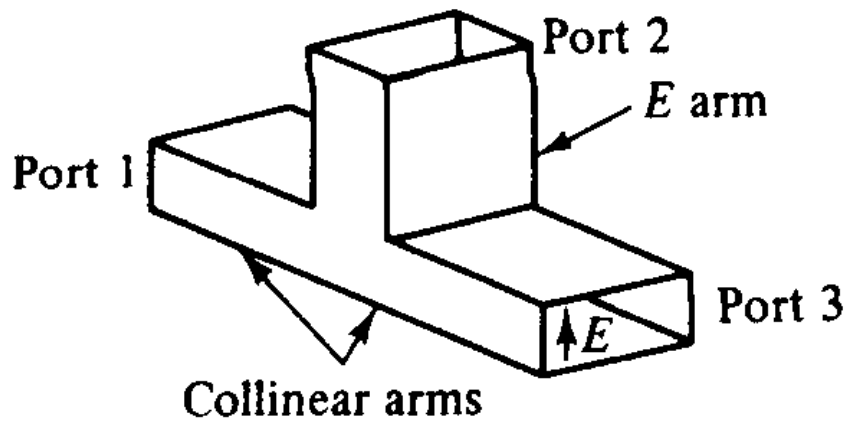


Microwave Hybrid Circuits

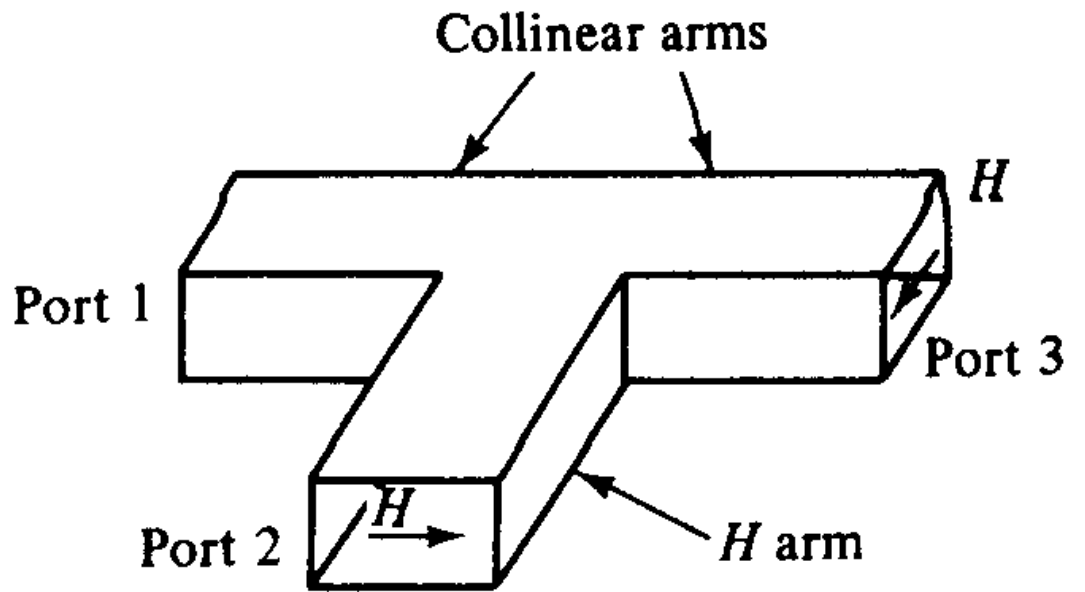
Microwave circuits consists of several microwave devices connected in some way to achieve the desired transmission of a microwave signal

The interconnection of two or more microwave devices may be regarded as a **microwave junction**.

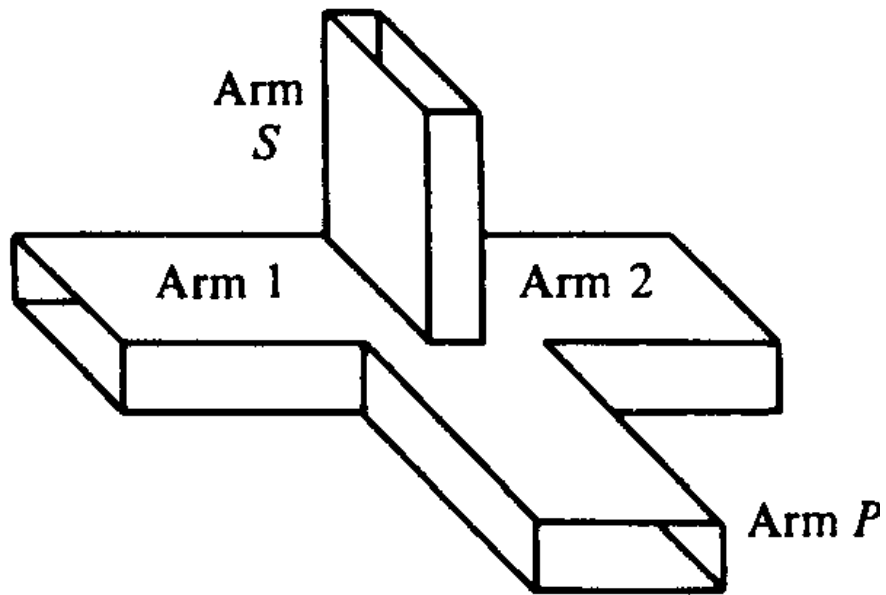
Waveguide Tees as the E-plane tee, H-plane tee, Magic tee, hybrid ring tee(rat-race circuit), directional coupler and the circulator



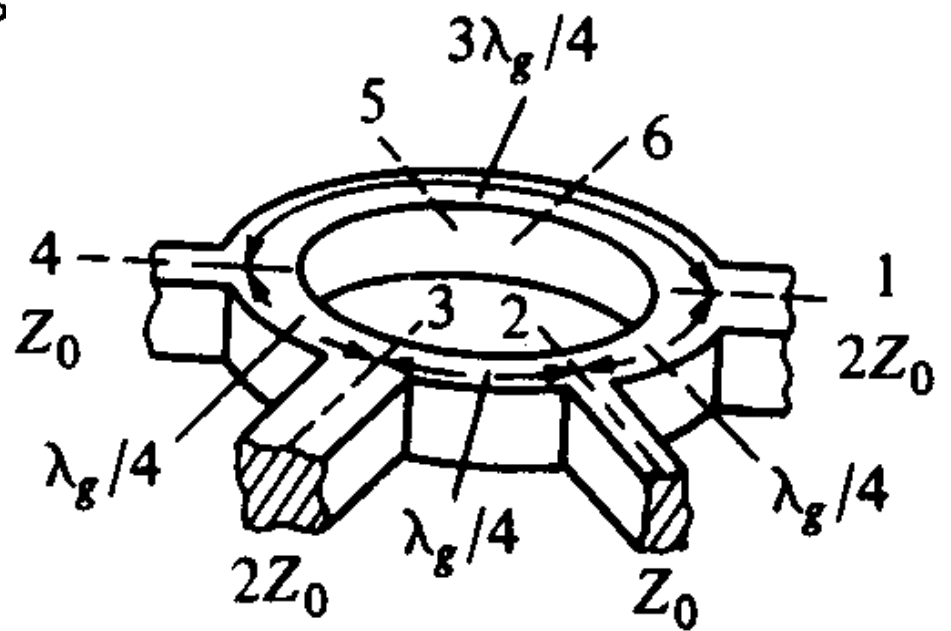
(a) *E*-plane tee



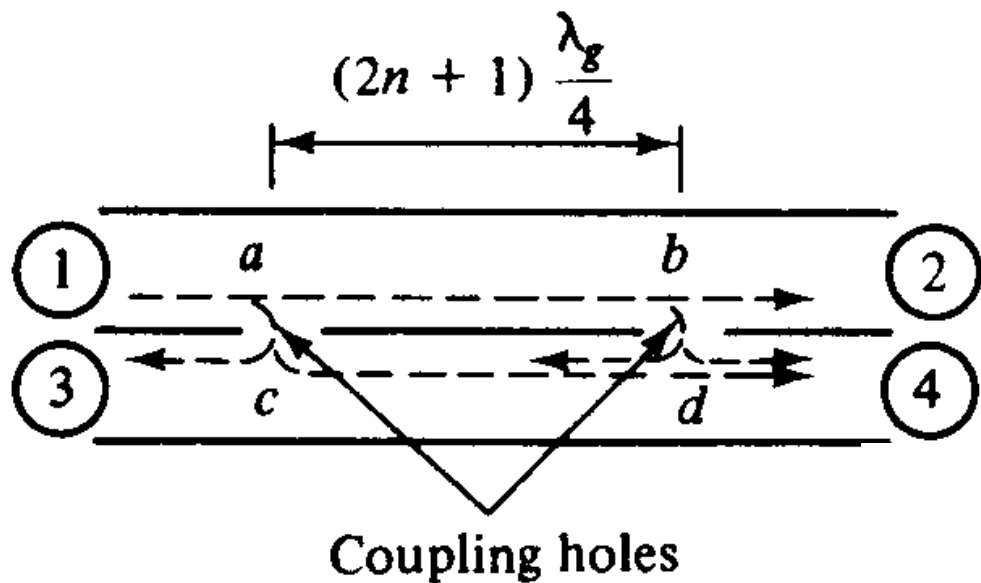
(b) *H*-plane tee



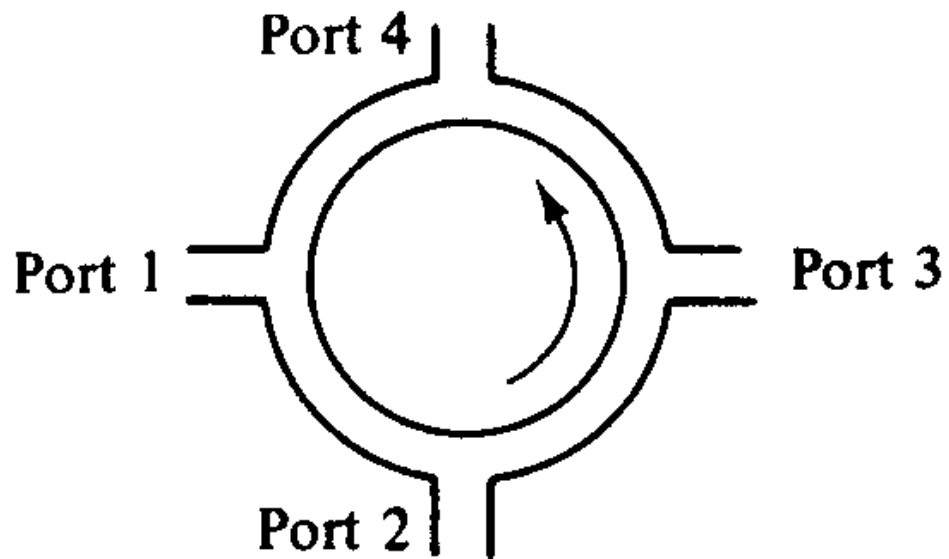
(c) Magic tee



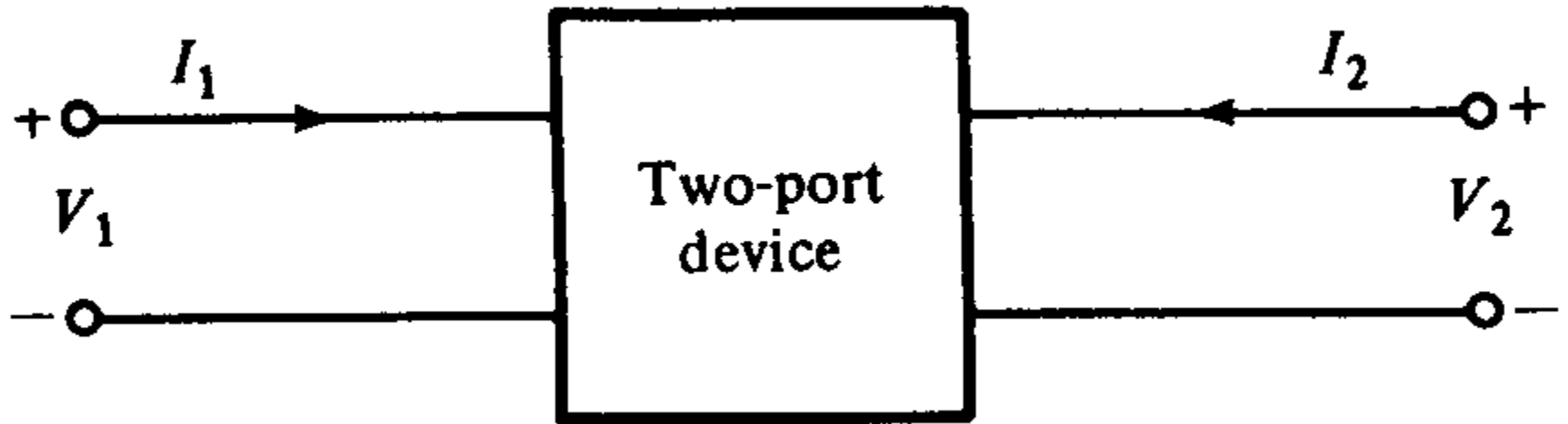
(d) Hybrid ring



(e) Directional coupler



(f) Circulator



Two-port network.

H,Y,Z and ABCD parameters

H parameters:
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}$$

Y parameters:
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_1 \end{aligned}$$

Z parameters:
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}$$

ABCD parameters:
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

All these network parameters relate total voltages and total currents at each of the two ports. For eg,

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad \text{(short circuit)}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} \quad \text{(open circuit)}$$

If the frequencies are in the microwave range, however.....

..... The H,Y and Z parameters cannot be measured for the following reasons:

1. Equipment is not readily available to measure total voltage and total current at the ports of the network.
2. Short and Open circuits are difficult to achieve over a broad band of frequencies.
3. Active devices, such as power transistors and tunnel diodes, frequently will not have stability for a short or open circuit.

New method of characterization is needed

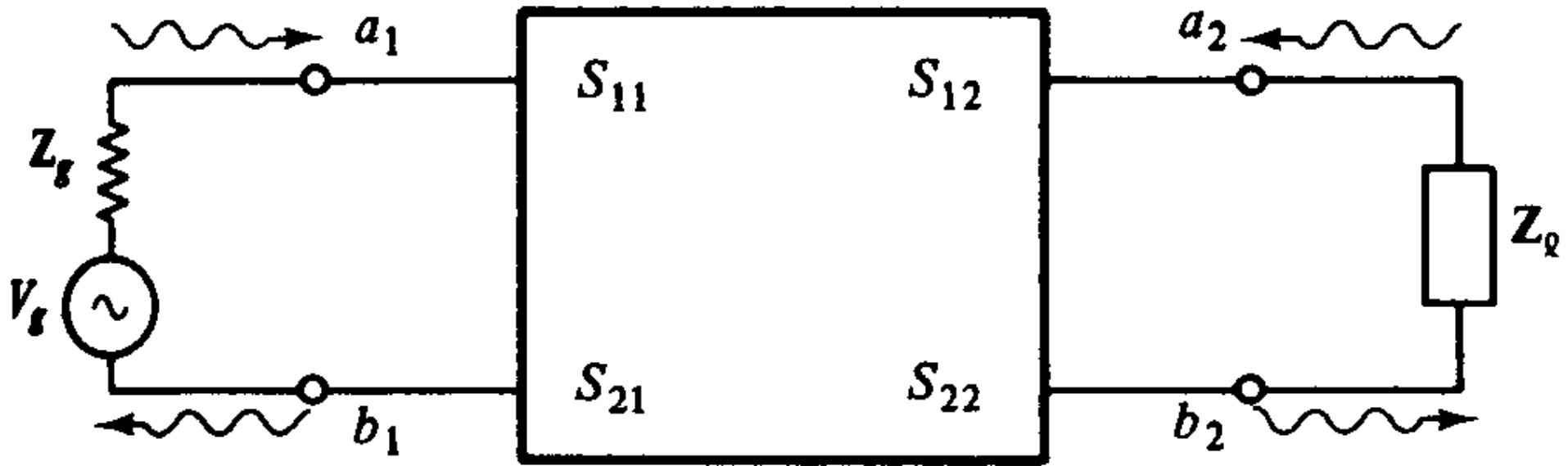
The logical variables to use at the microwave frequencies are travelling waves rather than total voltages and total currents.

These are the S parameters,

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

S parameters of a two port network



Waveguide Tee

1. H-plane Tee
2. E-plane Tee
3. Magic Tee
4. Hybrid Rings
5. Corners
6. Bends
7. Twists

Tee Junction

A waveguide or coaxial-line junction with three independent ports

Matrix of third order, containing nine elements, six of which should be independent.

The characteristics of a three port junction can be explained by three theorems of the tee junction.

These theorems are derived from the equivalent-circuit representation of the tee junction

Statements

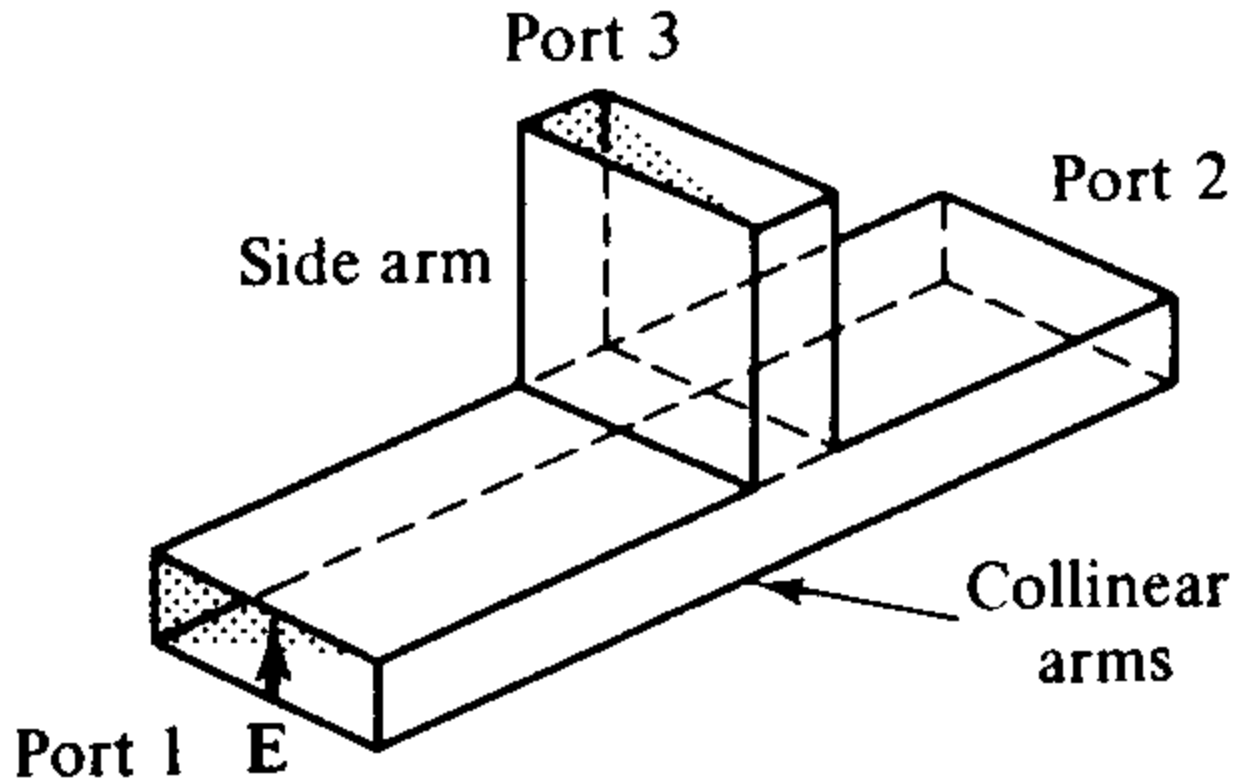
1. A short circuit may always be placed in one of the arms of a three-port junction in such a way that no power can be transferred through the other two arms.
2. If the junction is symmetric about one of its arms, a short circuit can always be placed in that arm so that no reflections occur in power transmission between the other two arms. (i.e the arms present matched impedances.)

3. It is impossible for a general three-port junction of arbitrary symmetry to present matched impedances at all three arms.

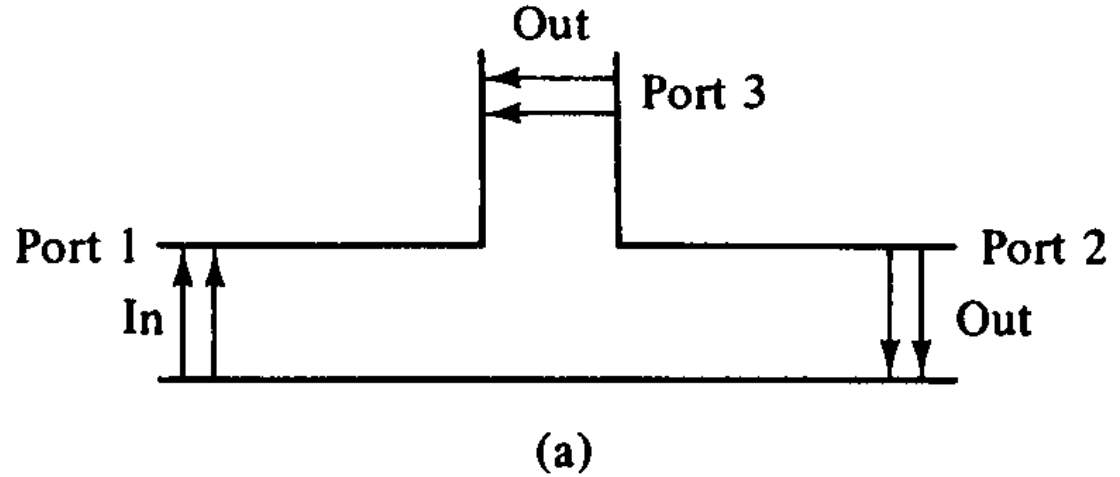
E-plane Tee

Series Tee

A waveguide tee in which the **axis** of its side arm is **parallel to the E-field** of the main guide.



If the collinear arms are symmetric about the side arm, there are two different transmission characteristics

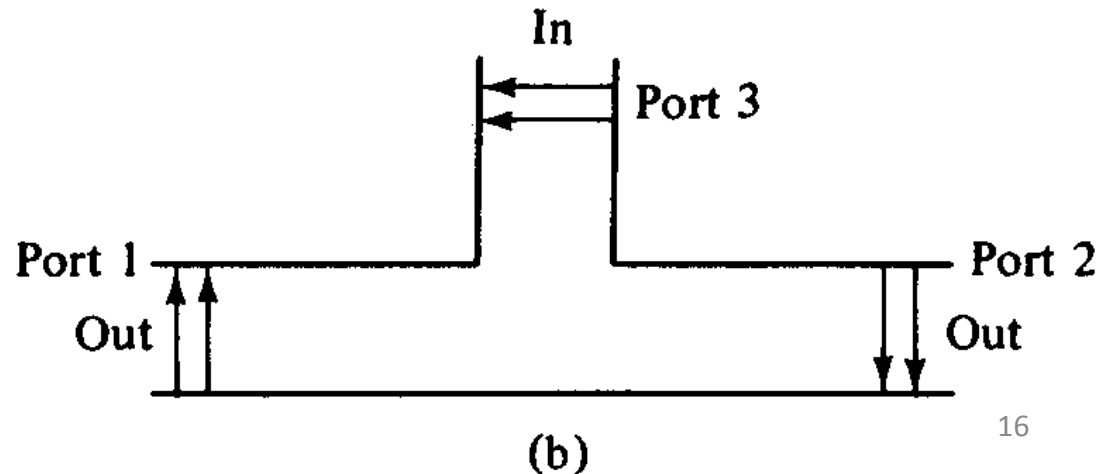


Two way

Transmission of
E-plane tee

a) i/p-main arm

b) i/p-side arm



If E-plane tee is perfectly matched with the aid of screw tuners or inductive or capacitive windows at the junction, the diagonal components of the S-matrix, S_{11} , S_{22} and S_{33} are zero because there will be no reflection.

When the waves are fed into the side arm (port 3), the waves appearing at port1 and port2 of the collinear arm will be in the opposite phase and in the same magnitude. Therefore,

$$S_{13} = -S_{23} \text{ (both have opposite signs)}$$

For a matched junction, the S matrix is given by

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

Symmetry property

$$S_{12} = S_{21}, S_{13} = S_{31} \text{ and } S_{23} = S_{32}$$

Zero property,

The sum of (each term of any column (row) multiplied by the complex conjugate of the corresponding terms of any column(row) is zero.)

$$S_{11}S_{12}^* + S_{21}S_{22}^* + S_{31}S_{32}^* = 0$$

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

Hence,

$$S_{13}S_{23}^* = 0$$

i.e $S_{13} = 0$ or $S_{23} = 0$ or both = 0

Unity property,

The sum of the products of each term of any one row (column) multiplied by its complex conjugate is unity

$$S_{21}S_{21}^* + S_{31}S_{31}^* = 1 \quad \text{I}$$

$$S_{12}S_{12}^* + S_{32}S_{32}^* = 1 \quad \text{II}$$

$$S_{13}S_{13}^* + S_{23}S_{23}^* = 1 \quad \text{III}$$

Subst zero property in unitary property

$$|S_{12}|^2 = 1 - |S_{13}|^2 = 1 - |S_{23}|^2 \quad \text{IV}$$

Eq III and IV are contradictory

If $S_{13} = 0$ then $S_{23} = 0$ and thus eqn III is false.

Similarly, if $S_{23} = 0$, then $S_{13} = 0$ and hence eq IV is also not true.

This inconsistency proves the statement that the tee junction cannot be matched to the three arms.

Diagonal elements are not all zero

When an E-plane is constructed of an empty waveguide, it is poorly matched at the tee junction.

Hence $S_{ij} \neq 0$ if $i = j$.

However since the collinear arm is usually symmetric about the side arm,

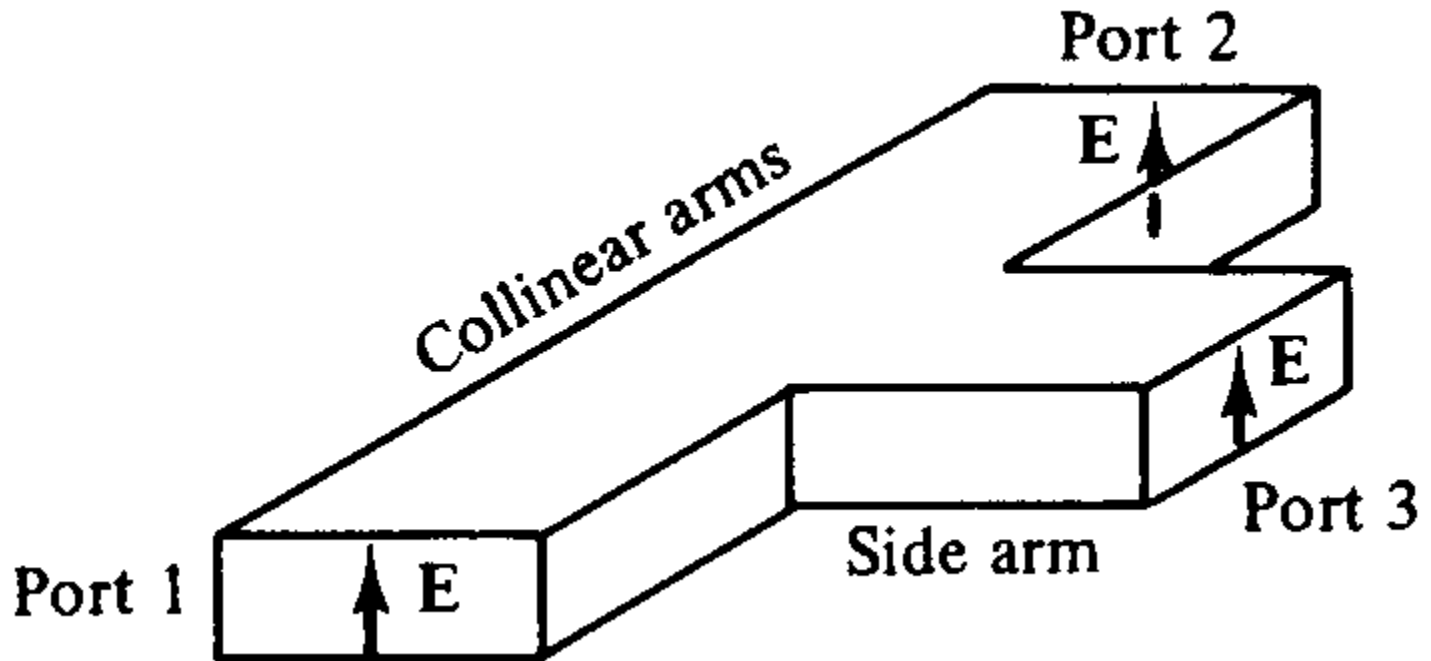
$$|S_{13}| = |S_{23}| \text{ and } S_{11} = S_{22} \quad \text{Thus,}$$

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{bmatrix}$$

H-Plane Tee

Shunt tee

A waveguide tee in which the axis of its side arm is “shunting” the E-field or parallel to the H-field of the main guide.



If two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive.

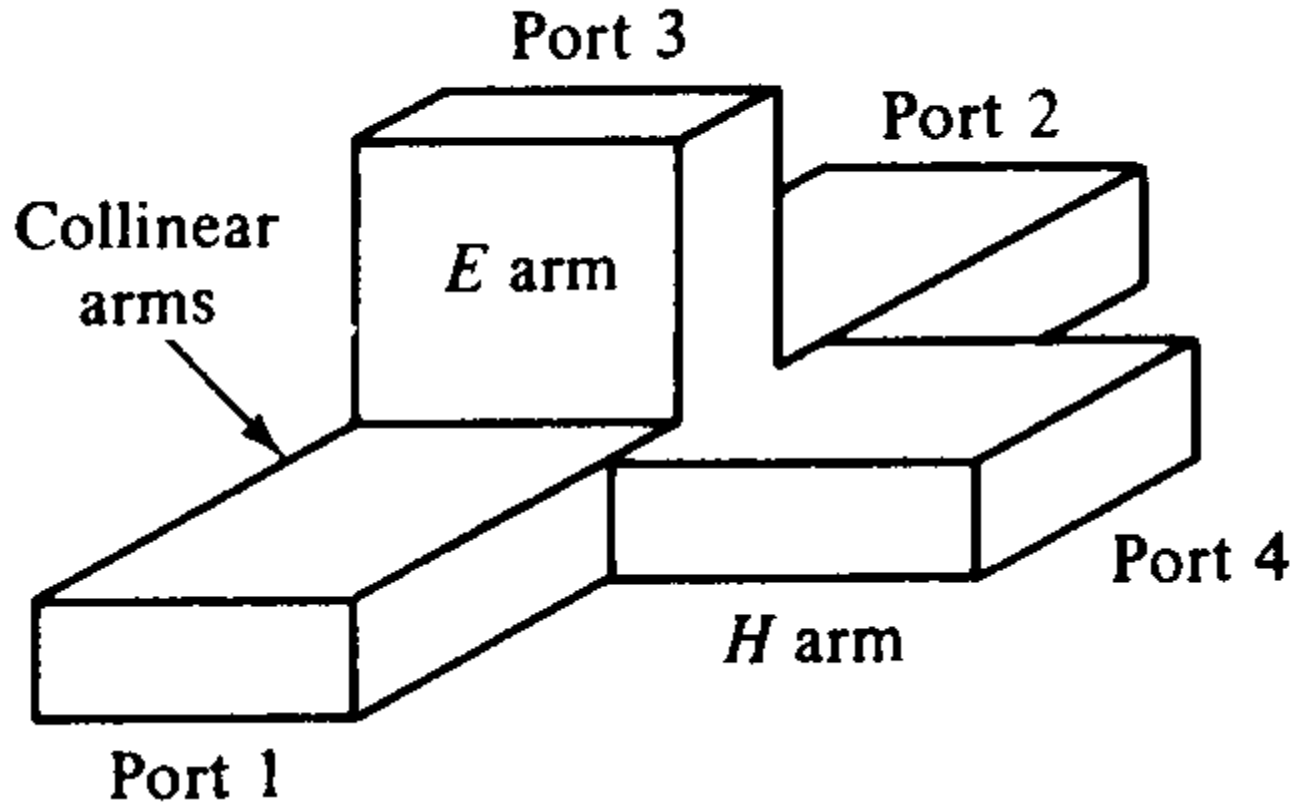
If the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in the same magnitude.

Therefore the S matrix of H-plane tee is similar to E-plane tee except

$$S_{13} = S_{23}$$

Magic Tees (Hybrid Tees)

Combination of E-plane tee and H-plane tee.



Characteristics

1. If two waves of equal magnitude and the same phase are fed into port 1 and port 2, the output will be zero at port 3 and additive at port 4
2. If a wave is fed into port 4 (H arm), it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3 (E arm).
3. If a wave is fed into port 3 (E arm), it will produce an output of equal magnitude and opposite phase at port 1 and port 2. Output at port 4 is zero i.e $S_{43} = S_{34} = 0$.

4. If a wave is fed into one of the collinear arms at port 1 or port 2, it will not appear in the other collinear arm at port 2 or port 1 because the E arm causes a phase delay while the H arm causes the phase advance. i.e $S_{12} = S_{21} = 0$.

S matrix of magic tee is

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

Application

Mixing

Duplexing

Impedance measurements.

Radar transmitters

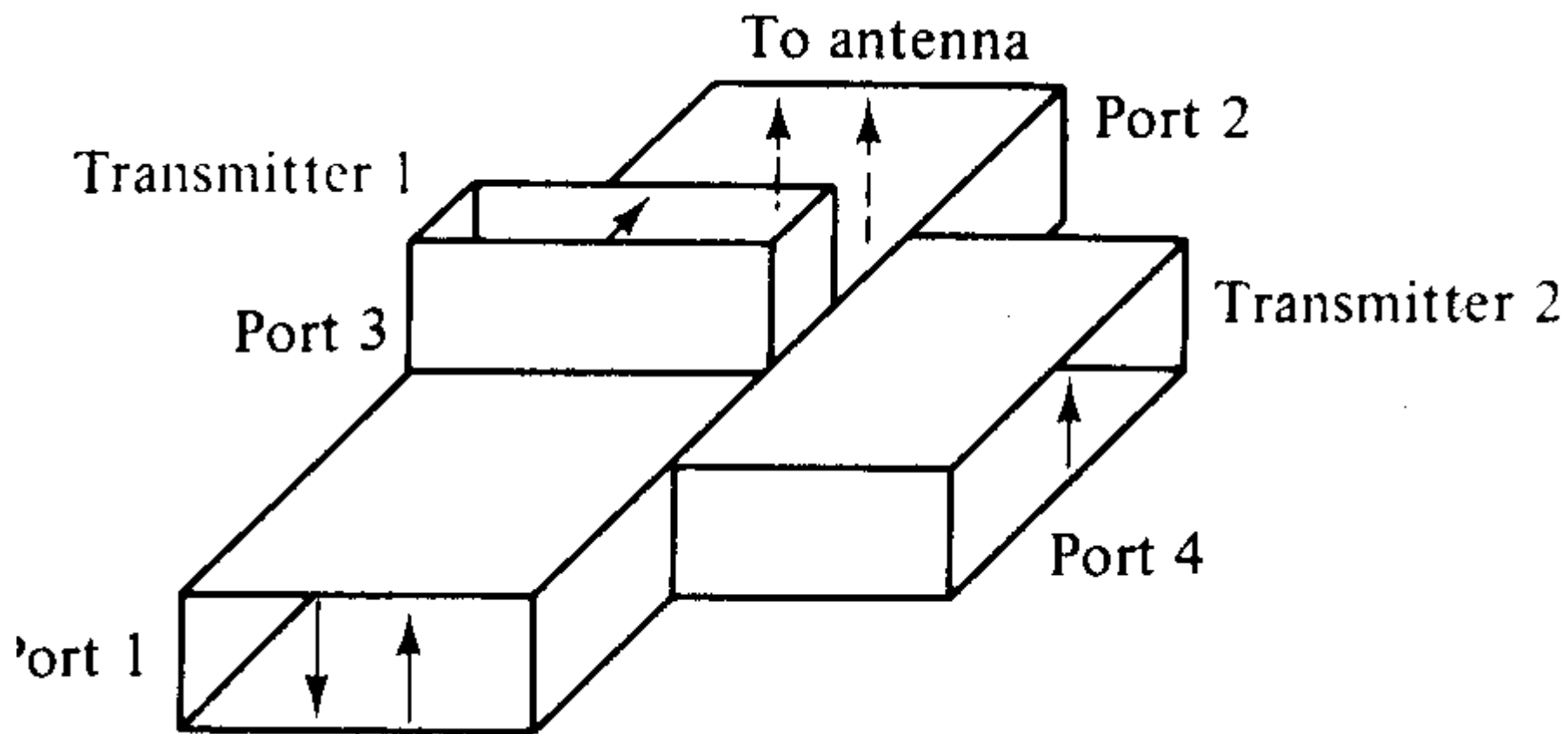
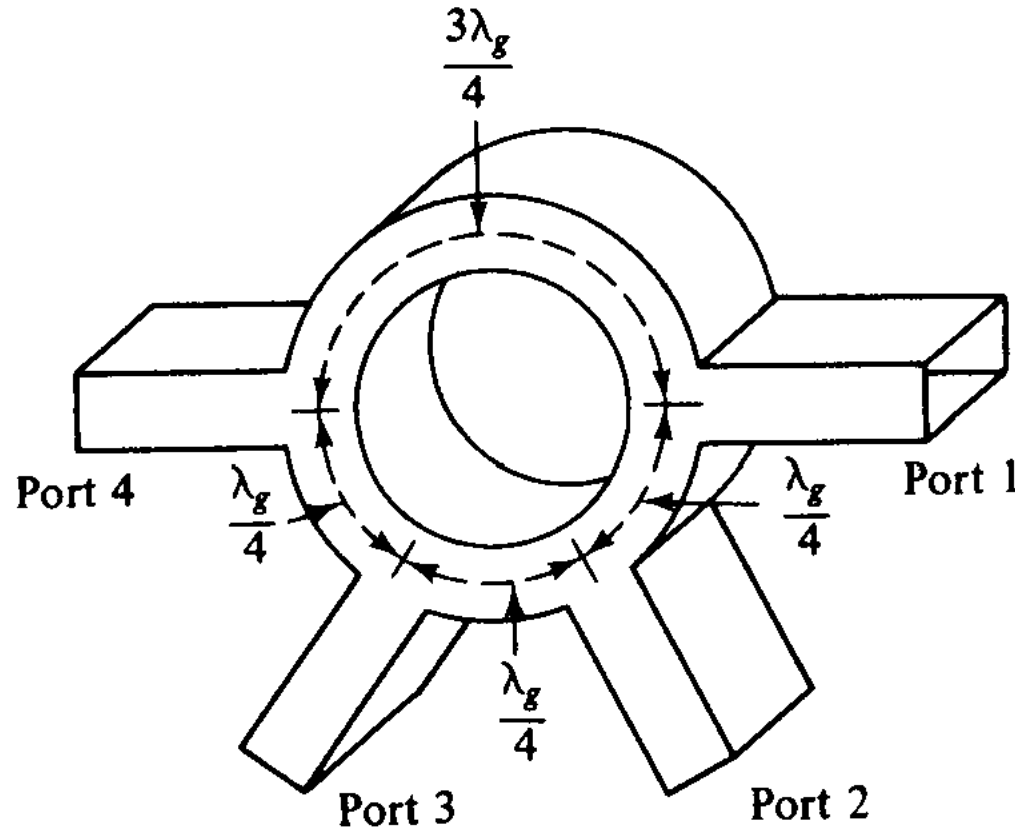


Figure 4-4-8 Magic tee-coupled transmitters to antenna.

The magic tee is commonly used for mixing, duplexing, and impedance measurements. Suppose, for example, there are two identical radar transmitters in equipment stock. A particular application requires twice more input power to an antenna than either transmitter can deliver. A magic tee may be used to couple the two transmitters to the antenna in such a way that the transmitters do not load each other. The two transmitters should be connected to ports 3 and 4, respectively, as shown in Fig. 4-4-8. Transmitter 1, connected to port 3, causes a wave to emanate from port 1 and another to emanate from port 2; these waves are equal in magnitude but opposite in phase. Similarly, transmitter 2, connected to port 4, gives rise to a wave at port 1 and another at port 2, both equal in magnitude and in phase. At port 1 the two opposite waves cancel each other. At port 2 the two in-phase waves add together; so double output power at port 2 is obtained for the antenna as shown in Fig. 4-4-8.

Hybrid Rings (Rat-Race Circuits)

Annular line of proper electrical length to sustain standing waves, to which four arms are connected at proper intervals by means of series or parallel junctions.



Hybrid ring
With series
junctions

Characteristics similar to hybrid tee.

When a wave is fed into port 1, it will not appear at port 3 because the difference of phase shifts for the waves travelling in the clockwise and anticlockwise directions is 180.

Thus the waves are cancelled at port 3.

Similarly the waves fed into port 2 will not emerge at port 4 and so on.

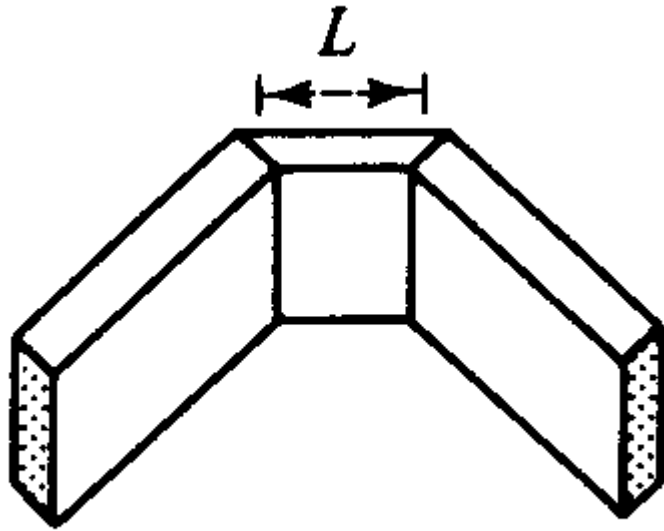
S matrix for an ideal hybrid ring

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Phase cancellation occurs only at designated frequency for an ideal hybrid ring.

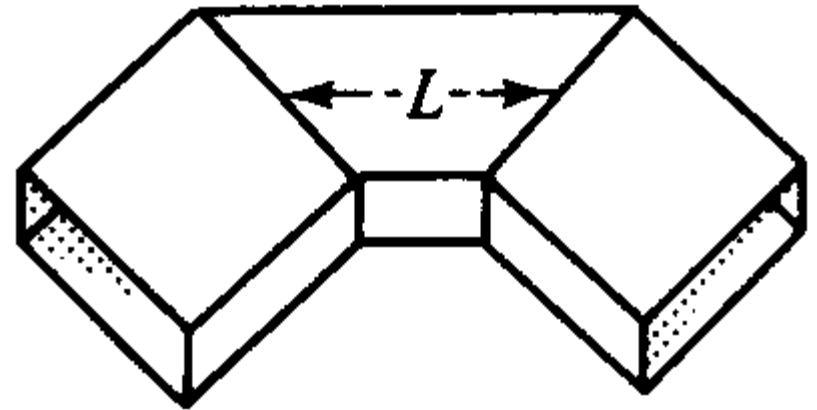
In actual hybrid rings there are small leakage couplings, and hence the zero elements in the matrix above are not quite equal to zero.

Waveguide Corners, Bends, and Twists



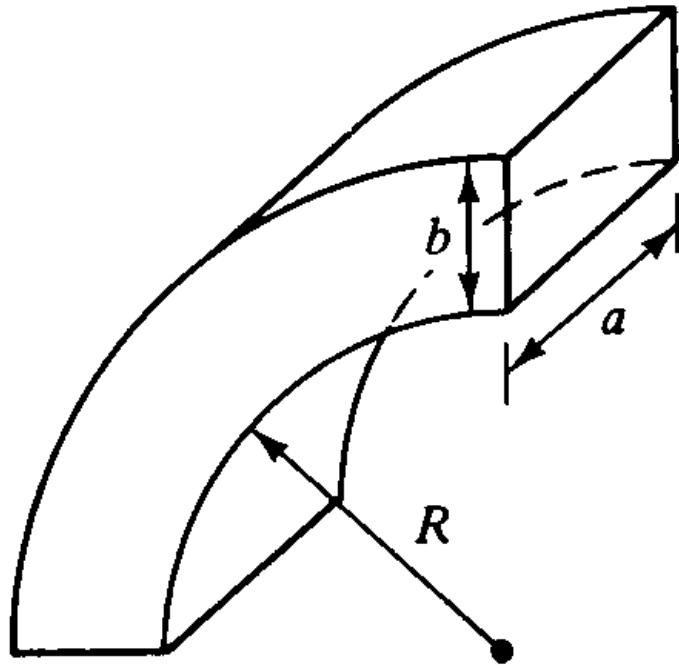
(a)

E plane Corner



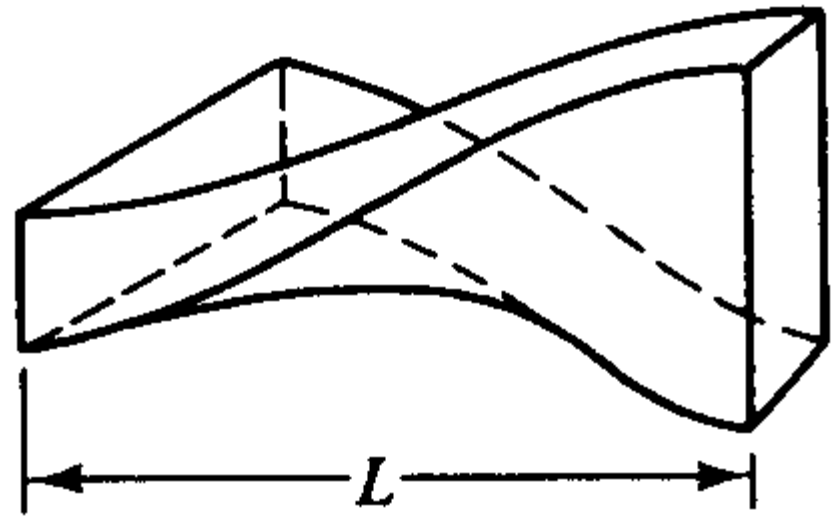
(b)

H-plane corner



(c)

Bend



(d)

Continuous twist

These waveguide components are normally used to change the direction of the guide through an arbitrary angle.

In order to minimize reflections from the discontinuities, it is desirable to have the mean length L between continuities equal to an odd number of quarter wavelengths.

i.e

$$L = (2n + 1) \frac{\lambda_g}{4}$$

where $n = 0, 1, 2, 3, \dots$, and λ_g is the wavelength in the waveguide.

If the mean length L is an odd number of quarter wavelengths, the reflected waves from both ends of the waveguide section are completely cancelled.

For the waveguide bend, the minimum radius of curvature for a small reflection is given by Southworth as

$$R = 1.5b \quad \text{for an } E \text{ bend}$$

$$R = 1.5a \quad \text{for an } H \text{ bend}$$

Where a and b are the dimensions of the bend.