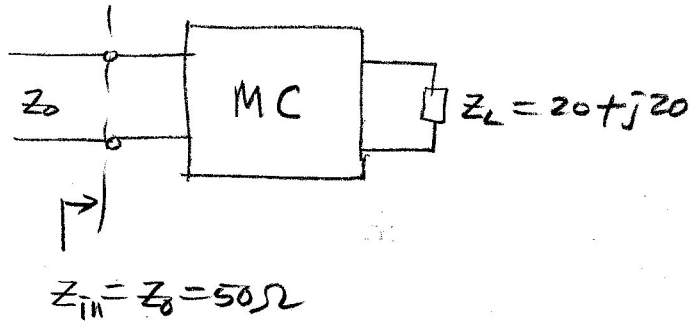


Ch. 임피던스 정합 (Impedance Matching)

1. 개요

0. 임피던스 정합 회로 (matching circuit; MC)



0. 임피던스 정합 회로 용도

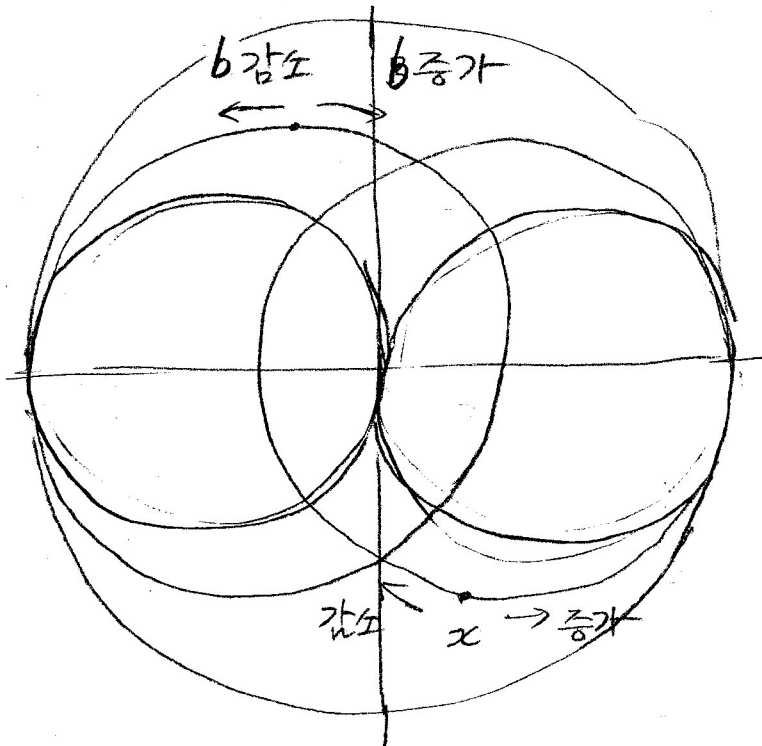
- 안테나와 송수신부간 임피던스 정합
- 증폭기 입출력 임피던스 정합
- 혼합기 임피던스 정합
- 무선충전

0. 임피던스 정합 회로의 종류

- 1) LC 임피던스 정합 회로 : 1단, 다단
- 2) stub 임피던스 정합 회로 : 1개, 2개, 다수
- 3) $1/\lambda$ -각장 임피던스 정합 회로 : 1단, 다단
- 4) 연속 임피던스 테이퍼 : Linear, exponential, Klopfenstein
- 5) 능동 임피던스 정합
- 6) 적응 임피던스 정합

2. 스미스도표와 임피던스 조합

0. 용량성/유도성 소자 연결



병렬연결 : Admittance 사용

직렬연결 : Impedance 사용

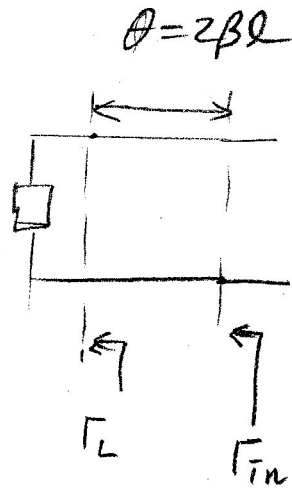
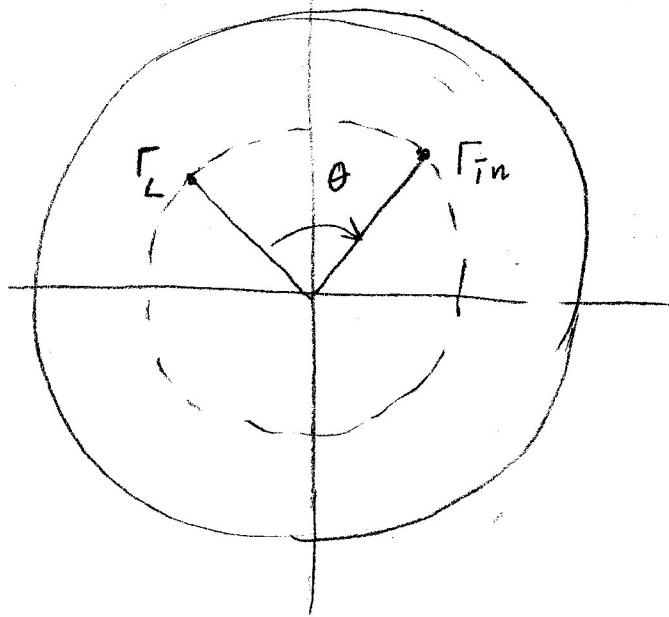
병렬 jB 연결 :
$$jB = \begin{cases} \frac{j\omega C}{Y_0} & \text{커패시터} \\ \frac{1/j\omega L}{Y_0} & \text{인덕터} \end{cases}$$

Constant g circle 상에서 이동

직렬 jX 연결 :
$$jX = \begin{cases} \frac{j\omega L}{Z_0} & \text{인덕터} \\ \frac{1/j\omega C}{Z_0} & \text{커패시터} \end{cases}$$

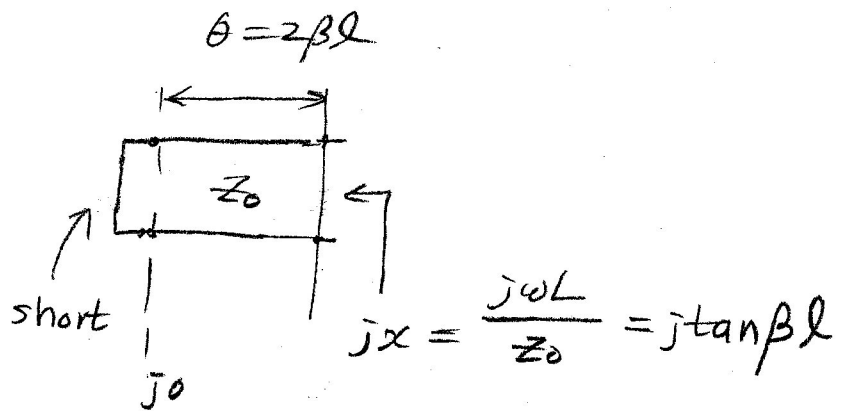
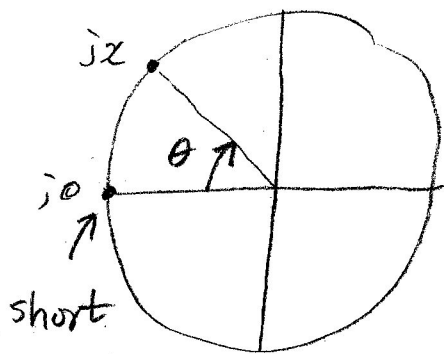
Constant r circle 상에서 이동

0. 전송선 상에서의 위치 이동

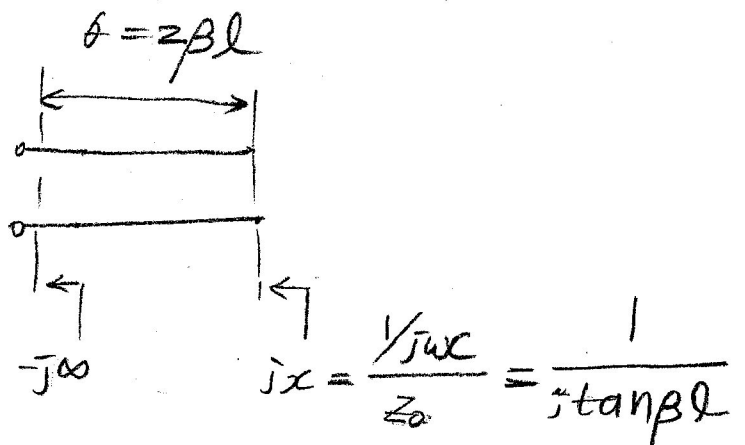
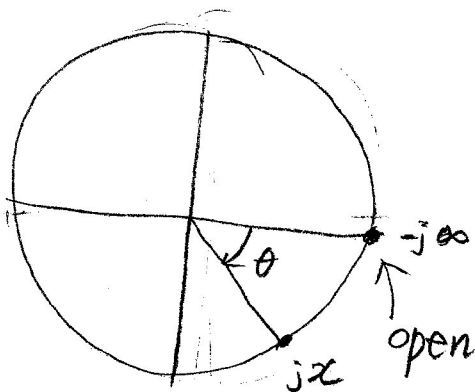


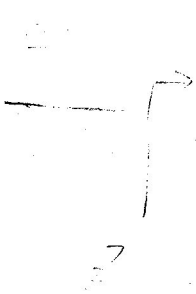
0. 전송선을 이용한 인덕터, 캐패시터 구현

- 인덕터

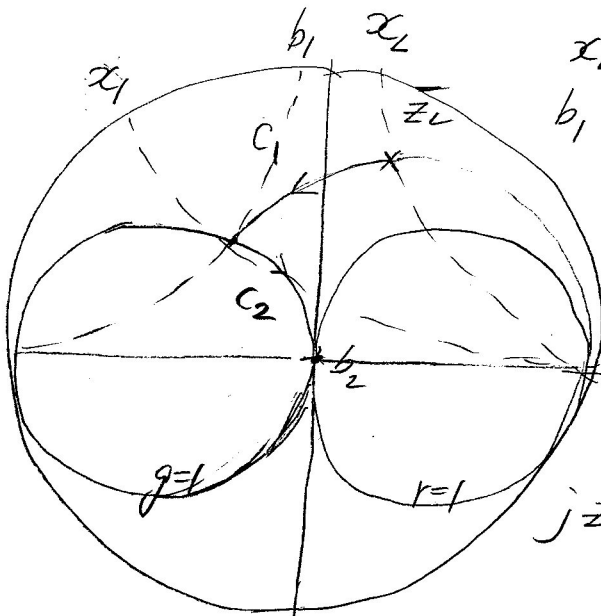


- 캐패시터

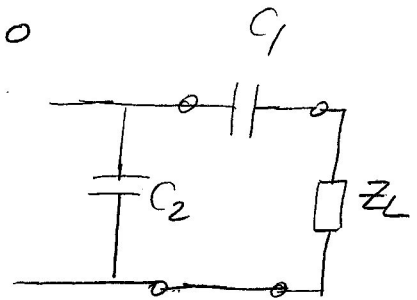




1.11 lumped element 정합, LC 정합



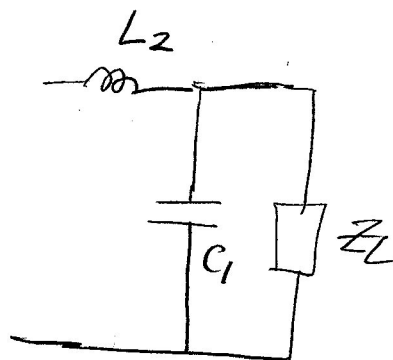
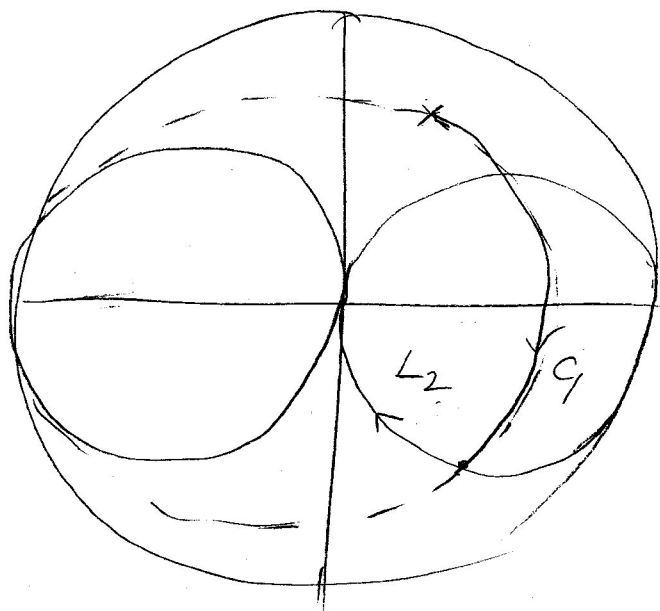
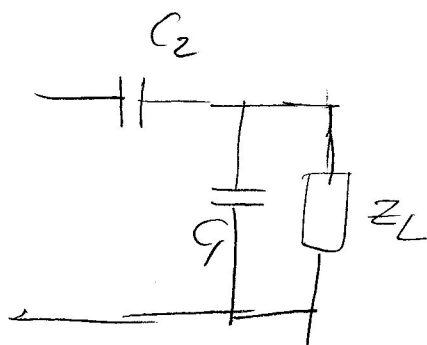
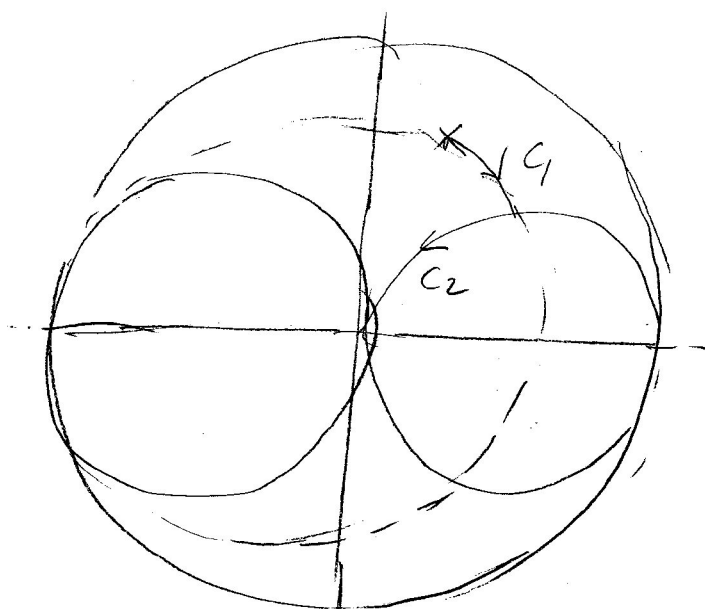
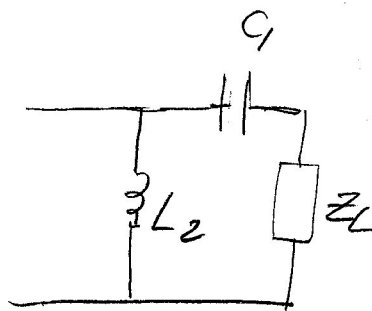
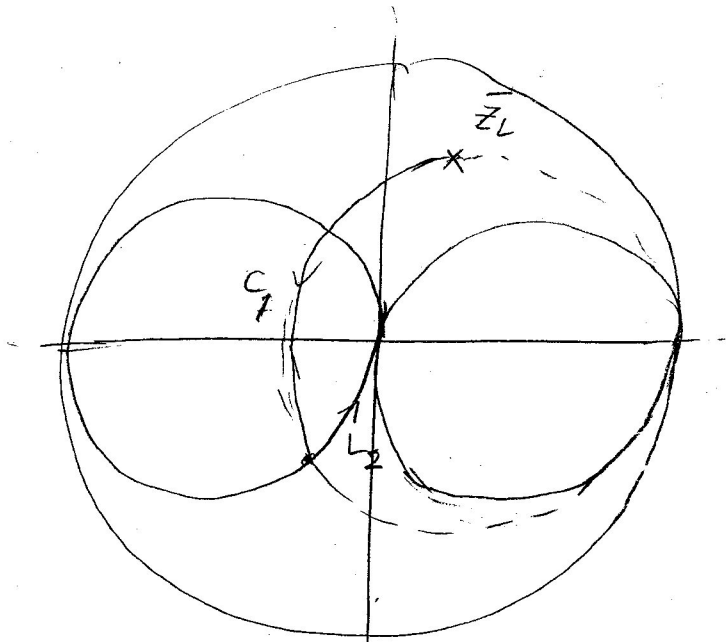
$$x_L > x_1 \\ b_1 < 0, b_2 = 0$$



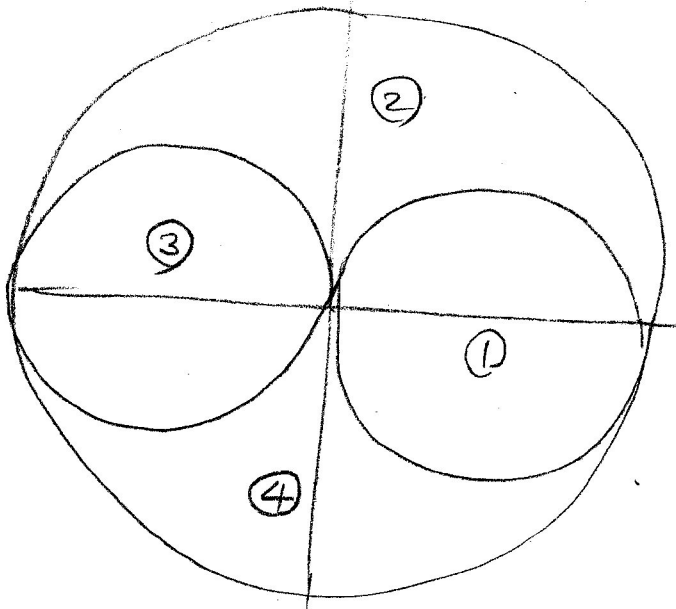
$$jz_0(x_L) - j\frac{1}{\omega C_1} = j(x_1)z_0$$

$$jY_0(b_1) + j\omega C_2 = 0 \quad (b_1 < 0)$$

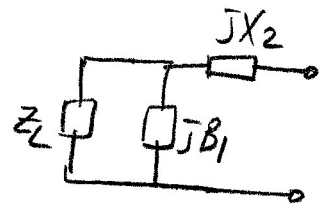
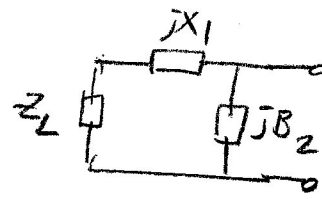
element	z	Y
	$-j\frac{1}{\omega C}$	$j\omega C$
	$j\omega L$	$-j\frac{1}{\omega L}$



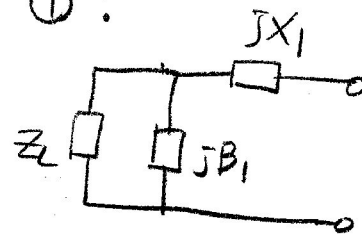
LC-Matching Network 75



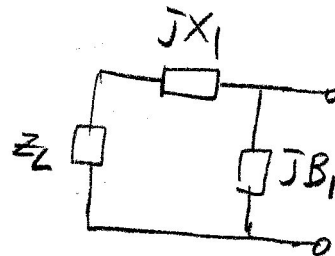
②, ④ :



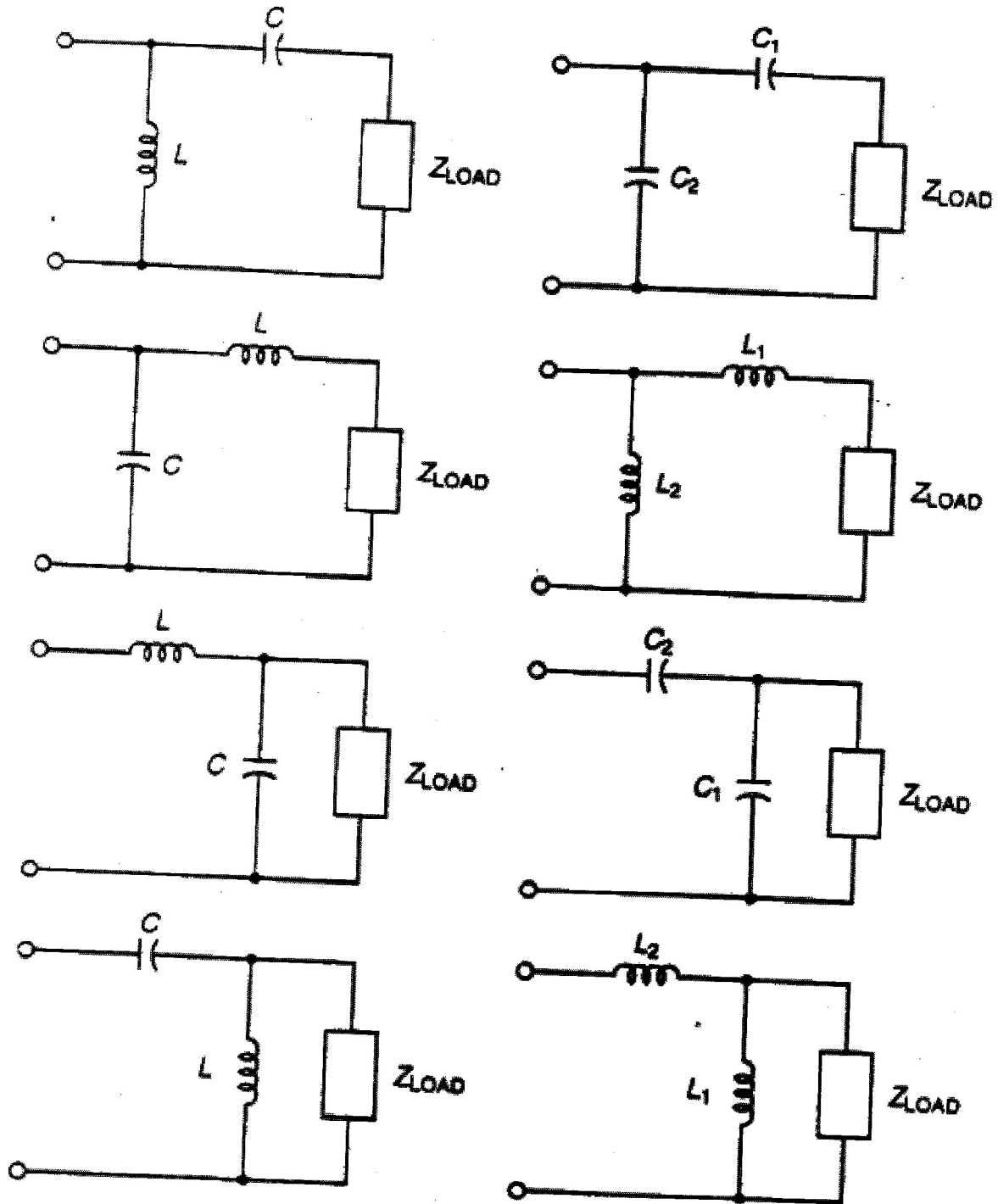
① :



③ :

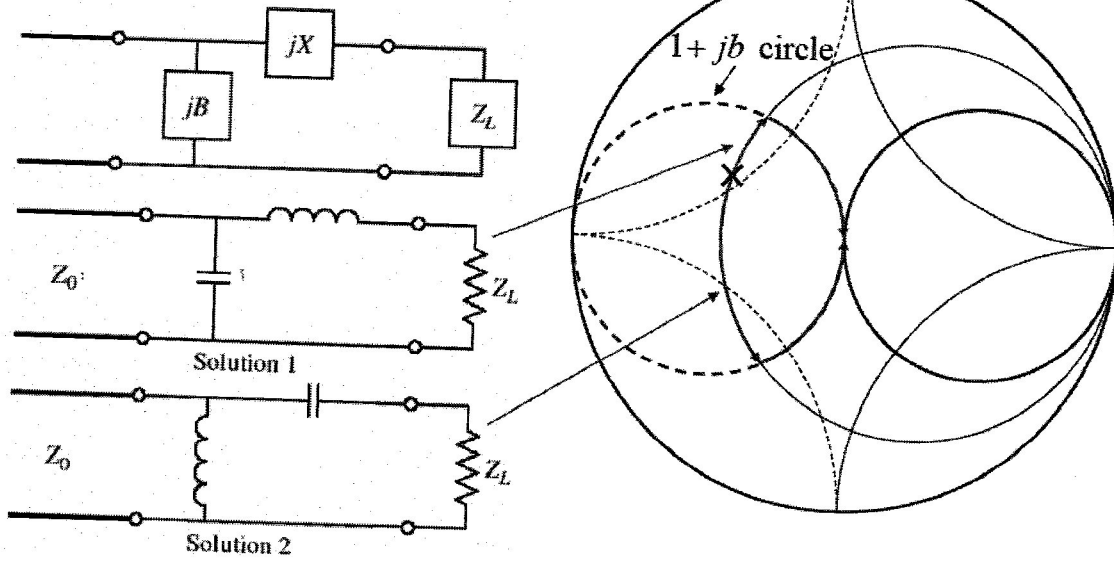


○ 1단 LC 정합회로 경우의 수: 총 8가지

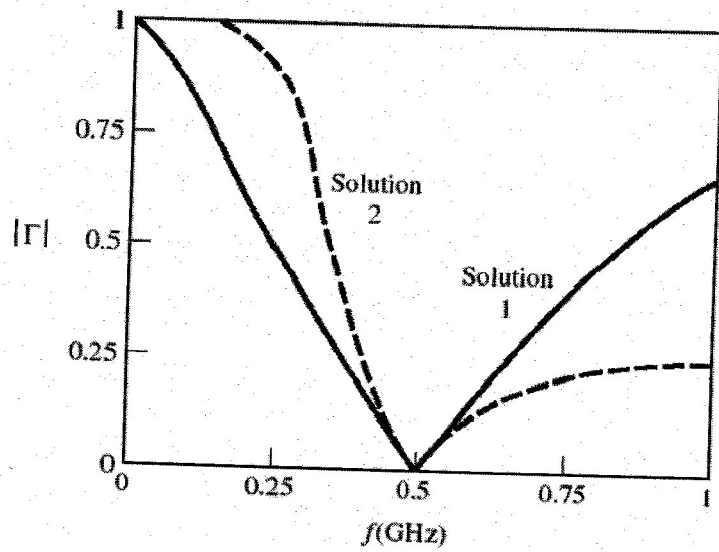
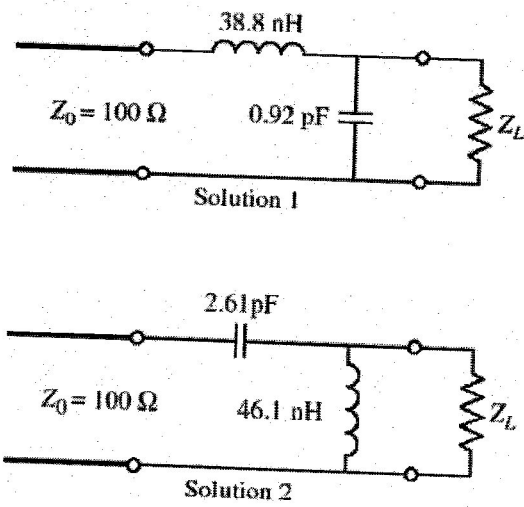


○ LC 정합 예제

Case 2: z_L is outside the $1 + jx$ circle



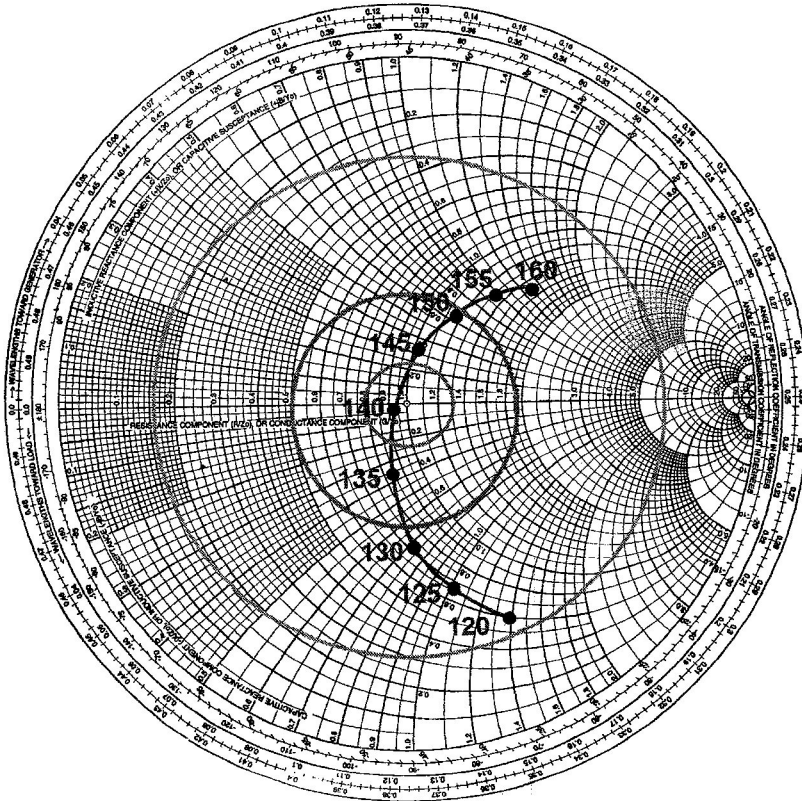
$Z_L = 200 - j100 (\Omega), f = 500 \text{ MHz}$



○ Foster의 리액턴스 정리

- 주파수 증가시 모든 부하의 리액턴스와 서셉턴스 증가
- 주파수 증가에 따라 스미스도표 상 임피던스 궤적: 시계방향으로 회전

Smith Chart



○ 다단 LC 정합

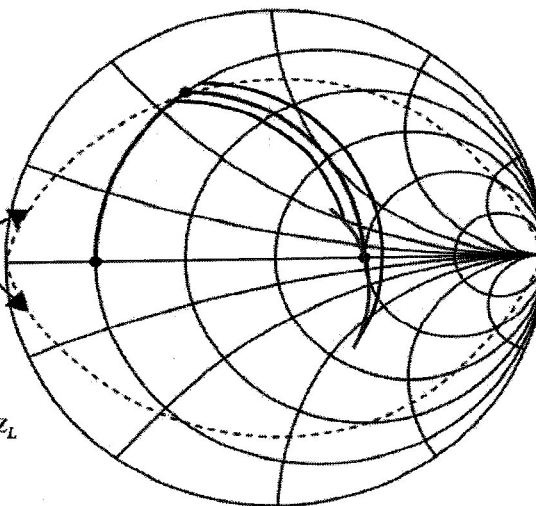
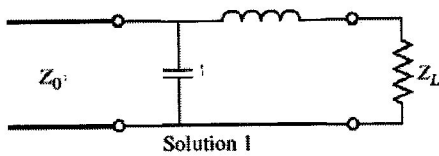
- Constant Q circles:

$$Q = \frac{X}{R} = \frac{B}{G}$$

$$1 + Q^2 = F_R$$

$$\left(F_R = \frac{R_0}{R_L} \right)$$

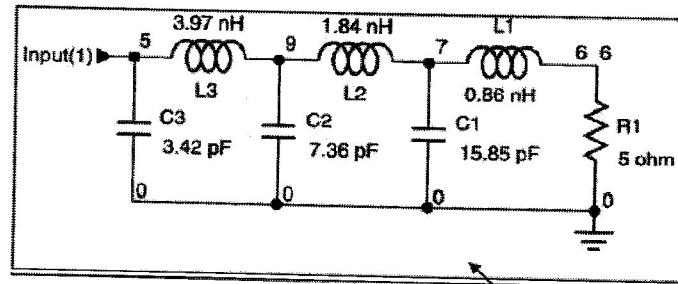
$Q = 3$



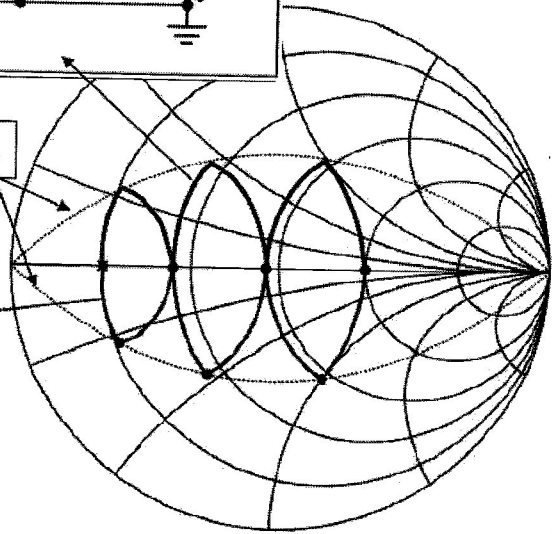
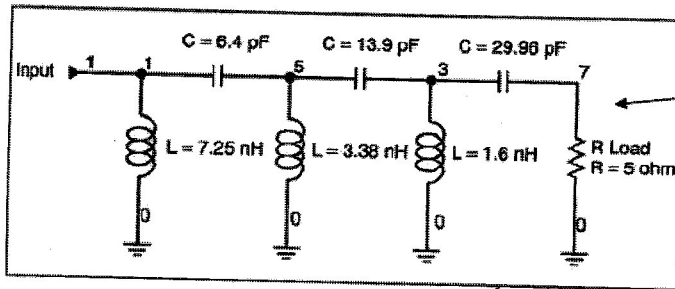
Broadband →

Low Q matching

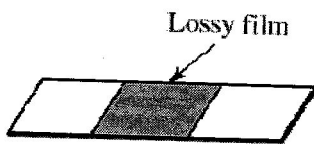
$$1 + Q^2 = \sqrt[n]{F_R}$$



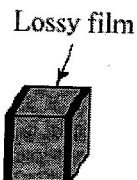
$Q = 1.074$



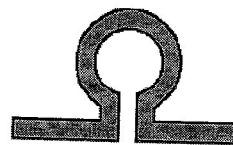
○ 고주파 집중회로 소자



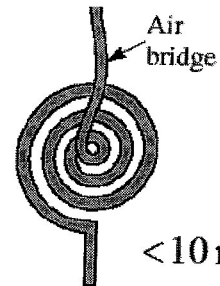
Planar resistor



Chip resistor



Loop inductor

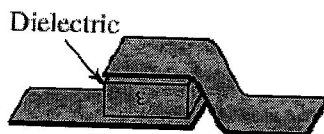


Spiral inductor

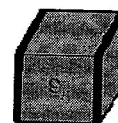
$< 10 \text{ nH}$



Interdigital gap capacitor $< 0.5 \text{ pF}$



Metal-insulator-metal capacitor $< 25 \text{ pF}$

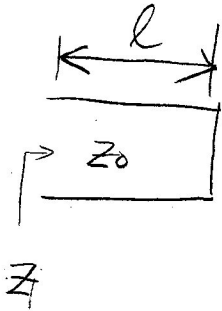


Chip capacitor

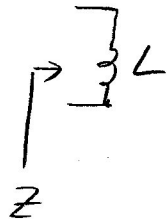
2.4.2 단일 스텝 매칭 (single-stub matching)

stub : 두루터기, 전송선에 짧은 전송선이 병결 또는 직렬로 연결된 것

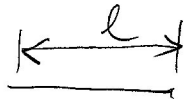
short stub



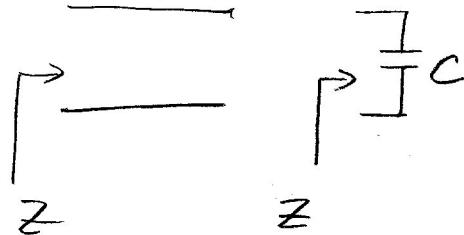
$$Z = jZ_0 \tan \beta l = j\omega L$$



open stub:

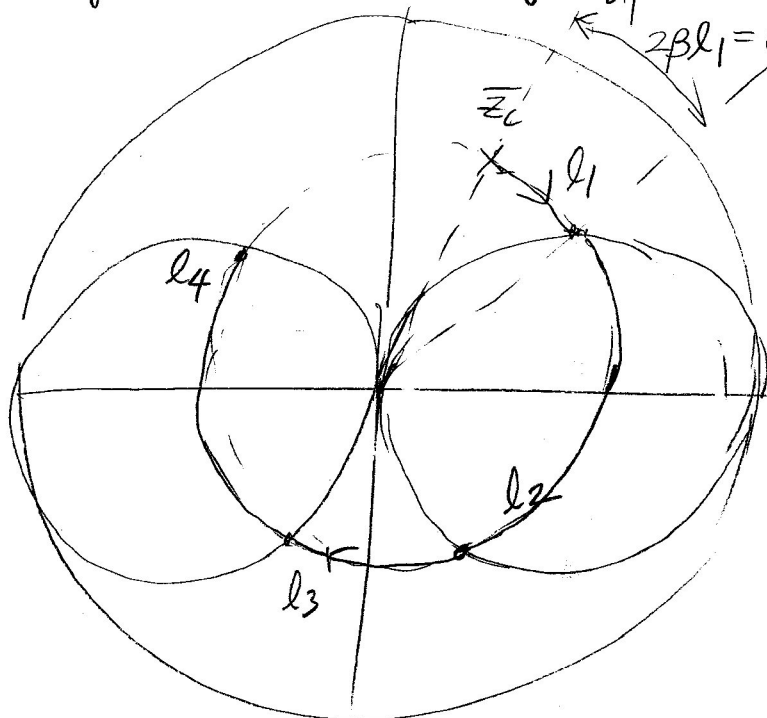


$$Z = -j \frac{Z_0}{\tan \beta l} = -j \frac{1}{\omega C}$$

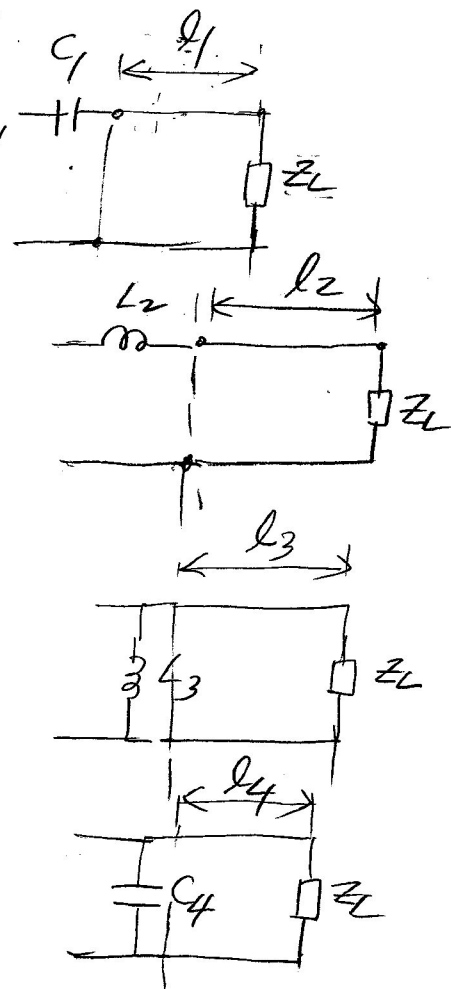


$$Y = jY_0 \tan \beta l = j\omega C$$

single-stub matching: 4가지 해

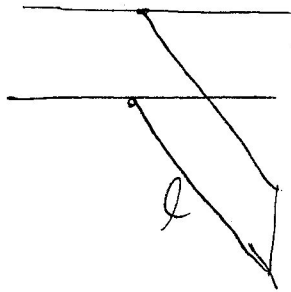


$$\theta_1 = 2\beta l_1 = \frac{4\pi}{\lambda} l_1$$

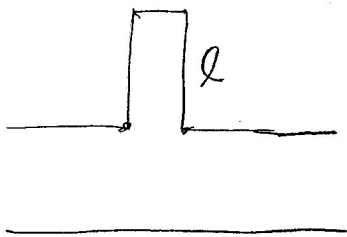


$C \rightarrow$ use open stub

$L \rightarrow$ use short stub



shunt L

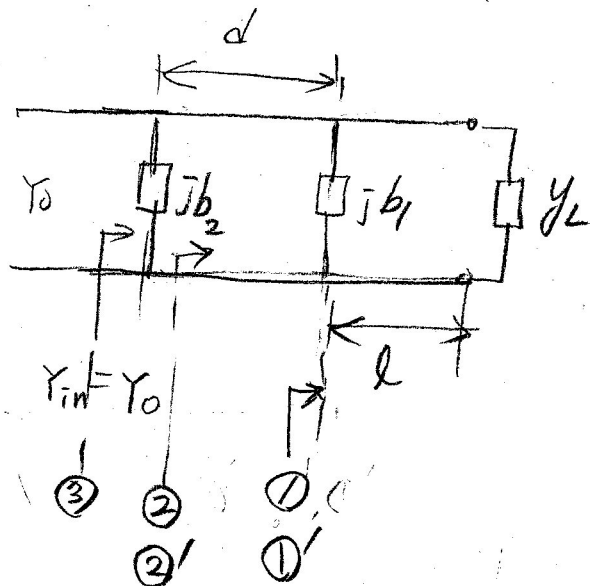


series L

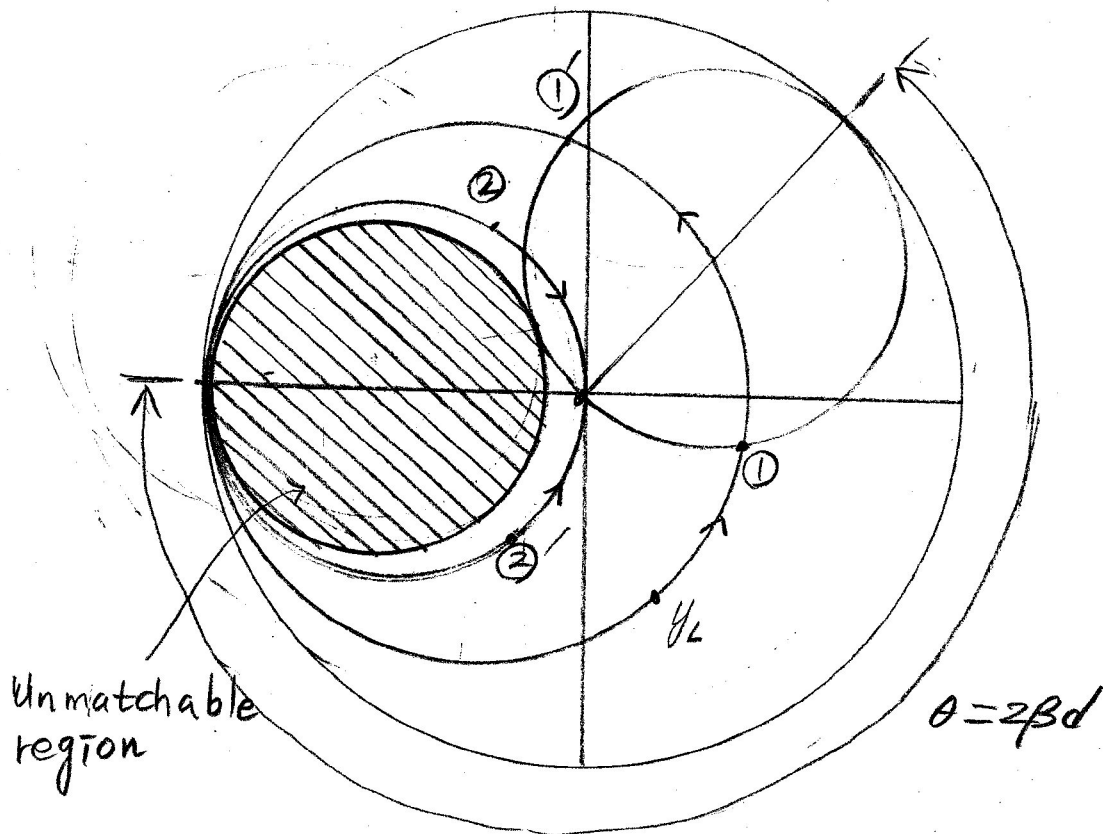
◦, 컨덕터스 리플렉의 단점: 부하 임피던스가 바뀌면 stub의 위치를 변경해야 함 (전송선에 다시 주영을 넣어야 한다)

→ 해결책: 이중 stub tuner, etc.

3. 이중스텝 정합 (Double-stub matching).

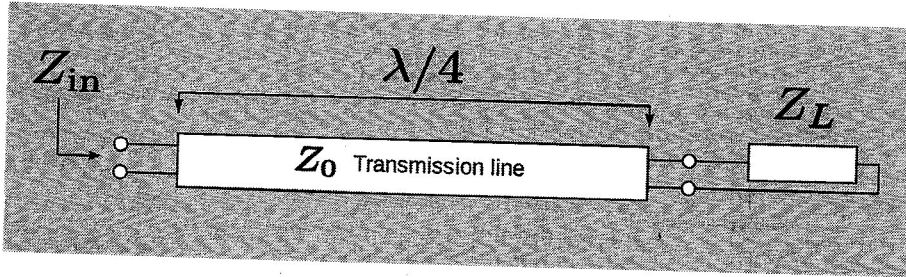


l : arbitrary
 d : fixed
 jB_1, jB_2 : design



4. 단일 1/4-파장 정합회로

1) 구조



2) 설계공식

$$Z_0 = \sqrt{Z_L Z_{in}}$$

(증명)

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell}$$

$$\ell = \frac{\lambda}{4} \rightarrow \beta \ell = \frac{\pi}{2} \rightarrow \tan \beta \ell = \infty \rightarrow Z_{in} = \frac{Z_0^2}{Z_L} \rightarrow Z_0 = \sqrt{Z_L Z_{in}}$$

3) 특성

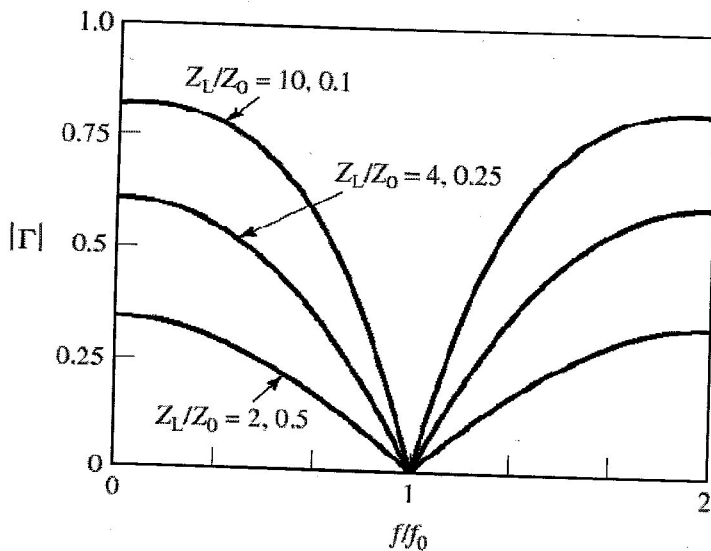


그림: Z_L/Z_0 에 따른 단일 1/4-파장 임피던스 정합회로의 주파수 특성

4) 예제

- 100Ω 마이크로스트립 선로를 50Ω 마이크로스트립 선로에 정합시키는 1단 1/4파장 변환기를 설계하라.

- 기판: FR-4, 유전상수 4.3, 유전체 두께 1.2mm, 도체 두께 0.035mm
- 기판 크기: 세로 = 30mm, 가로 = 길이 각 선로(1/4파장)의 합 = 128.4mm
- $f = 1\text{GHz}$
- 인터넷 마이크로스트립 계산기 mcalc 사용: 주파수 분산, 손실포함
<http://mcalc.sourceforge.net/>

- 설계결과:

<u>Characteristic imp.(ohm)</u>	<u>Line width (mm)</u>	<u>Effective dielectric constant</u>	<u>Quarter-wave (mm)</u>
50	2.30	3.2444	41.6
70.7	1.20	3.0693	42.8
100	0.50	2.9027	44.0

5. 다단 1/4 파장 임피던스 정합회로

1) 기초

- Why 다단 임피던스 정합? → 주파수 특성 확대역화
- 다단 1/4 파장 임피던스 정합회로의 종류
 - 동작 주파수 범위 내에서 반사계수의 변화 형상에 따라

Binomial (= maximally flat)

Chebyshev (= equi-ripple)

2) 구조

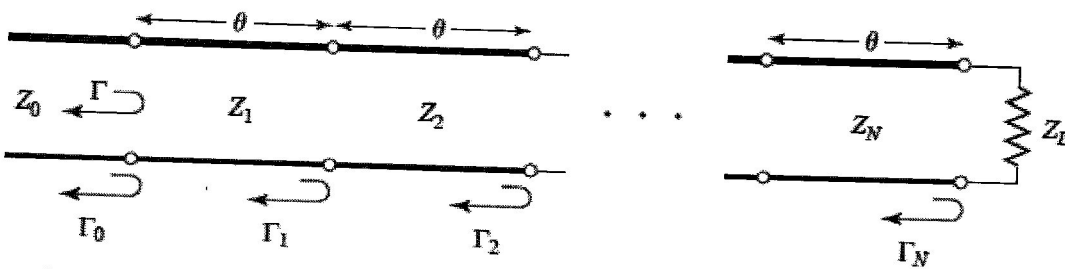


그림: 다단 1/4 파장 임피던스 정합회로

3) 설계공식

$$\Gamma(\theta) = f(\cos \theta, \Gamma_1, \dots, \Gamma_N) = \begin{cases} 2e^{-jN\theta} \left[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \dots + \frac{1}{2}\Gamma_{N/2} \right], & N \text{ even} \\ 2e^{-jN\theta} \left[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \dots + \Gamma_{(N-1)/2} \cos \theta \right], & N \text{ odd} \end{cases}$$

3.1) Binomial (or maximally flat) 1/4-파장 변환

$$\Gamma(\theta) = A(1 + e^{-j2\theta})^N, |\Gamma(\theta)| = 2^N |A| |\cos \theta|^N = f(\cos \theta, \Gamma_1, \dots, \Gamma_N)$$

$$\ln \frac{Z_{n+1}}{Z_n} \approx 2^{-N} C_n^N \ln \frac{Z_L}{Z_0}, n = 0, 1, \dots, N-1$$

$$C_n^N = \frac{N!}{n!(N-n)!}$$

$$\Gamma_m = \max |\Gamma(\theta)|, A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\theta_m = \cos^{-1} \left[\frac{1}{2} \left(\frac{\Gamma_m}{|A|} \right)^{1/N} \right]$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

표: Binomial 다단 1/4-파장 임피던스 변환회로 설계수치

Z_L/Z_0	$N=2$		$N=3$			$N=4$			
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

Z_L/Z_0	$N=5$					$N=6$					
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

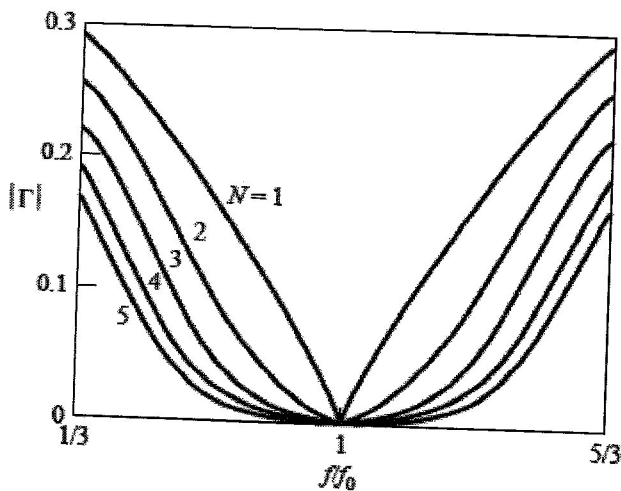


그림: $Z_L/Z_0 = 2$ 인 경우 binomial 다단 1/4-파장 임피던스 변환회로의 주파수 특성

3.2) Chebyshev (or equi-ripple response) 1/4-파장 임피던스 변환

- 대역폭 내에서 주어진 반사계수 조건 만족하는 해 중에서 가장 넓은 대역폭 제공

$$\Gamma(\theta) = Ae^{-jN\theta} T_N(\sec \theta_m \cos \theta) = f(\cos \theta, \Gamma_1, \dots, \Gamma_N)$$

$\Gamma_1, \dots, \Gamma_N$: obtained from the Chebyshev polynomial

$$\Gamma_m = \max |\Gamma(\theta)| = |A|$$

$$\sec \theta_m = \cosh \left[\frac{1}{N} \cosh^{-1} \left(\frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \right) \right]$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

표: Chebyshev 임피던스 변환기 설계값

Z_L/Z_0	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

$N = 4$

Z_L/Z_0	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

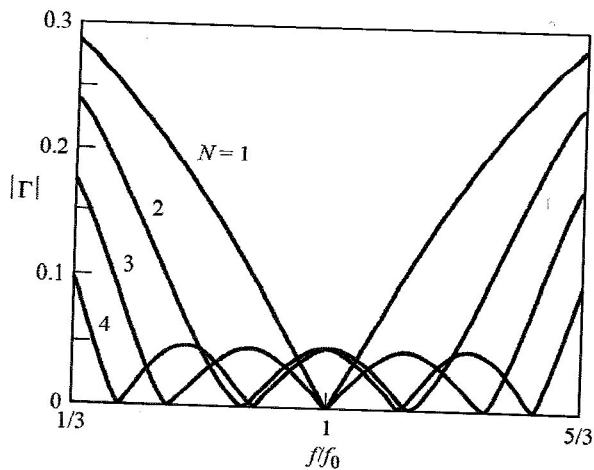


그림: Chebyshev 임피던스 변환기의 주파수 특성

4) 예제

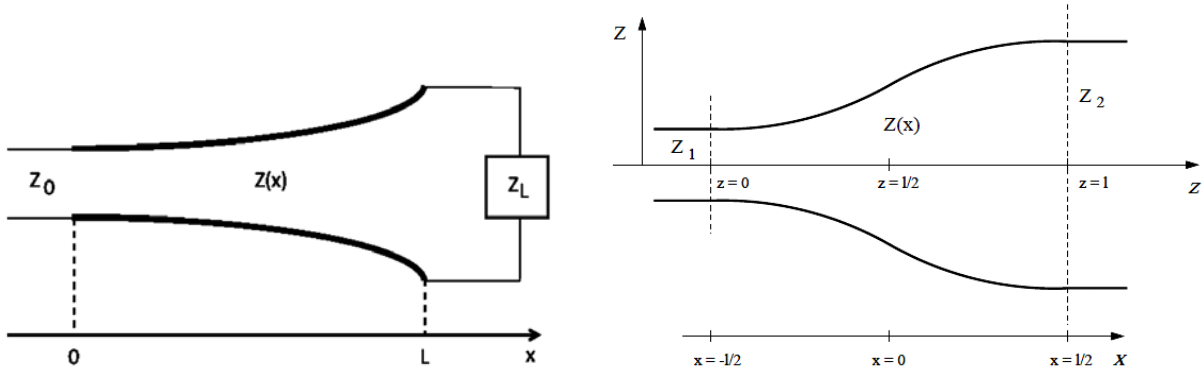
- 100Ω 마이크로스트립 선로를 50Ω 마이크로스트립 선로에 정합시키는 3단 1/4파장 Chebyshev 변환기를 설계하라.
- 기판: Isola I-Tera MT RF, $h = 0.762\text{mm}$, $t = 0.034\text{mm}$, $\epsilon_r = 3.56$, $\tan\delta = 0.0032$
- 기판크기: 높이 = 30mm, 폭 = 선로길이의 합
- $R_L = 100\Omega$, $\Gamma_m = 0.05$, $N = 3$, $f = 10\text{GHz}$
- 표로부터 $Z_1 = 1.1475Z_0 = 57.4\Omega$, $Z_2 = 1.4142Z_0 = 70.7\Omega$, $Z_3 = 1.7429Z_0 = 87.1\Omega$
- 인터넷 마이크로스트립 선로 계산기 mcalc 사용: 주파수 분산, 손실포함
<http://mcalc.sourceforge.net/>

Characteristic imp.(ohm)	Line width (mm)	Effective dielectric constant	Quarter-wave (mm)
50	1.68	2.8459	4.44
57.4	1.23	2.7825	4.49
70.7	0.89	2.6835	4.58
87.1	0.56	2.5875	4.66
100	0.39	2.5263	4.72

5. 연속 테이퍼 (Continuous Taper) 임피던스 정합

1) 구조

- 전송선의 임피던스가 Z_0 에서부터 Z_L 까지 연속적으로 변화



2) 사례

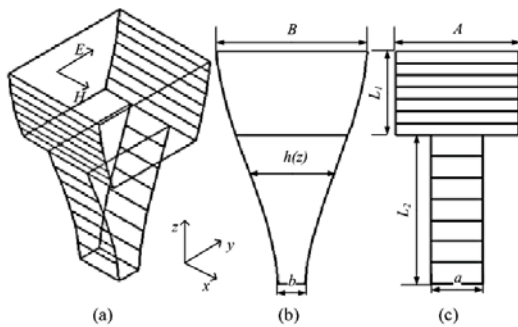
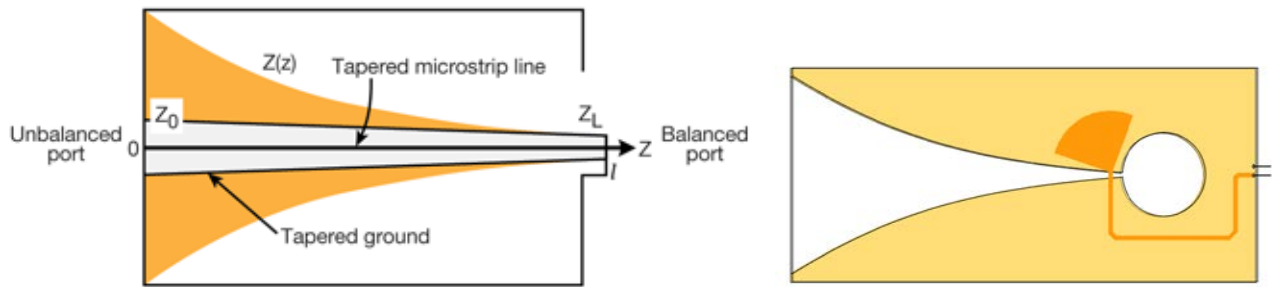
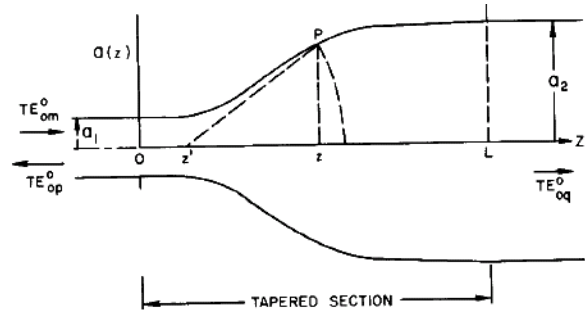


Fig. 1. (a) 3-D view of the tapered box horn element. (b) 2-D view in E-



3) 설계

$$\Gamma(\theta) = \frac{1}{2} \int_0^L e^{-j2\beta\ell} \left(\frac{d}{dz} \ln \frac{Z}{Z_0} \right) dz$$

- Exponential taper:

$$Z = Z_0 e^{az}, \quad a = \frac{\ln(Z_L / Z_0)}{L}$$

$$\Gamma(\theta) = \frac{\ln(Z_L / Z_0)}{2} e^{-j\beta L} \frac{\sin \beta L}{\beta L}$$

- Triangular taper:

$$Z = \begin{cases} Z_0 e^{2(Z_L/Z_0)^2 \ln(Z_L/Z_0)}, & 0 \leq z \leq L/2 \\ Z_0 e^{[4Z_L/Z_0 - 2(Z_L/Z_0)^2 - 1] \ln(Z_L/Z_0)}, & L/2 \leq z \leq L \end{cases}$$

$$\Gamma(\theta) = \frac{\ln(Z_L / Z_0)}{2} e^{-j\beta L} \left[\frac{\sin \beta L}{\beta L} \right]^2$$

- Klopfenstein

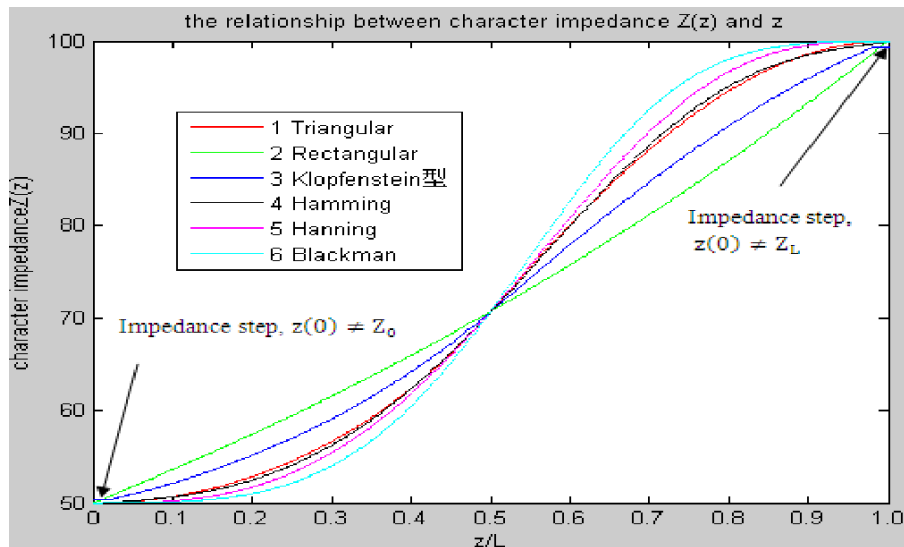
$$\ln Z = \frac{\ln(Z_L Z_0)}{2} + \frac{\Gamma_0}{\cosh A} A^2 \phi(2z/L - 1, A)$$

$$\Gamma(\theta) = \Gamma_0 e^{-j\beta \ell} \frac{\cos \sqrt{(\beta L)^2 - A^2}}{\cosh A}$$

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

4) 특성

S. Chen and Z. Liang, "The impedance matching analysis on different tapered line function", Proc. IEEE IC-BNMT2011, pp. 620-623.



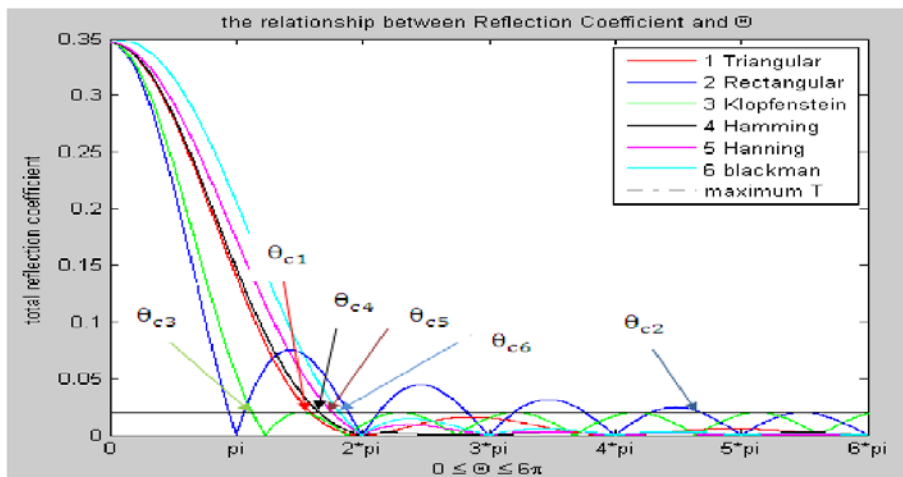
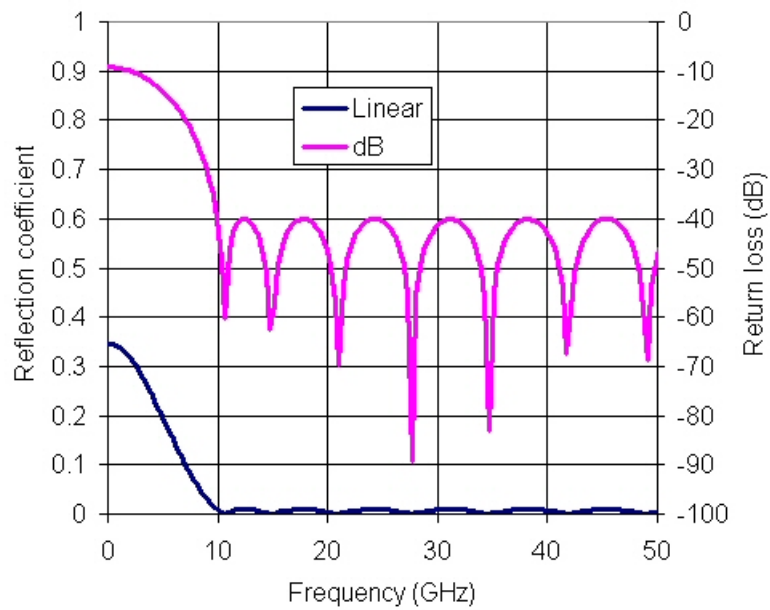


Figure 3 The relationship between the total $T(\theta)$ with θ by using MATLAB

R. P. Hecken, "A near-optimum matching section without discontinuities", IEEE T-MTT, 20(11), 734-739, 1972.

Frequency response of Dolph-Tchebycheff transmission line taper



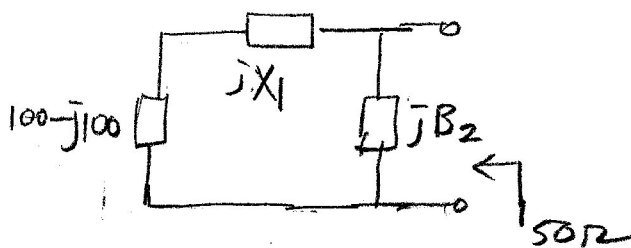
임피던스 정합 : 연습문제

1. $L = 200\text{nH}$, $f = 10\text{MHz}$

a) $Z =$

b) $Y =$

LC 정합회로, $f = 100\text{MHz}$



X_1, B_2 : un-normalized

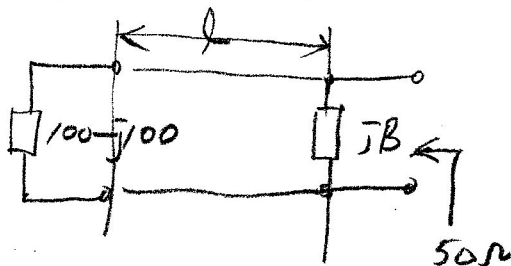
2. X_1 의 해 2개 =

3. X_1 에 해당하는 L_1 또는 C_1 의 값 =

4. B_2 의 해 2개 =

5. B_2 에 해당하는 L_2 또는 C_2 의 값 =

단일 stub 정합

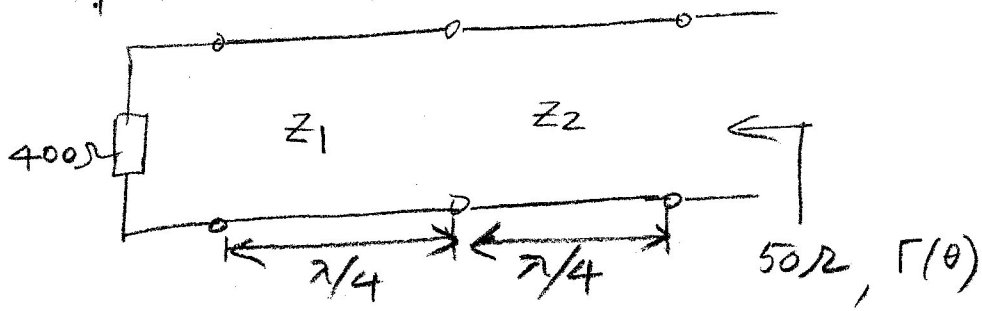


6. 가능한 l 값 중 가장 작은 $l(\lambda) =$

7. 가능한 B 값 중 가장 작은 $B(S) =$

8. l 에 해당하는 개방 또는 단락 전송선 ($Z_0 = 50\Omega$)
길이의 최소값 $(\lambda) =$

$\frac{1}{4}$ 파장 변환기



9. $Z_2 = 50\Omega$ 일 경우 $Z_1(\omega) =$

10. $\Gamma(\theta)$ 가 binomial (maximally flat) 특성을 가지기
위한

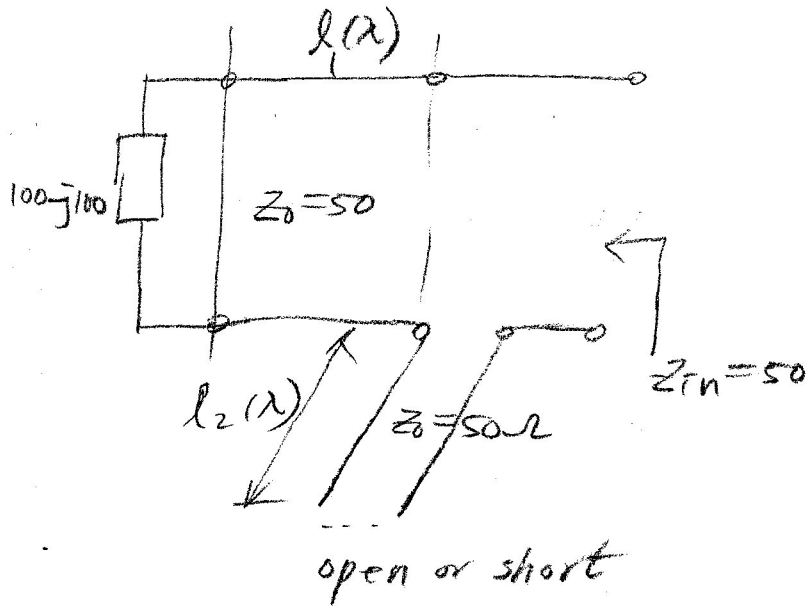
$$Z_1(\omega) =$$

$$Z_2(\omega) =$$

↙ 먼저 시행

임피던스 정합 : Quick Smith 를

1. $f = 100\text{MHz}$



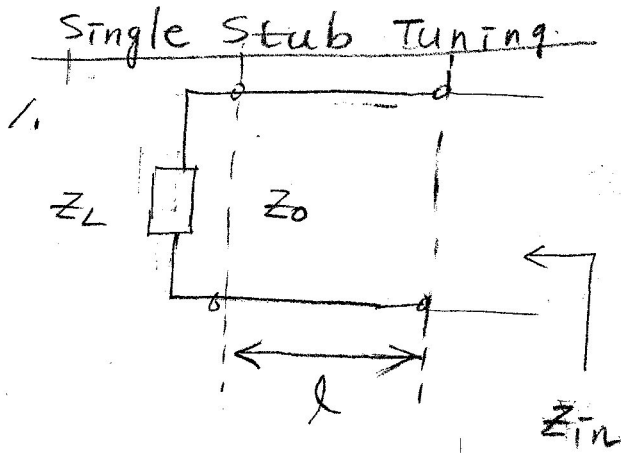
Quick Smith 를 이용하여 위 문제의 해를 구하라. l_1 과 l_2 는 최소값을 적용.

2. Stub을 L 또는 C로 대체하고 ($f = 100\text{MHz}$)

50-150MHz Sweep하여 결과 plot

4중시행

임리전스평합 : Python coding, 프로그램 1개로 작성



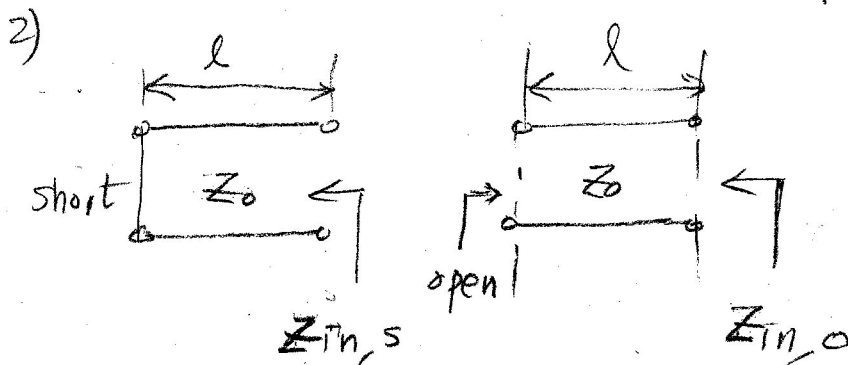
1) 입력 :

Z_L (complex), Z_0 (real) in ohm. =

출력 : $l(\lambda)$ (from 0 to 0.5 by 0.01

$l(\lambda)$	$Z_{in}(\text{ohm})$
0.00	...
...	...
0.50	...

수동으로 (by inspection) $\text{Re}(Z_{in}) = R_{in} = Z_0$ 인 값이 되는 l 을 구함. 이때 $\text{Im}(Z_{in}) = X_{in}$ 을 기록. 2가지가 있을 경우 l 이 작은 경우 선택



$Z_L = 1e-6$

$Z_L = 1e6$ (open)

이정 :

$Z_0(\text{ohm})$ for short and open circuit =

결과 : for $l(\lambda)$ 0 to 0.25 by 0.01

$l(\lambda)$	$Z_{in,s}(\text{ohm})$	$Z_{in,o}(\text{ohm})$
0.0	$1e-6j$	$1e6j$
$\vdots$	$\vdots$	$\vdots$
0.25	$1e6j$	$1e-6j$

수동으로 1)항에서 구한 X_{in} 을 상쇄하는 $-X_{in}$ 의 susceptance를 가지는 경우를 구함, 2가지가 있을 경우 2이 작은 것을 선택

3) 위 1), 2)의 결과를 회로도 표시

