Chapter 6: Microwave Resonators

1) Applications
2) Series and Parallel Resonant Circuits
3) Loaded and Unloaded Q
4) Transmission Line Resonators
5) Rectangular Wave guide Cavities
6) Q of the TE_{10l} Mode
7) Resonator Coupling
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9) Excitation of Resonators
10) Summary

Microwave Resonators Applications

Microwave resonators are used in a variety of applications such as:

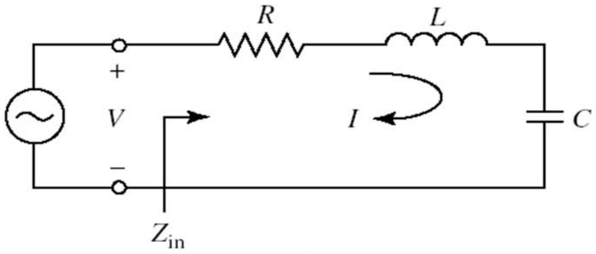
- Filters
- Oscillators
- Frequency meters
- Tuned Amplifier

The operation of microwave resonators are very similar to that of the lumped-element resonators of circuit theory, thus we will review the basic of series and parallel RLC resonant circuits first.

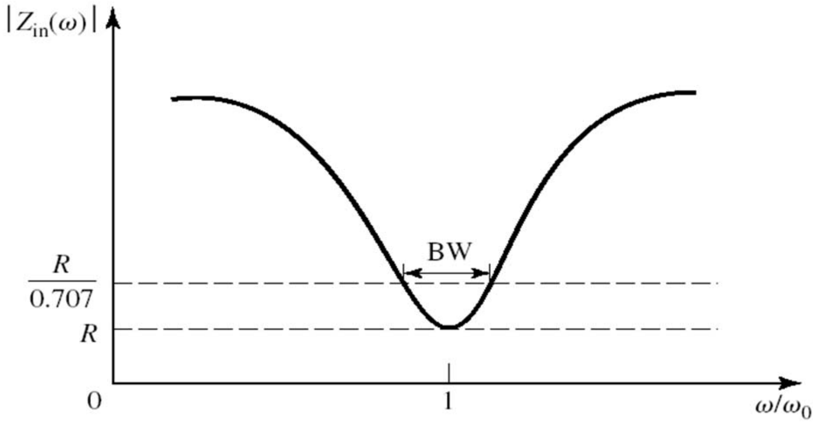
We will derive some of the basic properties of such circuits.

Series Resonant Circuit

Near the resonance frequency, a microwave resonator can be modeled as a series or parallel RLC lumped-element equivalent circuit.



(a)



(b)

A series *RLC* resonator and its response. (*a*) The series RLC circuit. (*b*) The input impedance magnitude versus frequency.

$$Z_{in} = R + j\omega L - j\frac{1}{\omega C}$$

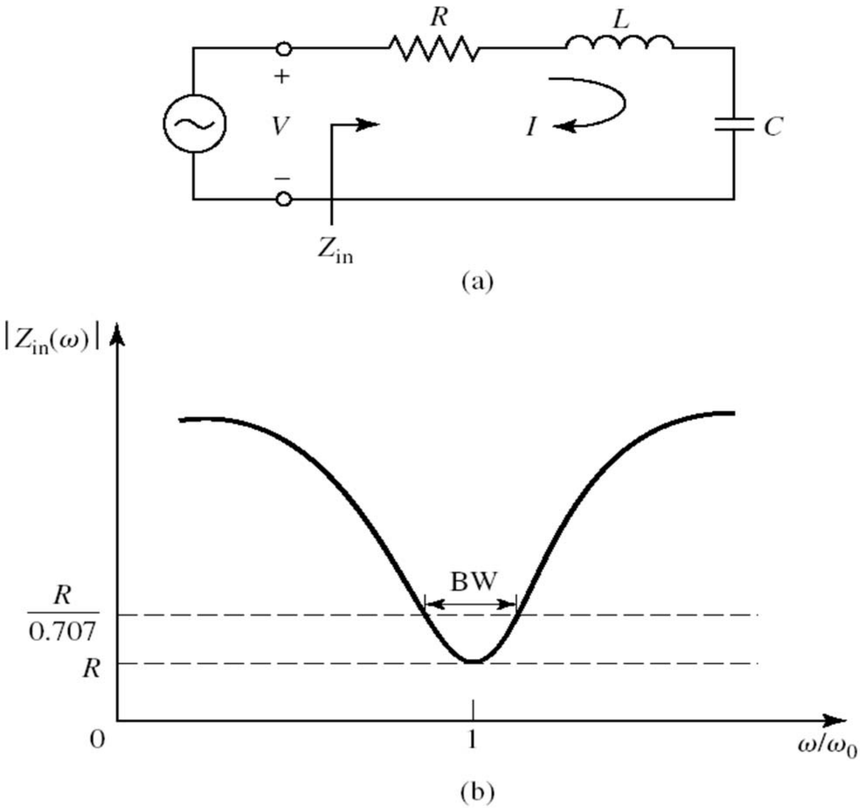
$$P_{in} = \frac{1}{2}|I|^2 Z_{in} = \frac{1}{2}|I|^2\left(R + j\omega L - j\frac{1}{\omega C}\right)$$

Average magnetic energy: $W_m = \frac{1}{4}|I|^2 L$

Average electric energy: $W_e = \frac{1}{4}|I|^2 \frac{1}{\omega^2 C}$

Resonance occurs when the average stored magnetic (W_m) and electric energies (W_e) are equal and Z_{in} is purely real.

Series Resonators



A series *RLC* resonator and its response. (*a*) The series RLC circuit. (*b*) The input impedance magnitude versus frequency.

• The frequency in which $\omega_o = \frac{1}{\sqrt{LC}}$ is called the **resonant frequency**.

• Another important factor is the **Quality Factor Q.**

$$Q = \omega \frac{Average\ Energy\ Stored}{Energy\ Loss\ /\ Second}$$

$$BW = 1/Q \qquad Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$$

Near the resonance $\omega = \omega_o + \Delta\omega$

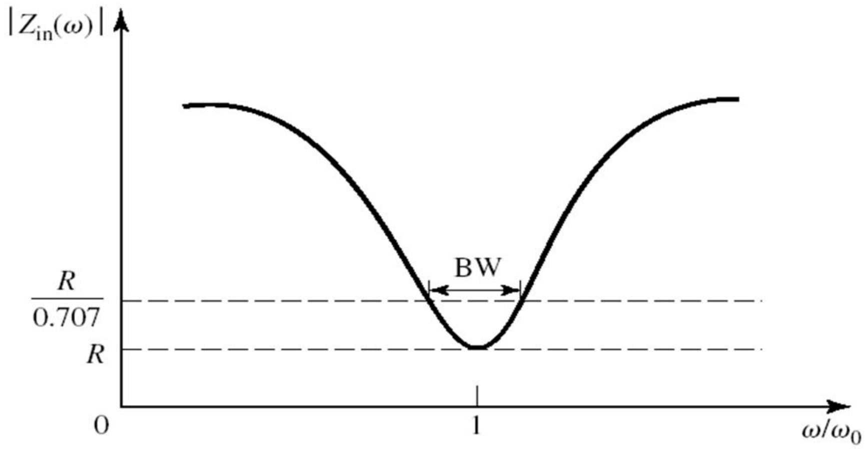
$$Z_{in} = R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right) = R + j\omega L\left(\frac{\omega^2 - \omega_o^2}{\omega^2}\right)$$

$$= R + j\frac{2RQ\Delta\omega}{\omega_o}$$

$$\omega^2 - \omega_o^2 = (\omega - \omega_o)(\omega + \omega_o) \cong 2\omega\Delta\omega$$

$$Z_{in} = R + j2L\Delta\omega$$

Series Resonators



The input impedance magnitude versus frequency.

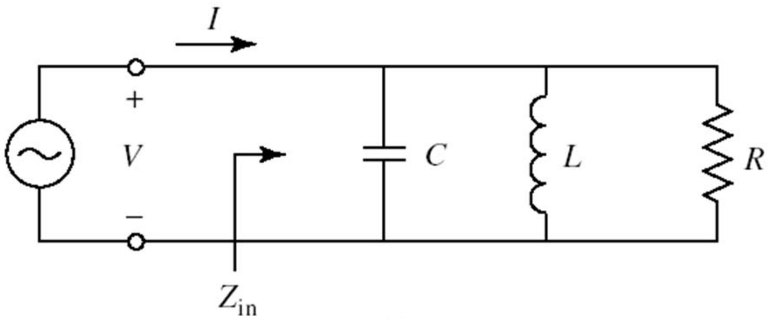
$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$$

Fractional Bandwidth is defined as: $\frac{\Delta\omega}{\omega_o} = \frac{BW}{2}$ $\quad BW = 1/Q$

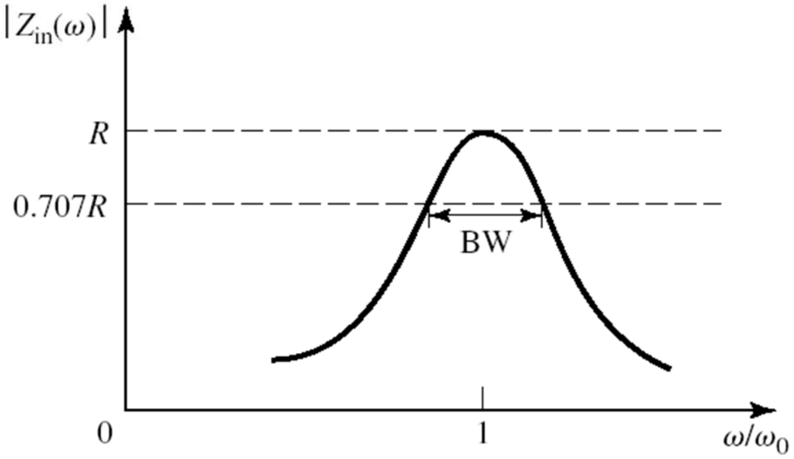
and happens when the average (real) power delivered to the circuit is one-half that delivered at the resonance.

- Bandwidth increases as R increases.
- Narrower bandwidth can be achieved at higher quality factor (Smaller R).

Parallel Resonators



(a)



(b)

A parallel RLC resonator and its response. (*a*) The parallel RLC circuit. (*b*) The input impedance magnitude versus frequency.

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1}$$

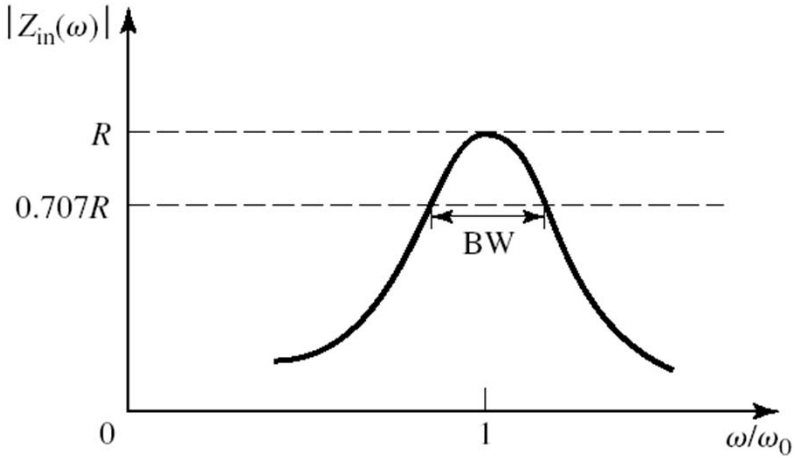
Resonance occurs when the average stored magnetic and electric energies are equal and Z_{in} is purely real. The input impedance at resonance is equal to R.

$$Q = \omega \frac{Average\ Energy\ Stored}{Energy\ Loss\ /\ Second}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad Q = \frac{R}{\omega_o L} = \omega_o RC$$

Note: Resonance frequency is equal to the series resonator case. Q is inversed.

Parallel Resonators



The input impedance magnitude versus frequency.

$$Q = \frac{R}{\omega_o L} = \omega_o RC \qquad BW = 1/Q$$

- Bandwidth reduces as R increases.
- Narrower bandwidth can be achieved at higher quality factor (Larger R).

Close to the resonance frequency:

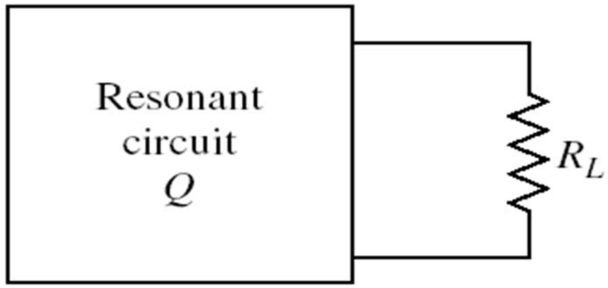
$$\omega = \omega_o + \Delta\omega$$

$$Z_{in} \cong \left(\frac{1}{R} + 2j\Delta\omega C\right)^{-1} = \frac{R}{1 + 2jQ\Delta\omega/\omega_o}$$

$$R \to \infty \qquad Z_{in} = \frac{1}{j2C(\omega - \omega_o)}$$

Loaded and Unloaded Q Factor

The Q factors that we have calculated were based on the characteristic of the resonant circuit itself, in the absence of any loading effect (Unloaded Q).



A resonant circuit connected to an external load, R_L.

In practice a resonance circuit is always connected to another circuitry, which will always have the effect of lowering the overall Q (Loaded Q).

$$Q_e = \begin{cases} \dfrac{\omega_o L}{R_L} & \text{for series connection} \\ \dfrac{R_L}{\omega_o L} & \text{for parallel connection} \end{cases}$$

Loaded Q Factor

If the resonator is a series RLC and coupled to an external load resistor R_L, the effective resistance is:

$$R_e = R + R_L$$

If the resonator is a parallel RLC and coupled to an external load resistor R_L, the effective resistance is:

$$R_e = R \cdot R_L / R + R_L$$

Then the loaded Q can be written as:

$$\frac{1}{Q_L} = \frac{1}{Q} + \frac{1}{Q_e}$$

Note: Loaded Q factor is always smaller than Unloaded Q.

TABLE 6.1 Summary of Results for Series and Parallel Resonators

Quantity	Series Resonator	Parallel Resonator
Input Impedance/admittance	$Z_{in} = R + j\omega L - j\dfrac{1}{\omega C}$ $\simeq R + j\dfrac{2RQ\Delta\omega}{\omega_0}$	$Y_{in} = \dfrac{1}{R} + j\omega C - j\dfrac{1}{\omega L}$ $\simeq \dfrac{1}{R} + j\dfrac{2Q\Delta\omega}{R\omega_0}$
Power loss	$P_{loss} = \dfrac{1}{2}\|I\|^2 R$	$P_{loss} = \dfrac{1}{2}\dfrac{\|V\|^2}{R}$
Stored magnetic energy	$W_m = \dfrac{1}{4}\|I\|^2 L$	$W_m = \dfrac{1}{4}\|V\|^2\dfrac{1}{\omega^2 L}$
Stored electric energy	$W_e = \dfrac{1}{4}\|I\|^2\dfrac{1}{\omega^2 C}$	$W_e = \dfrac{1}{4}\|V\|^2 C$
Resonant frequency	$\omega_0 = \dfrac{1}{\sqrt{LC}}$	$\omega_0 = \dfrac{1}{\sqrt{LC}}$
Unloaded Q	$Q = \dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 RC}$	$Q = \omega_0 RC = \dfrac{R}{\omega_0 L}$
External Q	$Q_e = \dfrac{\omega_0 L}{R_L}$	$Q_e = \dfrac{R_L}{\omega_0 L}$

Transmission Line Resonators (Short Circuited)

• Ideal lumped element (R, L and C) are usually impossible to find at microwave frequencies.

• We can design resonators with transmission line sections with different lengths and terminations (Open or Short).

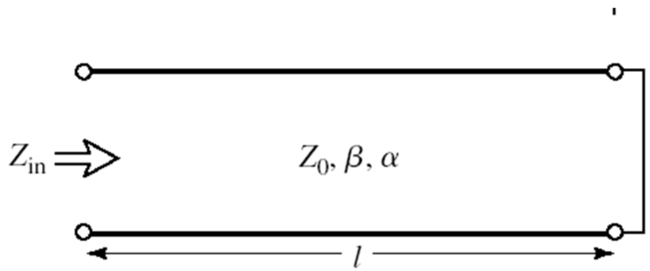
• Since we are interested in the Q of these resonators we will consider the Lossy Transmission Line.

For the special case of : $\ell = \lambda / 2$

$$Z_{in} = Z_o \tanh(\alpha + j\beta)\ell$$

$$Z_{in} = Z_o \frac{\tanh \alpha\ell + j \tan \beta\ell}{1 + j \tanh \beta\ell \cdot \tanh \alpha\ell}$$

$$Q = \frac{\omega_o L}{R} = \frac{\beta}{2\alpha}$$



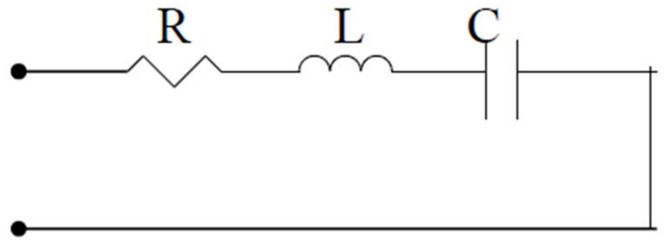
A short-circuited length of lossy transmission line.

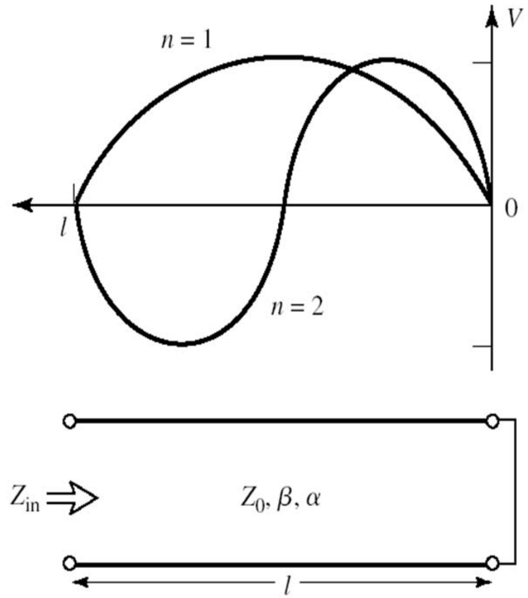
Transmission Line Resonators (Short Circuited)

- The resonance occurs for $\ell = \dfrac{n\lambda}{2}$ $\quad n = 1, 2, 3, \ldots$

$$Z_{in} = Z_o[\alpha\ell + j(\Delta\omega\pi/\omega_o)]$$

$$Q = \frac{\omega_o L}{R} = \frac{\beta}{2\alpha}$$





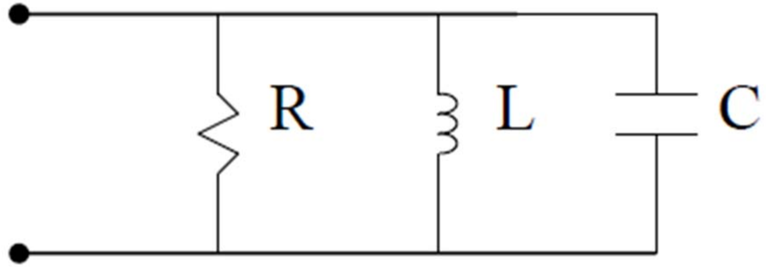
A short-circuited length of lossy transmission line and the voltage distributions for n=1, $\ell = \dfrac{\lambda}{2}$

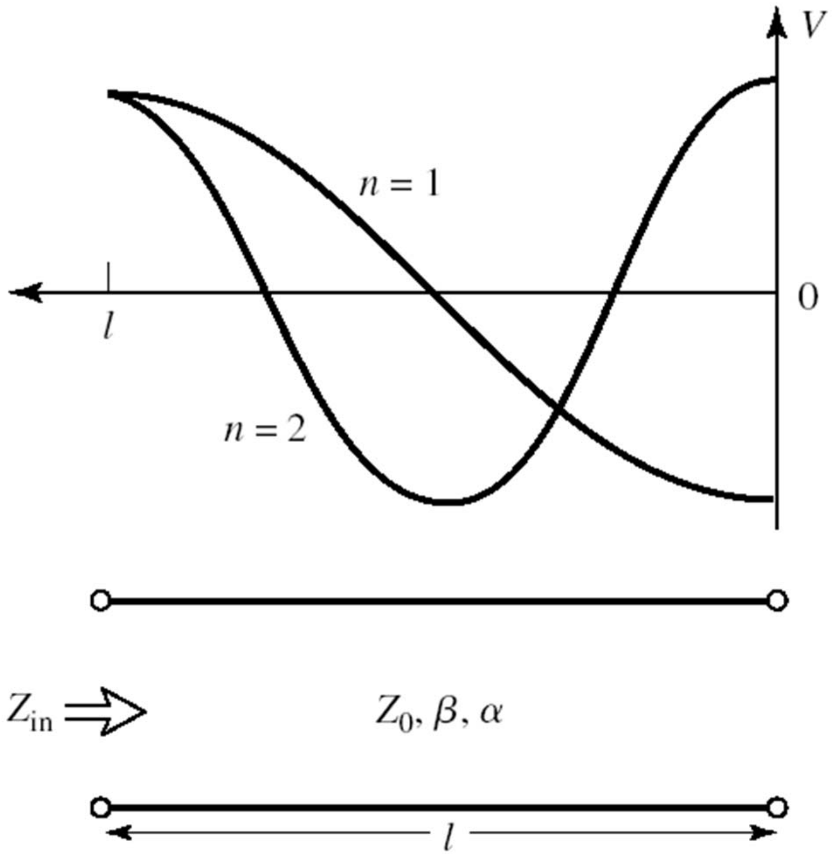
Transmission Line resonators

$$Z_{in} = \frac{Z_o}{\alpha\ell + j(\Delta\omega\pi/2\omega_o)}$$

$$R = \frac{Z_o}{\alpha\ell} \qquad C = \frac{\pi}{4\omega_o Z_o} \qquad L = \frac{1}{\omega_o^2 C}$$

$$Q = \frac{\beta}{2\alpha}$$





An open-circuited length of lossy transmission line, and the voltage distributions for n = 1 resonators.

Rectangular waveguide cavity resonator

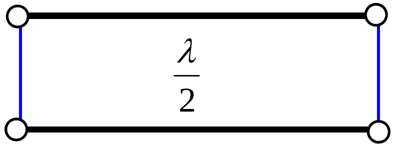
We can look at them as short circuit section of transmission line

$$\beta_{mn} = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Boundry conditions enforced

$$E_x = E_y = 0 \text{ for } z = 0, d$$

$$\Rightarrow \beta_{mn} d = l\pi \;\rightarrow \beta_{mn} = \frac{l\pi}{d} \qquad l = 1,2,3\ldots$$

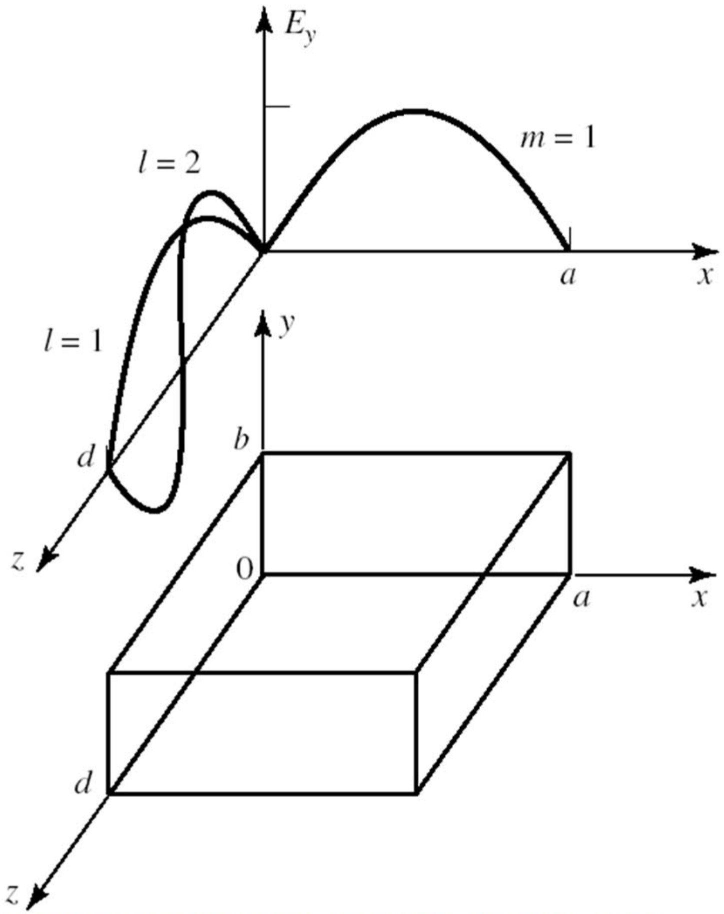


$$\Rightarrow \left(\frac{l\pi}{d}\right)^2 = \omega^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$$\frac{2\pi f}{c} = \sqrt{\left(\frac{l\pi}{d}\right)^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

then the resonance frequency

$$f = \frac{c}{2\pi}\sqrt{\left(\frac{l}{d}\right)^2 + \left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$



A rectangular resonant cavity, and the electric field distributions for the TE_{101} and TE_{102} resonant modes.

if $b < a < d$ the lowest and dominant resonant TE (resp TM) mode will be TE101 (resp. TM110)

Rectangular waveguide cavity resonator
Q factor for TE10*l*

$$Q = \left(\frac{1}{Q_c} + \frac{1}{Q_d} \right)^{-1}$$

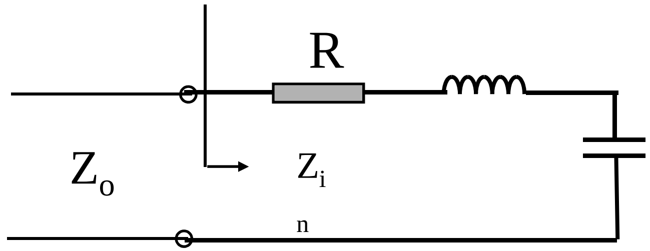
$$Q_d = \frac{1}{\tan \delta}$$

$$Q_c = \frac{(2\pi a d/\lambda)^3 \eta}{2\pi^2 R_s} \frac{1}{\left(2l^2 a^3 b + 2bd^3 + l^2 a^3 d + ad^3\right)}$$

$$R_s = \sqrt{\omega \mu_o / 2\sigma}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Resonators - coupling

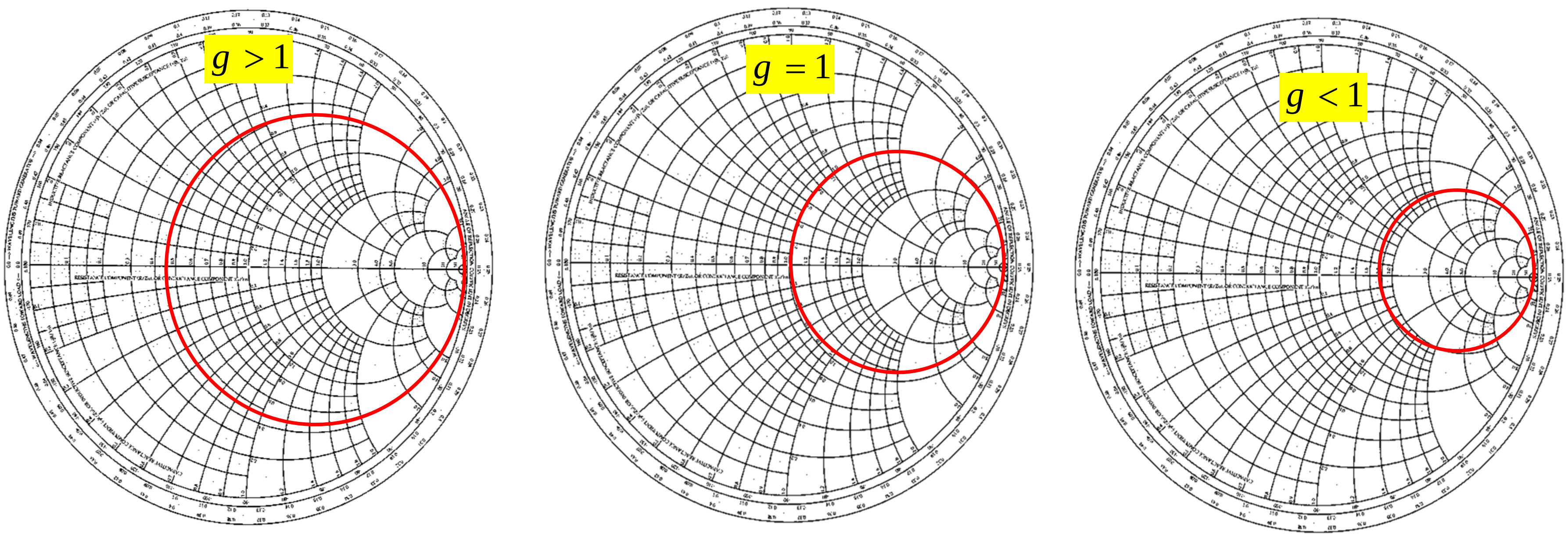


Critical coupling occurs when $2Q_L = Q_e = Q_u$

If we define coupling coefficient $g = \frac{Q_u}{Q_e}$ then we have

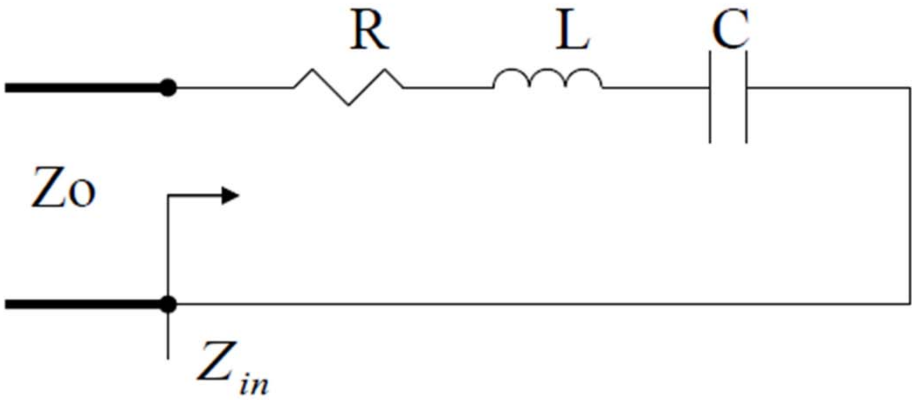
- undercoupled resonator if g<1
- critically coupled resonator if g=1 (resonator matched to the feed line)
- overcoupled resonator if g>1

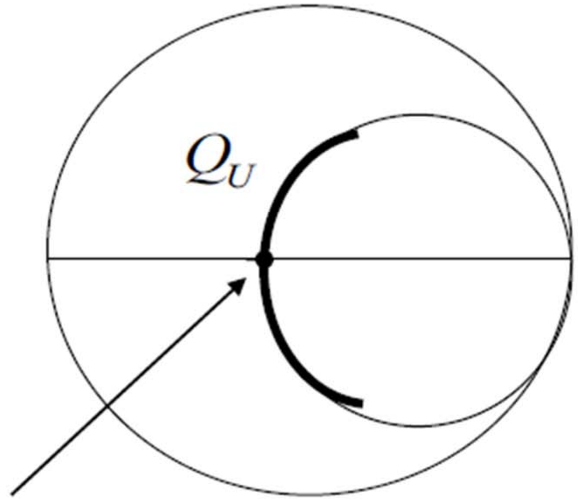
series RLC	parallel RLC
$g = \frac{Z_0}{R}$	$g = \frac{R}{Z_0}$



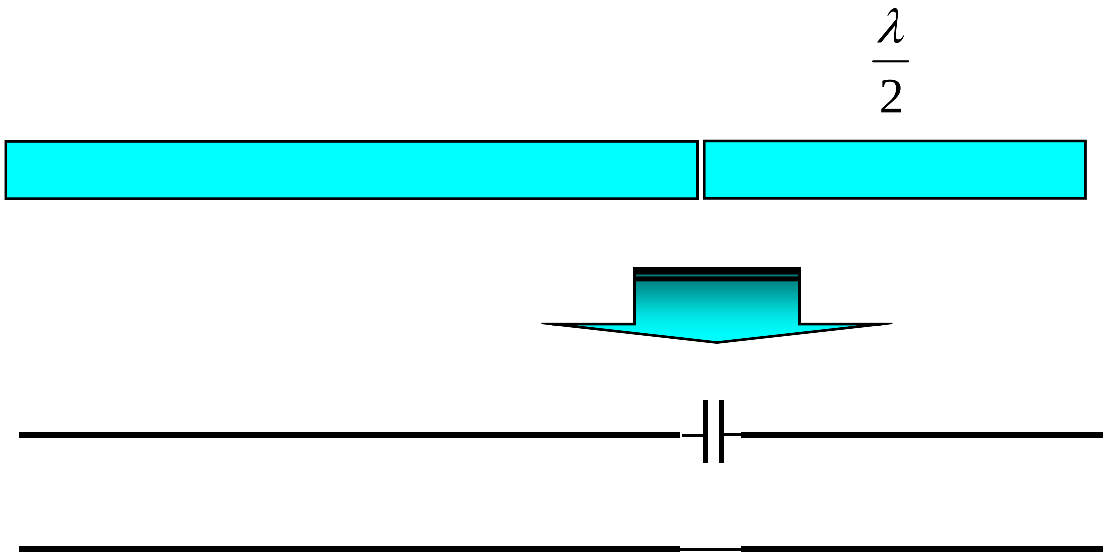
Series RLC circuit

• Critical coupling $Z_{in}(\omega_o) = Z_o$





Gap coupled microstrip resonator resonators



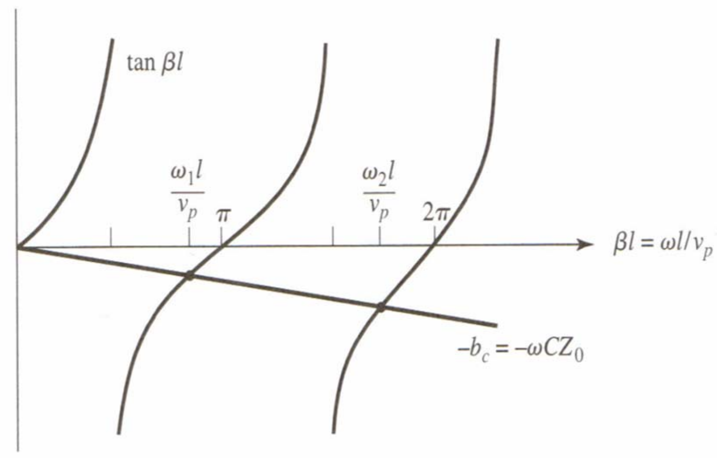
Serial RLC

coupling capacitor acts as impedance inverter

$$z = \frac{Z}{Z_o} = -j\left(\frac{\tan\beta l + b_c}{b_c \tan\beta l}\right) \quad \text{where } b_c = Z_o \omega C$$

Resonance occurs when $\tan\beta l + b_c = 0$

The coupling of the feed line to the resonator lowers its resonant frequency



Solutions to (6.78) for the resonant frequencies of the gap-coupled microstrip resonator.

Gap coupled microstrip resonator resonators

$$Z(\omega) = Z_o\left(\frac{\pi}{2Qb_c^2} + j\frac{\pi(\omega-\omega_o)}{\omega_o b_c^2}\right)$$

$$\Rightarrow R = Z_o\left(\frac{\pi}{2Qb_c^2}\right) \Rightarrow b_c = \sqrt{\left(\frac{\pi}{2Q}\right)}$$

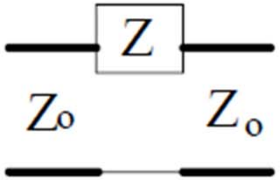
the coupling coeficient $g = \frac{Z_o}{R} = \frac{\pi}{2Q_u b^2{}_c}$

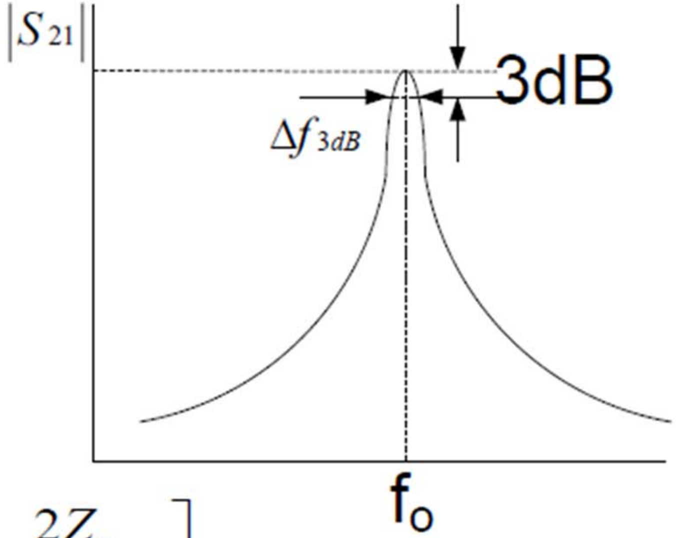
for critical coupling $Z_o = R \quad \Rightarrow b_c = \sqrt{\left(\frac{\pi}{2Q}\right)}$

for undercoupled resonator $\quad \Rightarrow b_c < \sqrt{\left(\frac{\pi}{2Q}\right)}$

for overcoupled resonator $\quad \Rightarrow b_c > \sqrt{\left(\frac{\pi}{2Q}\right)}$

.Determin Q_U from 2-port measurement





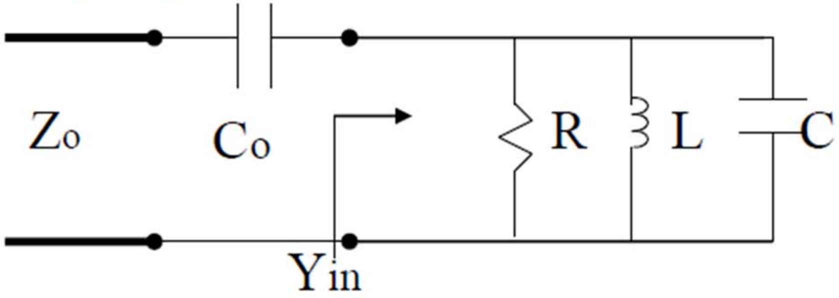
$$@\,f_o:[Z]=\begin{bmatrix} R & R \\ R & R \end{bmatrix},[S]=\begin{bmatrix} \dfrac{R}{R+2Z_o} & \dfrac{2Z_o}{R+2Z_o} \\ \dfrac{2Z_o}{R+2Z_o} & \dfrac{R}{R+2Z_o} \end{bmatrix}$$ .....prob 4.11

$$Q_U=\frac{\omega_o L}{R},\ Q_e=\frac{\omega_o L}{R_L}=\frac{\omega_o L}{2Z_o}\rightarrow g=\frac{Q_U}{Q_e}=\frac{2Z_o}{R}$$

$$S_{21}(f_o)=\frac{2Z_o}{R+2Z_o}=\frac{g}{1+g}\rightarrow g=\frac{S_{21}(f_o)}{1-S_{21}(f_o)}$$

$$\frac{1}{Q_L}=\frac{1}{Q_e}+\frac{1}{Q_U}=\frac{1}{Q_U}(1+g),\ \text{measure } Q_L=\frac{f_o}{\Delta f_{3sB}}\rightarrow Q_U=Q_L(1+g)$$

Solved problems: Prob. 6.22 A parallel resonator, calculate C_o for critical coupling and f_r.



R=1000 Ω, L=1.26nH, C=0.804pF, Z_o=50 Ω

$$Y_{in}(\omega) \approx \frac{1}{R} + j\frac{2Q_u\Delta\omega}{R\omega_o} = 10^{-3} + j50.5\times10^{-3}\frac{\Delta\omega}{\omega_o}, Q_U = \frac{R}{\omega_o L} = 25.3, \omega_o = \frac{1}{\sqrt{LC}} = 31.4\times10^{9}$$

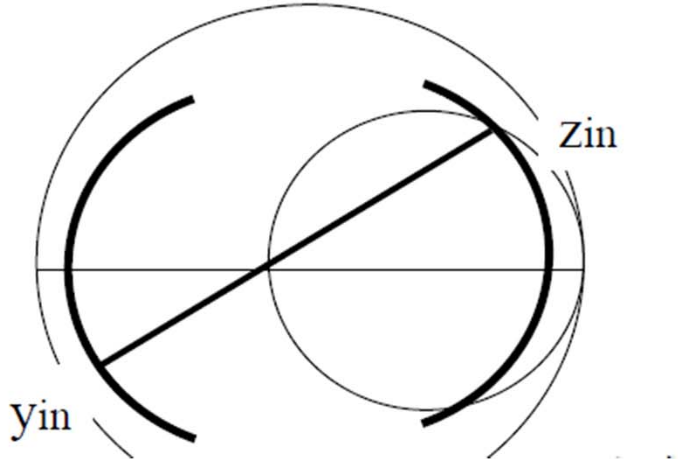
$$\rightarrow y_{in} = 50\times Y_{in} = 0.05 + j2.53\frac{\Delta\omega}{\omega_o} = \frac{1}{1+ja} = \frac{1}{1+a^2} - j\frac{a}{1+a^2} \rightarrow 0.05 = \frac{1}{1+a^2} \rightarrow a = 4.36$$

$$\because z_{in} + \frac{1}{j\omega C_o Z_o} = 1 \rightarrow z_{in} = 1 + j\frac{1}{\omega C_o Z_o} = 1 + ja$$

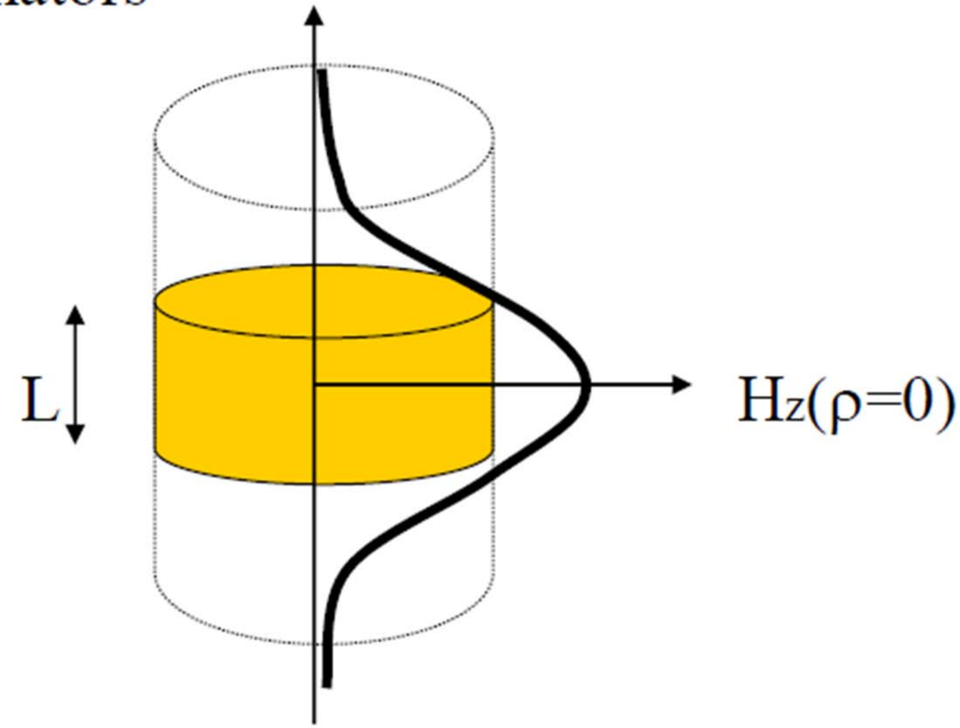
$$-\frac{a}{1+a^2} = -\frac{4.36}{20} = 2.53\frac{\Delta\omega}{\omega_o} \rightarrow \frac{\Delta\omega}{\omega_o} = -0.086$$

$$\rightarrow f_r = f_o - 0.086 f_o = 4.57GHz$$

$$\frac{1}{\omega_r C_o Z_o} = 4.36 \rightarrow C_o = 0.16pF$$



Dielectric resonators



$$10 < \varepsilon_r < 100, \quad TE_{01\delta}\ \text{mode}\ \delta = \frac{2L}{\lambda_g} < 1, \quad Q_d \approx \frac{1}{\tan\delta}$$

Ex.6.5 $\varepsilon_r = 95,\ \tan\delta = 0.001,\ a = 0.413cm$

$\rightarrow\ f = 3.4GHz,\ \ Q_d = 1000$

L09 - Microwave Resonator

1. Q=100 인 직렬 RLC 공진회로

a) 설계 : 공진주파수 → 설계자가 결정, 소자값 설계자가 결정

b) QuickSmith로 스미스도표상 임피던스 궤적도시

2. RLC 병렬공진회로에 대해 1 반복

3. 1의 회로의 주파수에 따른 입력임피던스 계산 Python 프로그램 작성

4. 2에 대해 3항 반복