. \quad (36)$$

Both $V(z)$ and v_0 have units of voltage and $I(z)$ and i_0 have units of current.

In order to extend the concept of voltage and current to the general waveguide structure, equations (34) and (35) must satisfy the same power relationships as equations (8) and (9). It can be shown that the power

flow in a waveguide across a transverse surface S is given by [ii]

$$P = \frac{1}{2} \operatorname{Re} \left\{ \int_S \mathbf{E} \times \mathbf{H}^* \cdot d\mathbf{S} \right\} = \frac{1}{2} \operatorname{Re} \left\{ \frac{V(z) I(z)^*}{v_0 i_0^*} P_0 \right\} \quad (37)$$

with modal power

$$P_0 = \int_S \mathbf{e}_t \times \mathbf{h}_t^* \cdot d\mathbf{S}. \quad (38)$$

Therefore, in order to retain the analogy with equations (8) and (9) we require

$$P = \frac{1}{2} \operatorname{Re} \{ V(z) I(z)^* \} \quad (39)$$

and thus

$$v_0 i_0^* = P_0. \quad (40)$$

We see that the magnitude of Z_0 is not uniquely defined since we are free to choose any value of v_0 and i_0 as long as equation (40) is satisfied. For example $|Z_0|$ is often set to the wave impedance of the propagating mode. Another popular choice, used in network analysers, is $|Z_0| = 1$. Note, however, that the phase of Z_0 is set by equation (40) and is an inherent characteristic of the propagating mode.

Since we cannot define a unique value of Z_0 we cannot define S-parameter measurements with respect to a nominal characteristic impedance. This is not really a problem for standard rectangular waveguide and coaxial cable which have set dimensions, since we can specify measurements with respect to WG-22 or APC7, etc. However, if we are using non-standard waveguides such as image or dielectric waveguide then all we can say is our S-parameters are with respect to the propagating mode on the structure.

Another important difference is that in general a waveguide will support more than one mode. Multimode structures can be analysed using multimodal S-parameters [iii]. Fortunately, under usual operating conditions, only the fundamental mode propagates. However, at a discontinuity, evanescent modes will always be present. These exponentially decaying fields will exist in the vicinity of the discontinuity and are required to completely explain the waveguide fields and network parameters.

It is important to remember that equations (34) and (35) are only equivalent waveguide voltages and currents which do not have all the properties of equations (1) and (2). For example, Z_0 is dependent on

normalisation and therefore we could define two different values of Z_0 for the same waveguide. Furthermore, even if we do use the same normalisation scheme it is possible for two different waveguides to have the same Z_0 . Clearly, a transition from one of these guides to the other will not result in a reflectionless transition, as conventional transmission theory would suggest. We also have to be very careful when converting to Z-parameters using equation (21) since Z is related to Z_0 . Since Z_0 is undefined the absolute value of Z-parameters cannot be determined. This is not surprising since impedance is intrinsically linked to current and voltage. However, even though absolute values cannot be defined, they can be useful in the development of equivalent circuit models for waveguide devices and junctions.

RENORMALISATION OF S-PARAMETERS

We have already seen that S-parameters are defined with respect to a reference characteristic impedance at each of the network's ports. Often we require that the S-parameters are renormalised to another set of port characteristic impedances. This is particularly important in measurement where the measured S-parameters are with respect to the transmission line Z_0 of the calibration items which will generally be a frequency dependent complex number, as in Figures 4 and 5. To convert an S-parameter matrix S that is with respect to the port impedance matrix:

$$Z_0 = \begin{bmatrix} Z_{01} & 0 & \cdots & 0 \\ 0 & Z_{02} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z_{0n} \end{bmatrix}$$

we firstly transform S to an impedance matrix using equation (21):

$$Z = Z_0^{1/2} (U - S)^{-1} (U + S) Z_0^{1/2}$$

Next the impedance matrix is transformed into the S-parameter matrix S'

$$S' = Z_0'^{-1/2} (Z - Z_0') (Z + Z_0')^{-1} Z_0'^{1/2}$$

where now a reference impedance matrix of Z_0' is used:

$$Z_0' = \begin{bmatrix} Z_{01}' & 0 & \cdots & 0 \\ 0 & Z_{02}' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z_{0n}' \end{bmatrix}$$

S' is then with respect to Z_0' . Often the renormalised S-parameters are with respect to 50 Ω in which case all the diagonal elements of Z_0' are equal to 50 Ω .

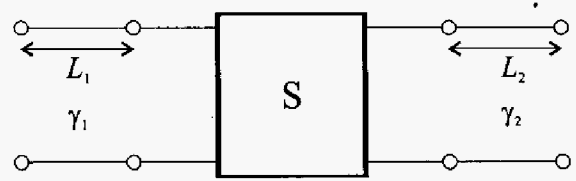


Fig. 6. Two-port network with feeding transmission lines at each port.

DE-EMBEDDING OF S-PARAMETERS

Another very important operation on an S-parameter matrix is the de-embedding of a length of transmission line from each of the ports. This is extremely important in measurement since often the device under test is connected to the measurement instrument by a length of transmission line and therefore the actual measured value includes the phase and attenuation of the line. It can be shown that the measured n -port S-parameters S' are related to the network's actual S-parameters S by

$$S' = \Theta S \Theta$$

where

$$\Theta = \begin{bmatrix} e^{-\gamma_1 L_1} & 0 & \cdots & 0 \\ 0 & e^{-\gamma_2 L_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-\gamma_N L_N} \end{bmatrix}$$

It is assumed that all of the lines are matched to their respective ports. If we know the length of line L_k at each port and the propagation constant γ_k then we can de-embed the effect of the lines. Thus,

$$S = \Theta^{-1} S' \Theta^{-1}$$

Due to the diagonal nature of Θ , the inverse operation Θ^{-1} simply changes the $-\gamma_k L_k$ terms to $+\gamma_k L_k$.

Fig. 6 shows a typical two-port network with lines connected to both ports. In this case the actual network parameters S are related to the un-embedded S-parameters S' by

$$S = \begin{bmatrix} S'_{11} e^{+2\gamma_1 L_1} & S'_{12} e^{+\gamma_1 L_1 + \gamma_2 L_2} \\ S'_{21} e^{+\gamma_1 L_1 + \gamma_2 L_2} & S'_{22} e^{+2\gamma_2 L_2} \end{bmatrix}$$

In the lossless case γ_k would degenerate to $j\beta_k$ and only a phase shift would be introduced by the lines.

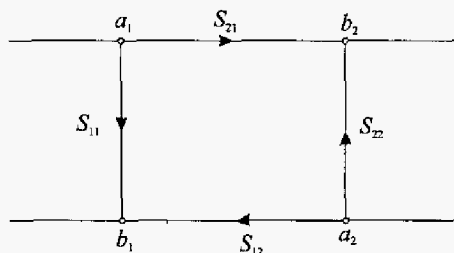


Fig. 7. Signal flow graph for two-port network

SIGNAL FLOW GRAPHS

The analysis so far has relied on matrix algebra. However, another important technique can also be used to analyse microwave circuits, or indeed their low frequency counterparts. This technique is known as the signal flow graph. Signal flow graphs express the network pictorially, see Fig. 7. The wave amplitudes are denoted by nodes, with the S-parameters being the gain achieved by the paths between nodes. To analyse signal flow graphs the following rules can be applied [iv]:

Rule 1: Two series branches, joined by a common node, can be replaced by one branch with gain equal to the product of the individual branches.

Rule 2: Two parallel branches joining two common nodes can be replaced with a single branch with gain equal to the sum of the two individual branches.

Rule 3: A branch that begins and ends on a single node can be eliminated by dividing the gains of all branches entering the node by one minus the gain of the loop.

Rule 4: Any node can be duplicated as long as all paths are retained.

These four rules are illustrated in Fig. 8.

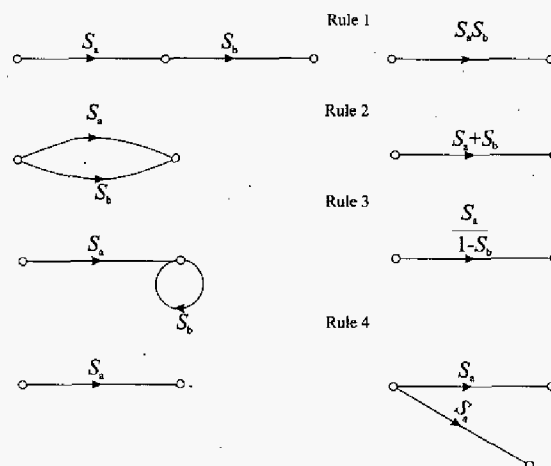


Fig. 8. Kuhn's rules for signal flow graph analysis.

Fig. 9 gives an example of applying the above rules to analyse a microwave circuit. The network is a simple two-port network terminated by an impedance with reflection coefficient Γ_L . We wish to calculate the reflection coefficient at the input terminal, i.e., b_1/a_1 . Firstly, we apply rule 4 to duplicate the a_2 node. Then, using rule 1, we eliminate both of the a_2 nodes. The closed loop, $S_{22}\Gamma_L$ is eliminated using rule 3. Next, rule 1 is applied to eliminate node b_2 . Finally, applying rule 2, we obtain a value for b_1/a_1 .

Clearly, for larger networks, the signal flow graph technique can be very difficult to apply. However, it can often be useful for analysing simple networks - giving a more intuitive approach to the problem.

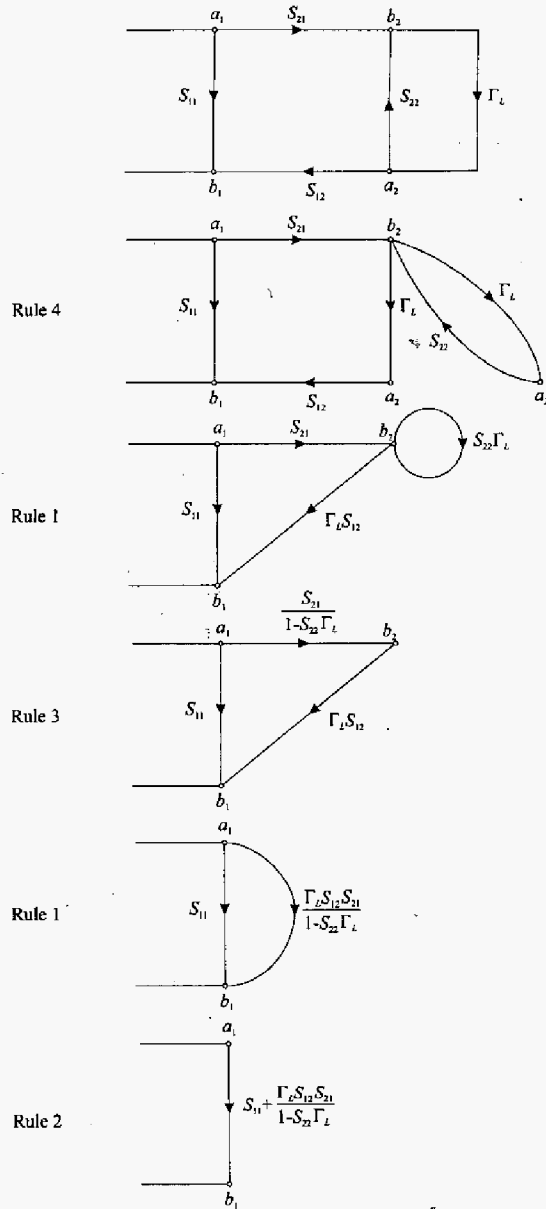


Fig. 9. Example of the use of signal flow graphs to analyse a microwave network.

APPENDIX

A. Reciprocity

Using the Lorentz reciprocity relation [v] it can be shown that, in general, $Z_{mn} = Z_{nm}$. This is only true for networks that do not contain anisotropic media such as ferrites. Therefore, in matrix notation we have

$$Z = Z^T$$

where Z^T is the transpose of Z . From equation (21) it becomes apparent that if the characteristic impedance is identical at every port then

$$(U - S)^{-1}(U + S) = (U + S^T)(U - S^T)^{-1}.$$

Therefore, $S = S^T$ and provided the impedance matrices are symmetrical, $S_{mn} = S_{nm}$.

B. Losslessness

An n -port network can be described an by $n \times n$ S-parameter matrix:

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad (B1)$$

$$\mathbf{b} = \mathbf{S}\mathbf{a}$$

If the network is lossless, then the power entering the network must be equal to the power flowing out of the network. Therefore, from equation (10) we have

$$\sum_{i=1}^n |b_i|^2 = \sum_{i=1}^n |a_i|^2 \quad (B2)$$

But $|b_i|^2 = b_i b_i^*$, therefore,

$$\sum_{i=1}^n |b_i|^2 = (b_1, b_2, \dots, b_n) \begin{bmatrix} b_1^* \\ b_2^* \\ \vdots \\ b_n^* \end{bmatrix}$$

We see that the column matrix in the above equation is given by the conjugate of the right hand side of equation (B1), i.e. $\mathbf{S}^* \mathbf{a}^*$. Similarly, the row matrix is given by the transpose of the right hand side of equation (B1), i.e. $(\mathbf{S}\mathbf{a})^T$. Therefore, we have

$$\sum_{i=1}^n |b_i|^2 = (\mathbf{S}\mathbf{a})^T \mathbf{S}^* \mathbf{a}^* = \mathbf{a}^T \mathbf{S}^T \mathbf{S}^* \mathbf{a}^* \quad (B3)$$

where we have used the fact that the transpose of the product of two matrices is equal to the product of the transposes in reverse. Substituting (B3) into equation (B2) yields

$$\mathbf{a}^T \mathbf{S}^T \mathbf{S}^* \mathbf{a}^* = \mathbf{a}^T \mathbf{a}^* \quad (B4)$$

where we have used the fact that

$$\sum_{i=1}^n |a_i|^2 = \mathbf{a}^T \mathbf{a}^*$$

Equation (B4) can be written as

$$\mathbf{a}^T (\mathbf{S}^T \mathbf{S}^* - \mathbf{U}) \mathbf{a}^* = 0$$

where $\mathbf{U}$ is the unit matrix. Therefore,

$$\mathbf{S}^T \mathbf{S}^* = \mathbf{U}$$

In words, the product of the transpose of the S-parameter matrix $\mathbf{S}$ with its complex conjugate is equal to the unit matrix $\mathbf{U}$. In the case of a two-port we have

$$S_{11}S_{11}^* + S_{21}S_{21}^* = 1,$$

$$S_{12}S_{12}^* + S_{22}S_{22}^* = 1,$$

$$S_{11}S_{12}^* + S_{21}S_{22}^* = 0$$

and

$$S_{12}S_{11}^* + S_{22}S_{21}^* = 0.$$

C. Two-port Transforms

Using equation (21) for a two-port network gives

$$\mathbf{S} = \frac{1}{\Delta Z} \begin{bmatrix} \Delta Z_1 & 2\sqrt{Z_{01}Z_{02}}Z_{12} \\ 2\sqrt{Z_{01}Z_{02}}Z_{21} & \Delta Z_2 \end{bmatrix}$$

where

$$\Delta Z = (Z_{11} + Z_{01})(Z_{22} + Z_{02}) - Z_{12}Z_{21},$$

$$\Delta Z_1 = (Z_{11} - Z_{01})(Z_{22} + Z_{02}) - Z_{12}Z_{21}$$

and

$$\Delta Z_2 = (Z_{11} + Z_{01})(Z_{22} - Z_{02}) - Z_{12}Z_{21}.$$

Similarly, using equation (22)

$$\mathbf{Z} = \frac{1}{\Delta S} \begin{bmatrix} Z_{01}\Delta S_1 & 2Z_{01}S_{21} \\ 2Z_{02}S_{12} & Z_{02}\Delta S_2 \end{bmatrix}$$

where

$$\Delta S = (1 - S_{11})(1 - S_{22}) - S_{12}S_{21},$$

$$\Delta S_1 = (1 + S_{11})(1 - S_{22}) + S_{12}S_{21}$$

and

$$\Delta S_2 = (1 - S_{11})(1 + S_{22}) + S_{12}S_{21}.$$

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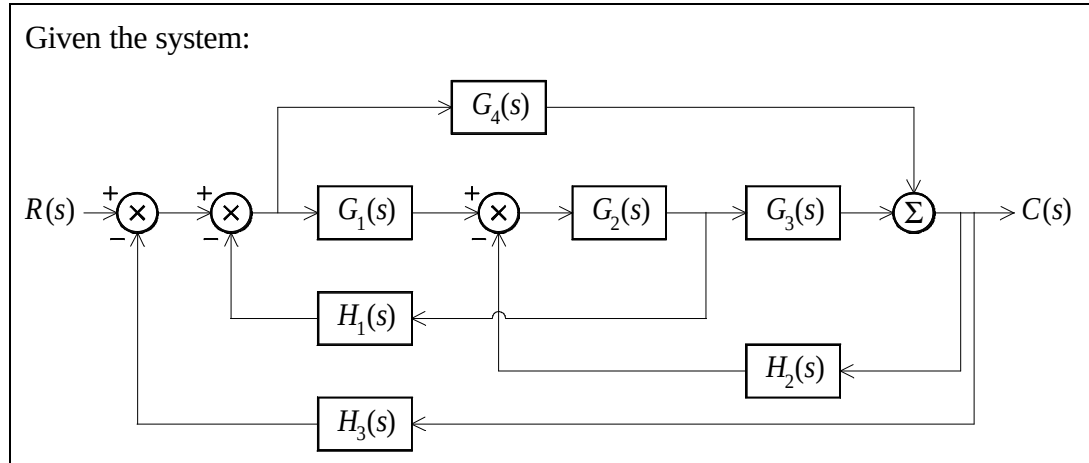
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MASON'S GAIN RULE

An example of finding the transfer function of a system represented by a block diagram using Mason's rule. This problem has also been worked using a matrix solution; see the file MatrixSolution.pdf.

The Problem:



Mason's Gain Rule:

$$M = \frac{\sum_j M_j \Delta_j}{\Delta}$$

M = transfer function or gain of the system

M_j = gain of one forward path

j = an integer representing the forward paths in the system

$\Delta_j = 1$ – the loops remaining after removing path j . If none remain, then $\Delta_j = 1$.

$\Delta = 1 - \Sigma$ loop gains $+ \Sigma$ nontouching loop gains taken two at a time $- \Sigma$ nontouching loop gains taken three at a time $+ \Sigma$ nontouching loop gains taken four at a time $- \dots$

1) Find the forward paths and their gains:

A **forward path** is a path from $R(s)$ to $C(s)$ that does not cross the same point more than once. There are two forward paths in this example, so we have a $j = 1$ and a $j = 2$. The two paths are:

$$M_1 = G_1 G_2 G_3 \quad \text{and} \quad M_2 = G_4$$

2) Find the loops and their gains:

A loop is a closed path that can be negotiated without crossing the same point more than once. There are five loops in this example:

$$\text{Loop 1} = -G_1 G_2 H_1$$

$$\text{Loop 4} = -G_4 H_3$$

$$\text{Loop 2} = -G_2 G_3 H_2$$

$$\text{Loop 5} = G_4 H_2 G_2 H_1$$

$$\text{Loop 3} = -G_1 G_2 G_3 H_3$$

3) Find the Δ_j s:

If we eliminate the path $M_1 = G_1 G_2 G_3$ from the system, no complete loops remain so: $\Delta_1 = 1$	If we eliminate the path $M_2 = G_4$ from the system, no complete loops remain so: $\Delta_2 = 1$
--	--

4) Find Δ :

Since there are no nontouching loop pairs in this example, Δ will just be one minus the sum of the loop gains:

$$\Delta = 1 - [(-G_1 G_2 H_1) + (-G_2 G_3 H_2) + (-G_1 G_2 G_3 H_3) + (-G_4 H_3) + (G_4 H_2 G_2 H_1)]$$

The Solution:

$$\text{Mason's Rule: } M = \frac{\sum_j M_j \Delta_j}{\Delta}$$

Applying the formula for Mason's rule, we have the transfer function:

$$\frac{C(s)}{R(s)} = M = \frac{G_1 G_2 G_3 + G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 H_3 + G_4 H_3 - G_4 H_2 G_2 H_1}$$

Mason's Rule:

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^n P_i \Delta_i}{\Delta}$$

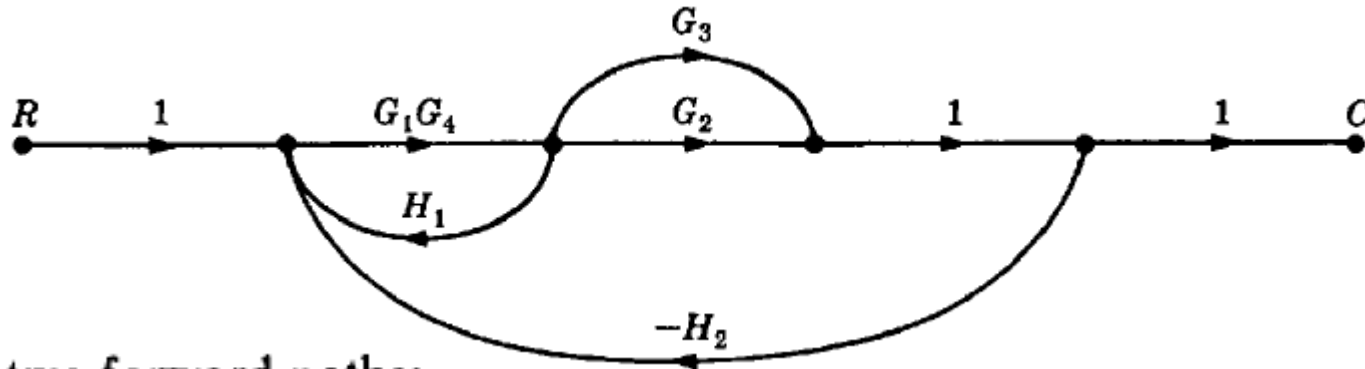
$\Delta = 1 -$ (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + ... and so forth with sums of higher number of non-touching loop gains

$\Delta_i =$ value of Δ for the part of the block diagram that does not touch the i -th forward path ($\Delta_i = 1$ if there are no non-touching loops to the i -th path.)

Systematic approach

1. Calculate forward path gain P_i for each forward path i .
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. etc
6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_i as portion of Δ not touching forward path i

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



There are two forward paths:

$$P_1 = G_1 G_2 G_4 \quad P_2 = G_1 G_3 G_4$$

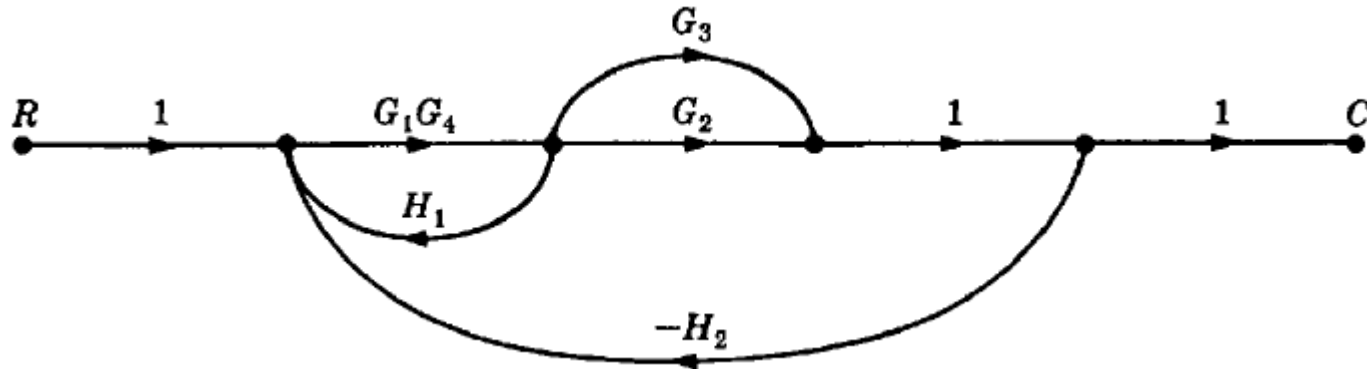
Therefore,

$$\frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1 G_4 H_1, \quad L_2 = -G_1 G_2 G_4 H_2, \quad L_3 = -G_1 G_3 G_4 H_2$$

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



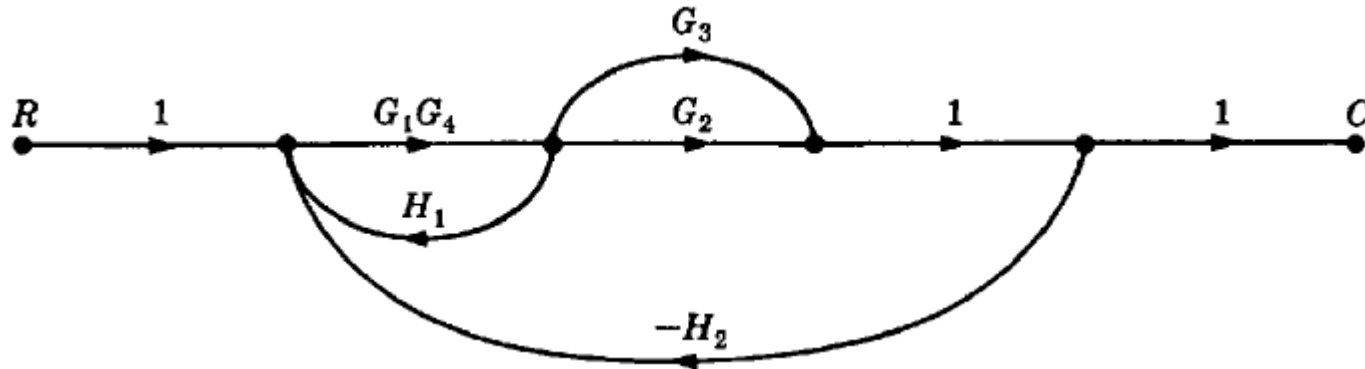
There are no non-touching loops, therefore

$$\Delta = 1 - (\text{sum of all individual loop gains})$$

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 - (G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2)$$

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



Eliminate forward path-1

$$\Delta_1 = 1 - (\text{sum of all individual loop gains}) + \dots$$

$$\Delta_1 = 1$$

Eliminate forward path-2

$$\Delta_2 = 1 - (\text{sum of all individual loop gains}) + \dots$$

$$\Delta_2 = 1$$

Example#1: Continue

$$\begin{aligned}\frac{C}{R} &= \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2} \\ &= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}\end{aligned}$$

6. 회로의 산란행렬 구하는 법

$$a = \frac{V^+}{\sqrt{Z_0}} = \sqrt{Z_0} I^+$$

$$b = \frac{V^-}{\sqrt{Z_0}} = -\sqrt{Z_0} I^-$$

병렬 회로: $V_1 = V_2 \rightarrow V_1^+ + V_1^- = V_2^+ + V_2^-$

$$\rightarrow \sqrt{Z_0} (a_1 + b_1) = \sqrt{Z_0} (a_2 + b_2)$$

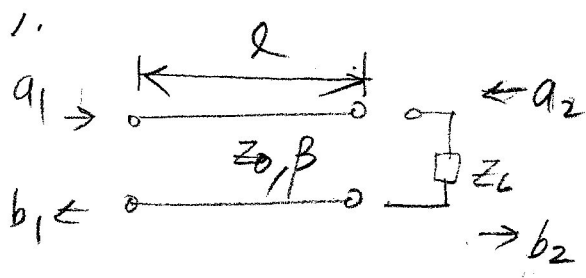
$$\rightarrow a_1 + b_1 = a_2 + b_2, \quad 1 + S_{11} = S_{21}$$

직렬회로: $I_1 = I_2 \rightarrow I_1^+ + I_1^- = I_2^+ + I_2^-$

$$\rightarrow \frac{1}{\sqrt{Z_0}} (a_1 - b_1) = \frac{1}{\sqrt{Z_0}} (a_2 - b_2)$$

$$\rightarrow a_1 - b_1 = a_2 - b_2 \rightarrow 1 - S_{11} = -S_{21}$$

Derivations

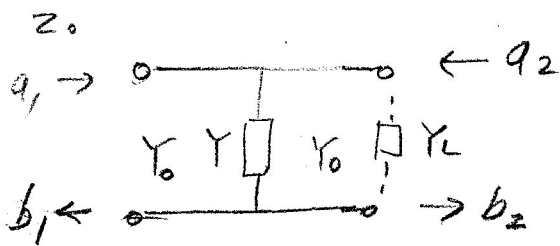


$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}, \quad a_2=0 \rightarrow Z_L = Z_0 \rightarrow \text{no reflection} \rightarrow S_{11}=0$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} = e^{-j\beta l}, \quad \text{phase delay (from transmission line theory)}$$

$$S_{12} = S_{21}$$

$$S_{22} = S_{11}$$



$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}, \quad a_2=0 \rightarrow Y_L = Y_0 \rightarrow \Gamma = \frac{Y_0 - (Y + Y_0)}{Y_0 + (Y + Y_0)} = \frac{-Y}{Y + 2Y_0}$$

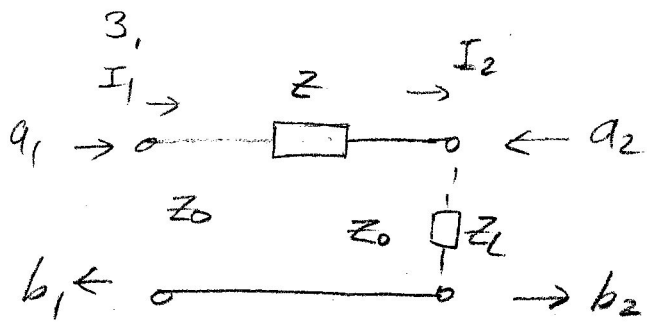
$$S_{11} = \Gamma = \frac{-Y}{Y + 2Y_0}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}, \quad \underbrace{a_1 + b_1}_{V_1/\sqrt{Z_0}} = \underbrace{a_2 + b_2}_{V_2/\sqrt{Z_0}}, \quad a_2=0, \quad \frac{b_2}{a_1} = 1 + \frac{b_1}{a_1}$$

$$S_{21} = 1 + S_{11} = \frac{2Y_0}{Y + 2Y_0}$$

$$S_{12} = S_{21}$$

$$S_{22} = S_{11}$$



$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}, \quad a_2=0 \rightarrow Z_L=Z_0 \rightarrow \Gamma = \frac{Z+Z_0-Z_0}{Z+Z_0+Z_0} = \frac{Z}{Z+2Z_0}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}, \quad I_1 = I_2 \rightarrow I_1^+ + I_1^- = I_2^+ + I_2^-$$

$$\rightarrow \sqrt{Z_0} (a_1 - b_1) = \sqrt{Z_0} (a_2 - b_2), \quad a_2=0$$

$$\rightarrow \frac{b_2}{a_1} = \frac{b_1}{a_1} - 1 = S_{11} - 1$$

$$S_{21} = S_{11} - 1 = \frac{2Z_0}{Z+2Z_0}$$

$$S_{12} = S_{21}$$

$$S_{22} = S_{11}$$

연습문제

1. 식 (24) 를 유도하라.
2. 식 (25) 를 유도하라.
3. p. 2/4 표의 4 번째 회로의 admittance matrix 를 유도하라.
4. p. 2/4 표의 5 번째 회로의 impedance matrix 를 유도하라.
5. p. 2/10 첫 번째 신호 흐름도에서 $\frac{b_1}{a_1}$ 를 Mason 정리를 이용하여 구하라.

Python Lab #1

[S] given

Find [T]

From [T] find [S] and compare it with given [S]

입력:

Given $S_{11}, S_{12}, S_{21}, S_{22}(\text{polar}) =$

출력:

Calculated $T_{11}, T_{12}, T_{21}, T_{22}(\text{polar}) =$

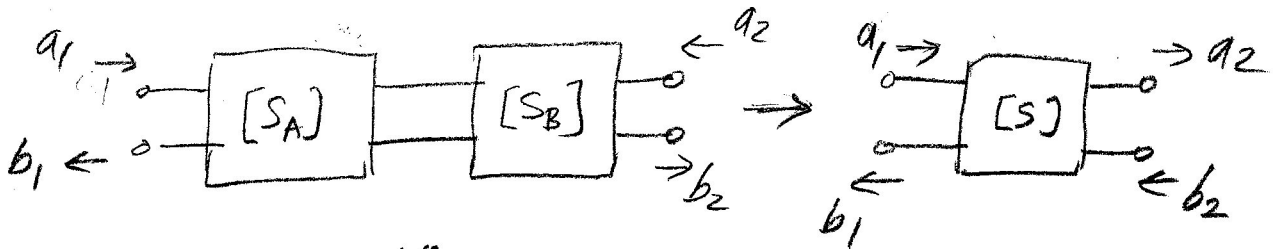
Calculated $S_{11}, S_{12}, S_{21}, S_{22}(\text{polar}) =$

Python Lab #2

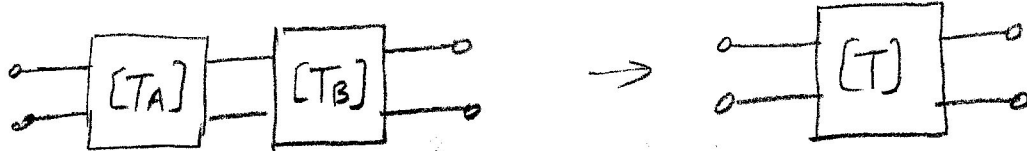
두 장치 A: 산란행렬 $[S_A]$

장치 B: 산란행렬 $[S_B]$

두 장치가 일렬로 연결되었을 경우 (cascaded) 산란행렬 $[S]$ 를 구하라.



힌트: T-matrix 이용



$$[T] = [T_A][T_B]$$

$[T] \leftrightarrow [S]$ 변환공식 $\rightarrow$ 강의노트 참조

입력:

$$S_{11A}, S_{12A} (\text{polar}) =$$

$$S_{21A}, S_{22A} (\text{polar}) =$$

$$S_{11B}, S_{12B} (\text{polar}) =$$

$$S_{21B}, S_{22B} (\text{polar}) =$$

출력:

$$S_{11}, S_{12} (\text{polar}) =$$

$$S_{21}, S_{22} (\text{polar}) =$$