# ECE 5317-6351
# Microwave Engineering

**Fall 2017**

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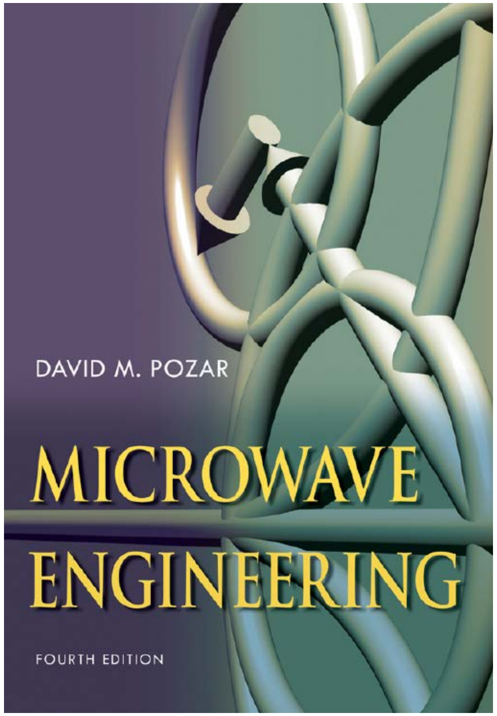
Dept. of ECE




## Notes 19

## Quadrature Coupler and Rat-Race Coupler

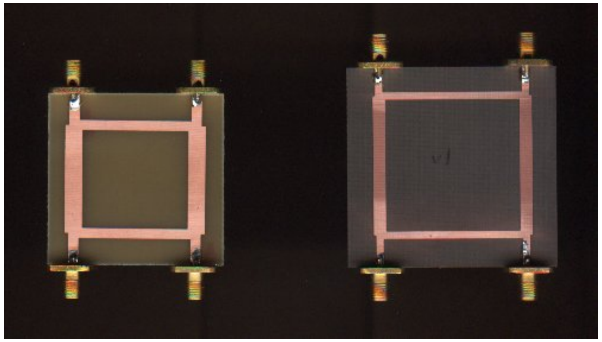
# Quadrature (90°) Coupler

"A quadrature coupler is one in which the input is split into two signals (usually with a goal of equal magnitudes) that are 90 degrees apart in phase. Types of quadrature couplers include branchline couplers (also known as quadrature hybrid couplers), Lange couplers and overlay couplers."

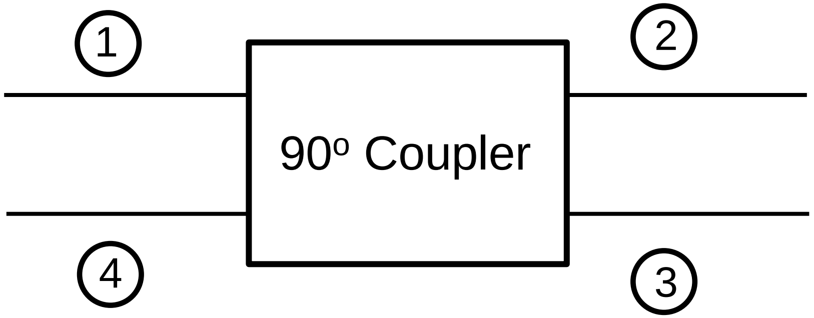
Taken from "Microwaves 101"

http://www.microwaves101.com/encyclopedia/Quadrature_couplers.cfm

This coupler is very useful for obtaining circular polarization: There is a 90° phase difference between ports 2 and 3.

# Quadrature Coupler (cont.)

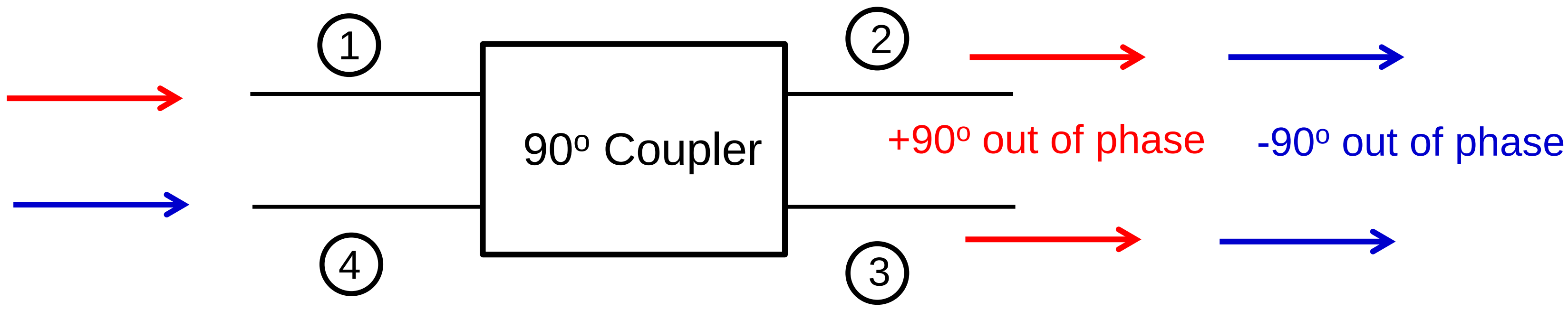



$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

- The quadrature hybrid is a lossless 4-port (the $S$ matrix is unitary ).
- All four ports are matched.
- The device is reciprocal (the $S$ matrix is symmetric.)
- Port 4 is isolated from port 1, and ports 2 and 3 are isolated from each other.

# Quadrature Coupler (cont.)

The quadrature coupler is usually used as a <u>splitter</u>:




- The signal from port 1 splits evenly between ports 2 and 3, with a 90° phase difference.

$$S_{21} = jS_{31}$$

Can be used to produce right-handed circular polarization.

- The signal from port 4 splits evenly between ports 2 and 3, with a -90° phase difference.

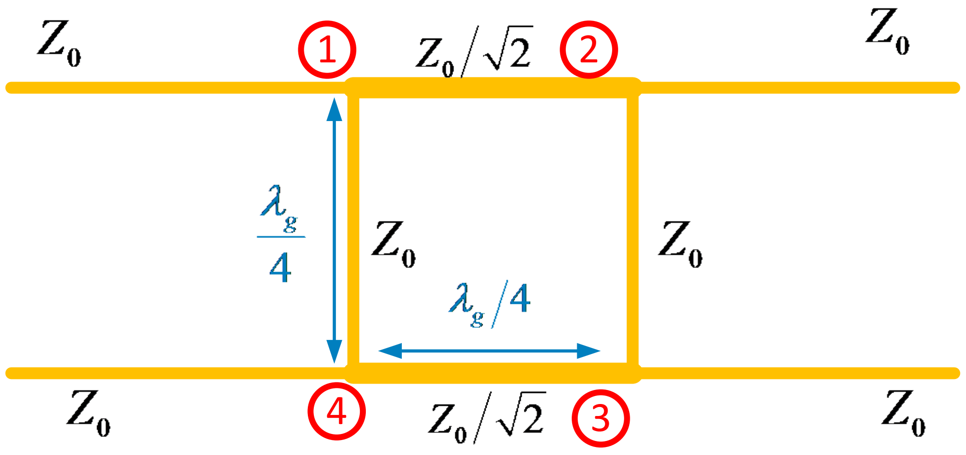
$$S_{24} = -jS_{34}$$

Can be used to produce left-handed circular polarization.

# Quadrature Coupler (cont.)

## Branch-line coupler

A microstrip realization of a branch-line coupler is shown here.




**Notes:**

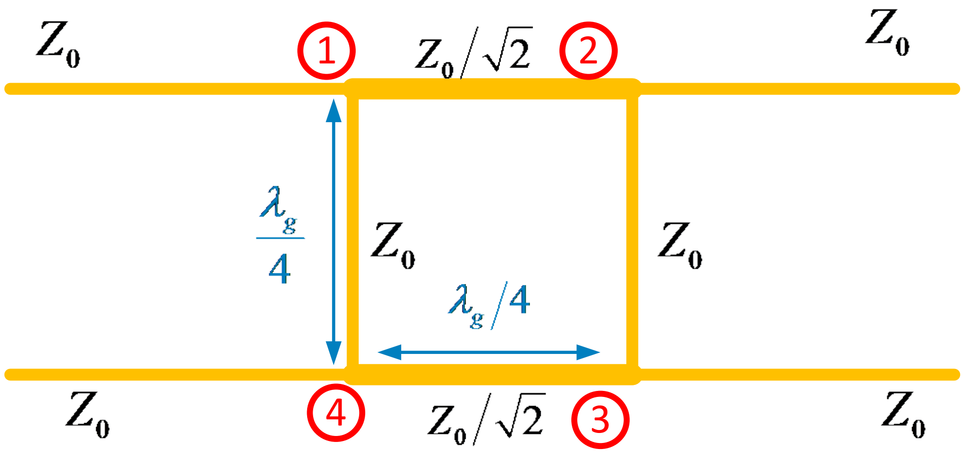
We only need to study what happens when we excite port 1, since the structure is symmetric.

We use even/odd mode analysis (exciting ports 1 and 4) to figure out what happens when we excite port 1.

This analysis is given in the Appendix.

# Quadrature Coupler (cont.)

## Summary




$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

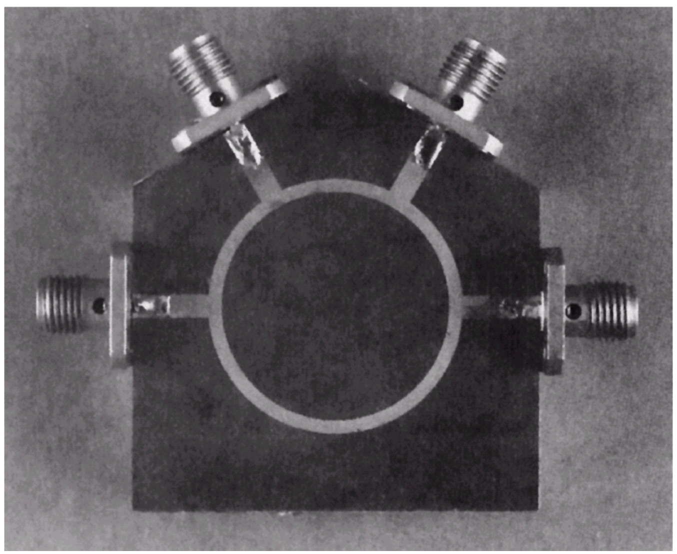
The input power to port 1 divides evenly between ports 2 and 3, with ports 2 and 3 being 90$^{o}$ out of phase.

# Rat-Race Ring Coupler (180$^o$ Coupler)

"Applications of rat-race couplers are numerous, and include mixers and phase shifters. The rat-race gets its name from its circular shape, shown below."

Taken from "Microwaves 101"

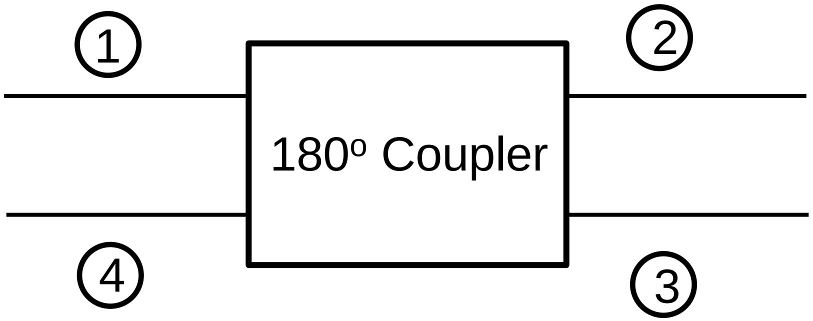
http://www.microwaves101.com/encyclopedia/ratrace_couplers.cfm

Photograph of a microstrip ring coupler

Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory

# Rat-Race Coupler (cont.)

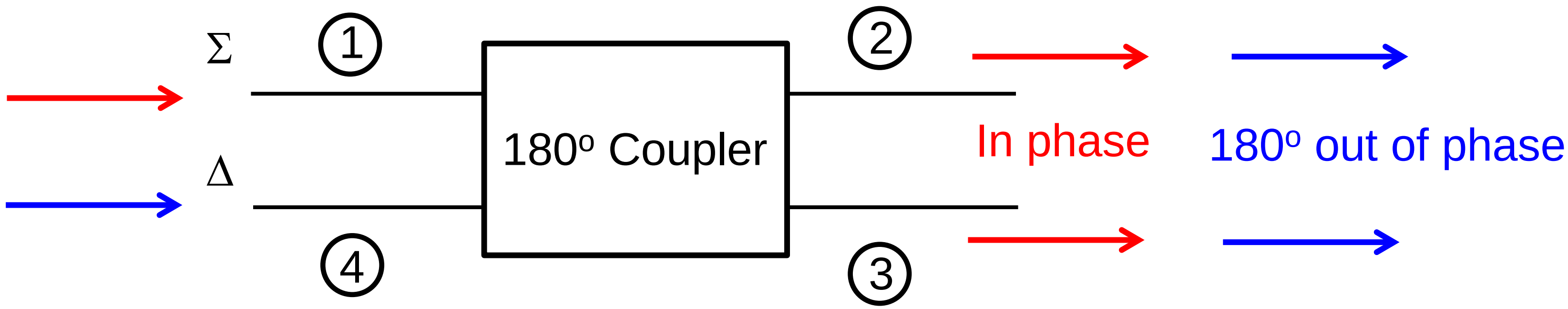



$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

- The rat race is a lossless 4-port (the $S$ matrix is unitary).
- All four ports are matched.
- The device is reciprocal (the $S$ matrix is symmetric).
- Port 4 is isolated from port 1 and ports 2 and 3 are isolated from each other.

# Rat-Race Coupler (cont.)

The rat race can be used as a splitter:




**Note:** A matched load is usually placed on port 4.

- The signal from the "sum port" Σ (port 1) splits evenly between ports 2 and 3, in phase. This could be used as a power splitter (alternative to Wilkenson).

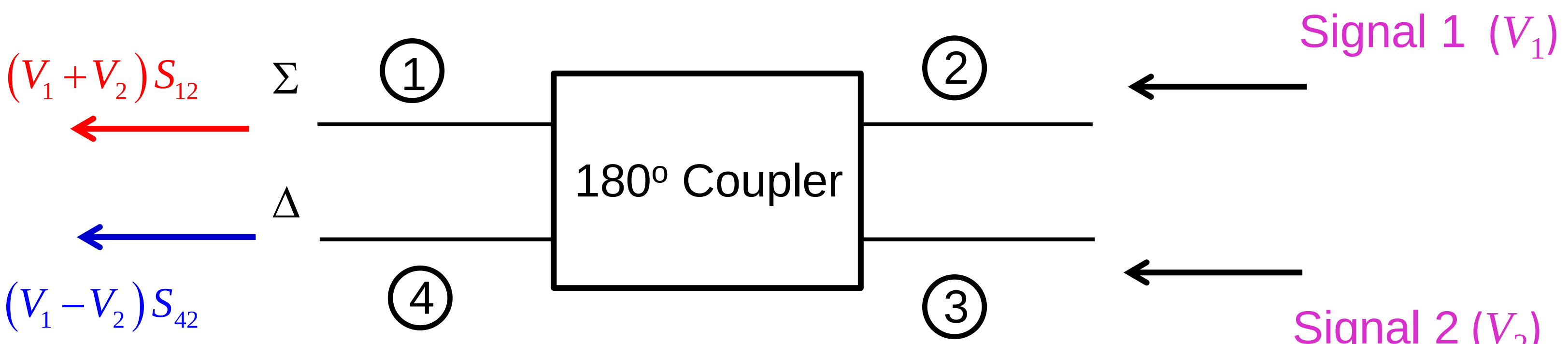
$$S_{21} = S_{31}$$

- The signal from the "difference port" Δ (port 4) splits evenly between ports 1 and 2, 180º out of phase. This could be used as a balun.

$$S_{24} = -S_{34}$$

# Rat-Race Coupler (cont.)

The rat race can be used as a combiner:




- The signal from the sum port Σ (port 1) is the sum of the input signals 1 and 2.

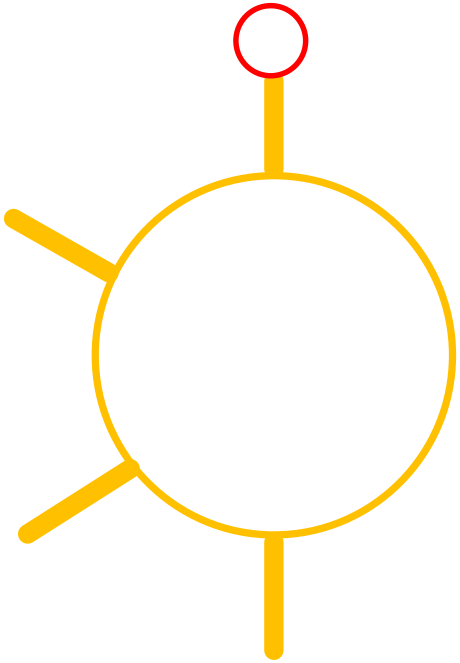
$$S_{12} = S_{13}$$

- The signal from the difference port Δ (port 4) is the difference of the input signals 1 and 2.

$$S_{42} = -S_{43}$$

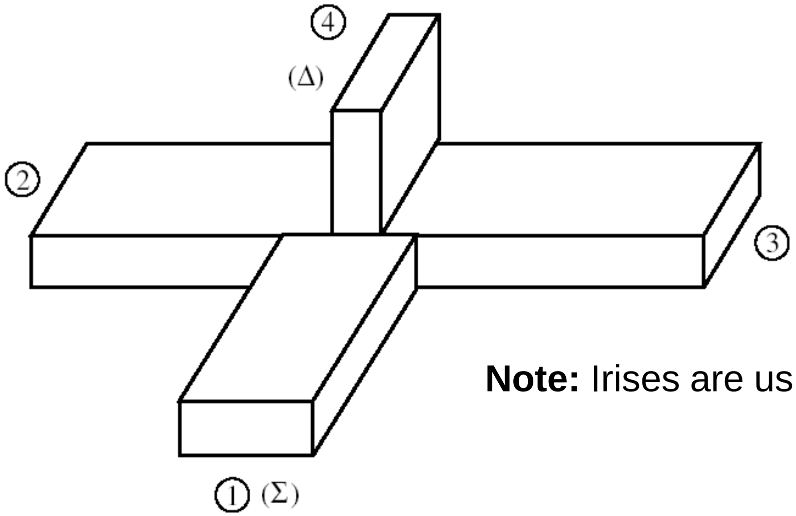
A microstrip realization is shown here.

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$



# Magic T

A waveguide realization of a 180° coupler is shown here, called a "Magic T."




"Magic T"

**Note:** Irises are usually used to obtain matching at the ports.

IEEE Microwave Theory and Techniques Society

$$[S]=\frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

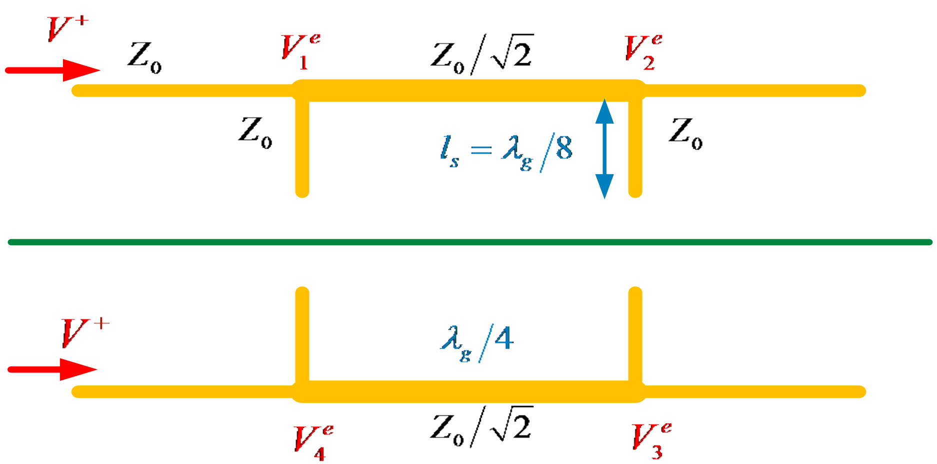
MTT-S®

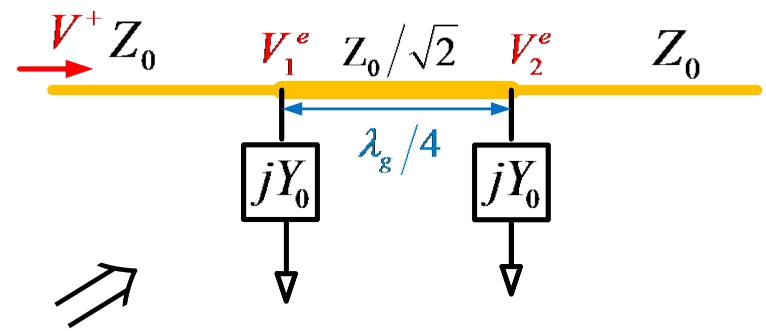
Note the logo!

# Appendix

**Here we analyze the quadrature coupler.**

## Even Analysis




$$Y_0 = 1/Z_0$$

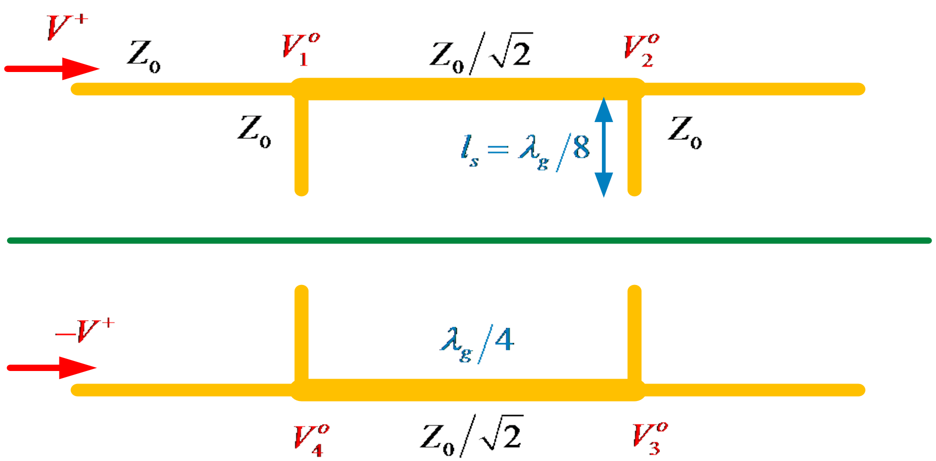
$$\begin{aligned} Y_s &= jY_0 \tan(\beta_s l_s) \\ &= jY_0 \tan(\pi/4) \\ &= jY_0 \end{aligned}$$

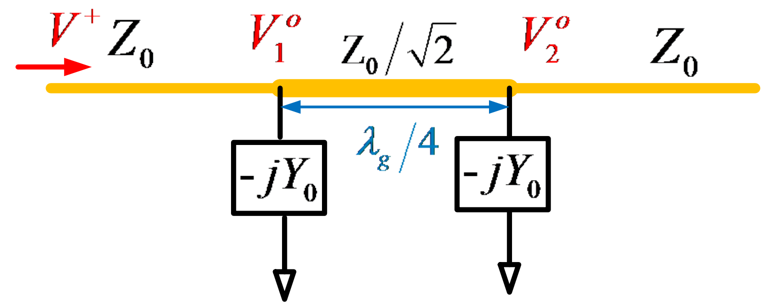
$$V_3^e = V_2^e$$

$$V_4^e = V_1^e$$

# Appendix (cont.)

## Odd Analysis




$$V_3^o = -V_2^o$$

$$V_4^o = -V_1^o$$

$$Y_0 = 1/Z_0$$

$$\begin{aligned} Y_s &= -jY_0 \cot(\beta_s l_s) \\ &= -jY_0 \cot(\pi/4) \\ &= -jY_0 \end{aligned}$$

# Consider the general case:

$$Y = \pm jY_0 \quad \begin{pmatrix} + \text{ for even} \\ - \text{ for odd} \end{pmatrix}$$

.

In general:

$$[ABCD]_Y = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Shunt load on line

$$[ABCD]_{\frac{\lambda}{4}} = \begin{bmatrix} 0 & \dfrac{jZ_0}{\sqrt{2}} \\ \dfrac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix}$$

Quarter-wave line

$$\left[ABCD^{line}\right] = \begin{bmatrix} \cos(\beta\ell) & jZ_0^{line}\sin(\beta\ell) \\ \left(j / Z_0^{line}\right)\sin(\beta\ell) & D = \cos(\beta\ell) \end{bmatrix}$$

Here: $Z_0^{line} = Z_0 / \sqrt{2}$

$\beta\ell = \pi / 2$

$$\Rightarrow \ [ABCD] = [ABCD]_Y \, [ABCD]_{\frac{\lambda}{4}} \, [ABCD]_Y$$

# Appendix (cont.)

Hence we have:

$$[ABCD]=\begin{bmatrix}1 & 0\\ Y & 1\end{bmatrix}\begin{bmatrix}0 & \dfrac{jZ_0}{\sqrt{2}}\\ \dfrac{j\sqrt{2}}{Z_0} & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ Y & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ Y & 1\end{bmatrix}\begin{bmatrix}\dfrac{jZ_0Y}{\sqrt{2}} & \dfrac{jZ_0}{\sqrt{2}}\\ \dfrac{j\sqrt{2}}{Z_0} & 0\end{bmatrix}$$

$$=\begin{bmatrix}\dfrac{jZ_0Y}{\sqrt{2}} & \dfrac{jZ_0}{\sqrt{2}}\\ \dfrac{jZ_0Y^2}{\sqrt{2}}+\dfrac{j\sqrt{2}}{Z_0} & \dfrac{jZ_0Y}{\sqrt{2}}\end{bmatrix}$$

$$Y=\pm jY_0=\pm\frac{j}{Z_0}\quad\begin{pmatrix}+\text{ for even}\\ -\text{ for odd}\end{pmatrix}$$

# Appendix (cont.)

Continuing with the algebra, we have:

$$[ABCD] = \frac{1}{\sqrt{2}} \begin{bmatrix} jZ_0\left(\pm\dfrac{j}{Z_0}\right) & jZ_0 \\ jZ_0\left(\pm\dfrac{j}{Z_0}\right)^2 + \dfrac{j2}{Z_0} & jZ_0\left(\pm\dfrac{j}{Z_0}\right) \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} j(\pm j) & jZ_0 \\ -j\left(\dfrac{1}{Z_0}\right) + \dfrac{j2}{Z_0} & j(\pm j) \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \mp 1 & jZ_0 \\ j\left(\dfrac{1}{Z_0}\right) & \mp 1 \end{bmatrix}$$

# Appendix (cont.)

Hence we have:

$$[ABCD]_{\substack{e\\0}} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mp 1 & jZ_0 \\ j\left(\frac{1}{Z_0}\right) & \mp 1 \end{bmatrix}$$

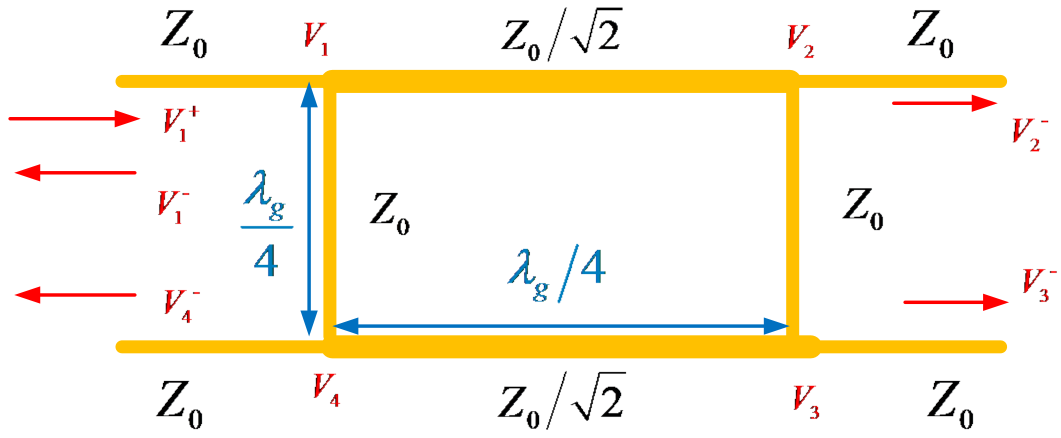
Convert this to $S$ parameters (use Table 4.2 in Pozar):

$$[S]_{\substack{e\\0}} = \begin{bmatrix} 0 & \frac{\mp 1 - j}{\sqrt{2}} \\ \frac{\mp 1 - j}{\sqrt{2}} & 0 \end{bmatrix}$$

Note: We are describing a two-port device here, in the even and odd mode cases.

# Appendix (cont.)

$V_1^+ = V^+ + V^+$



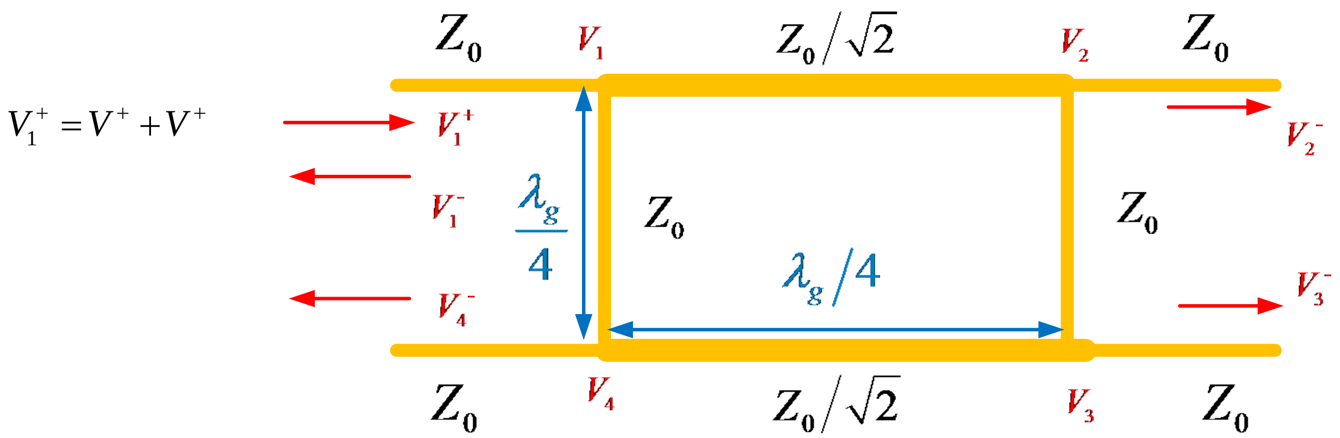

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{a_2=a_3=a_4=0} \Rightarrow \quad S_{11} = \frac{V_1^{-e} + V_1^{-o}}{V^+ + V^+} = \frac{V_1^{-e} + V_1^{-o}}{2V^+} = \frac{1}{2}\left(\frac{V_1^{-e}}{V^+} + \frac{V_1^{-o}}{V^+}\right)$$

$$= \frac{1}{2}\left(S_{11}^e + S_{11}^o\right) = 0 + 0$$

Hence $S_{11} = 0$

By symmetry: $S_{11} = S_{22} = S_{33} = S_{44} = 0$

# Appendix (cont.)




$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{a_2=a_3=a_4=0} \Rightarrow$$

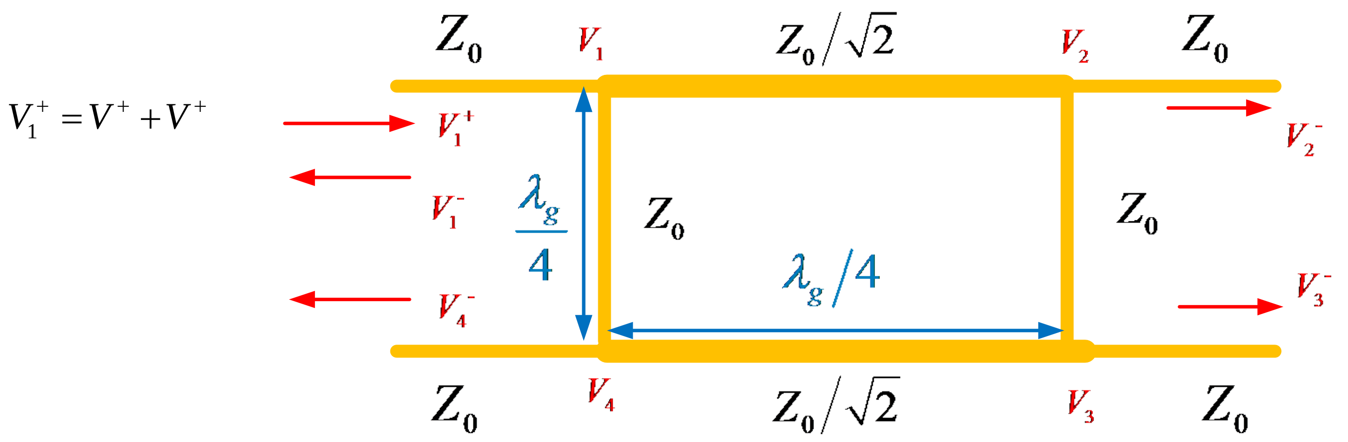
$$S_{21} = \frac{V_2^{-e} + V_2^{-o}}{V^+ + V^+} = \frac{V_2^{-e} + V_2^{-o}}{2V^+} = \frac{1}{2}\left(S_{21}^e + S_{21}^o\right)$$

$$= \frac{1}{2}\left[\left(\frac{-1-j}{\sqrt{2}}\right) + \left(\frac{1-j}{\sqrt{2}}\right)\right]$$

$$= \frac{-j}{\sqrt{2}}$$

By symmetry and reciprocity: $$S_{21} = S_{12} = S_{43} = S_{34} = \frac{-j}{\sqrt{2}}$$

# Appendix (cont.)




$$S_{31} = \left.\frac{V_3^-}{V_1^+}\right|_{a_2=a_3=a_4=0} \Rightarrow$$

$$S_{31} = \frac{V_3^{-e} + V_3^{-o}}{V^+ + V^+} = \frac{V_3^{-e} + V_3^{-o}}{2V^+} = \frac{V_2^{-e} - V_2^{-o}}{2V^+} = \frac{1}{2}\left(S_{21}^e - S_{21}^o\right)$$

$$= \frac{1}{2}\left[\left(\frac{-1-j}{\sqrt{2}}\right) - \left(\frac{1-j}{\sqrt{2}}\right)\right]$$

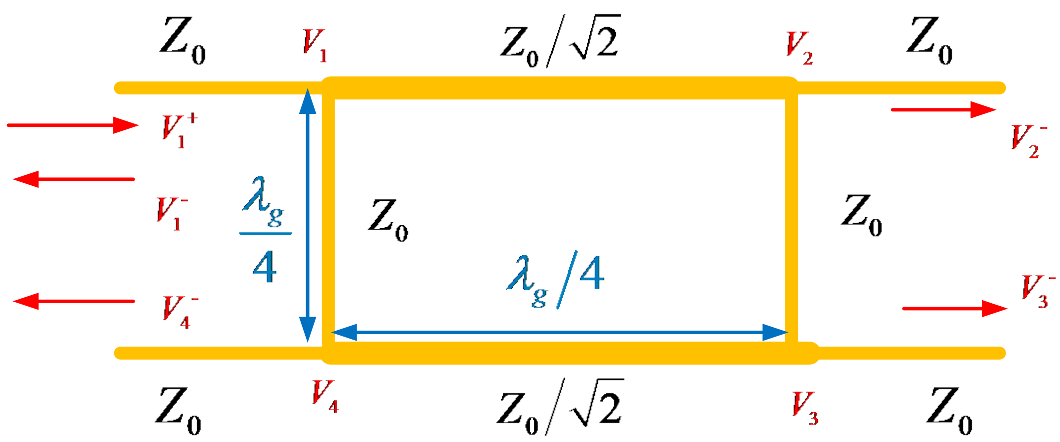
$$= \frac{-1}{\sqrt{2}}$$

By symmetry and reciprocity:

$$S_{31} = S_{13} = S_{24} = S_{42} = \frac{-1}{\sqrt{2}}$$

# Appendix (cont.)

$V_1^+ = V^+ + V^+$

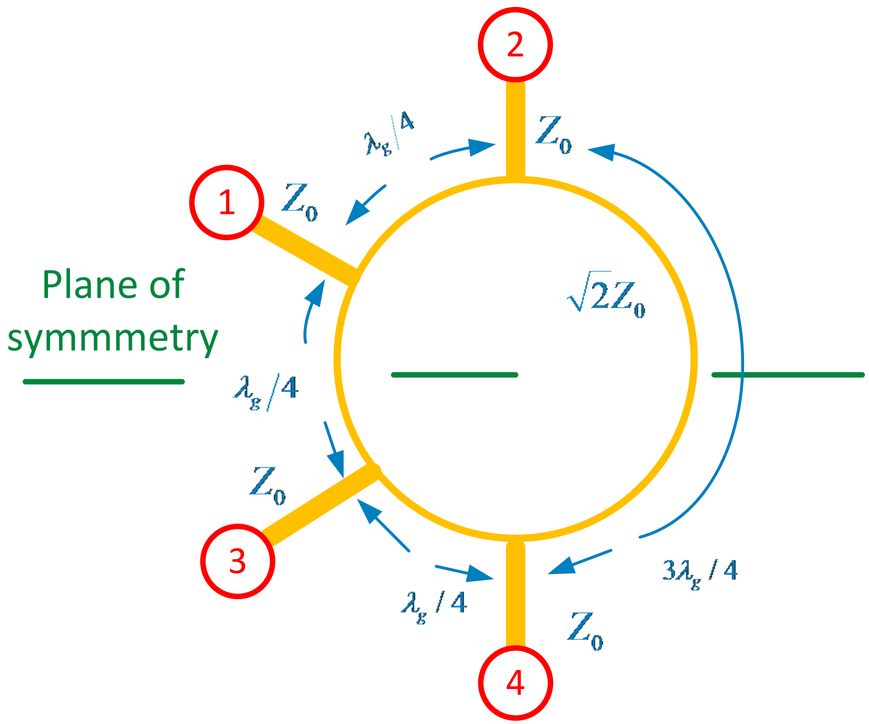



$$S_{41} = \left.\frac{V_4^-}{V_1^+}\right|_{a_2=a_3=a_4=0} \Rightarrow \quad S_{41} = \frac{V_4^{-e} + V_4^{-o}}{V^+ + V^+} = \frac{V_4^{-e} + V_4^{-o}}{2V^+} = \frac{V_1^{-e} - V_1^{-o}}{2V^+} = \frac{1}{2}\left(S_{11}^e - S_{11}^o\right) = 0$$

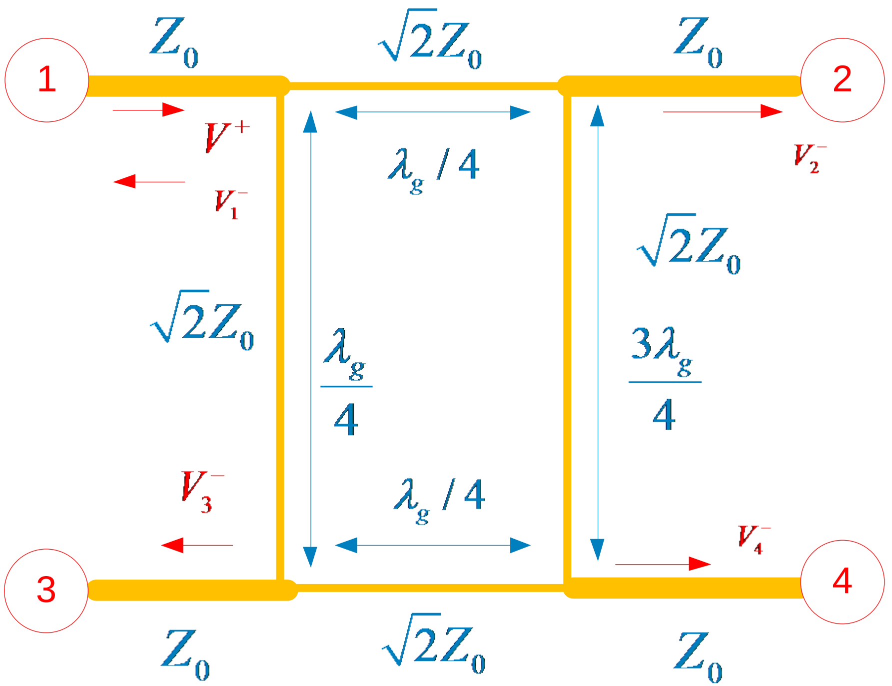
By symmetry and reciprocity: $S_{41} = S_{14} = S_{23} = S_{32} = 0$

# Appendix (cont.)

**Here we analyze the Rat-Race Ring coupler.**



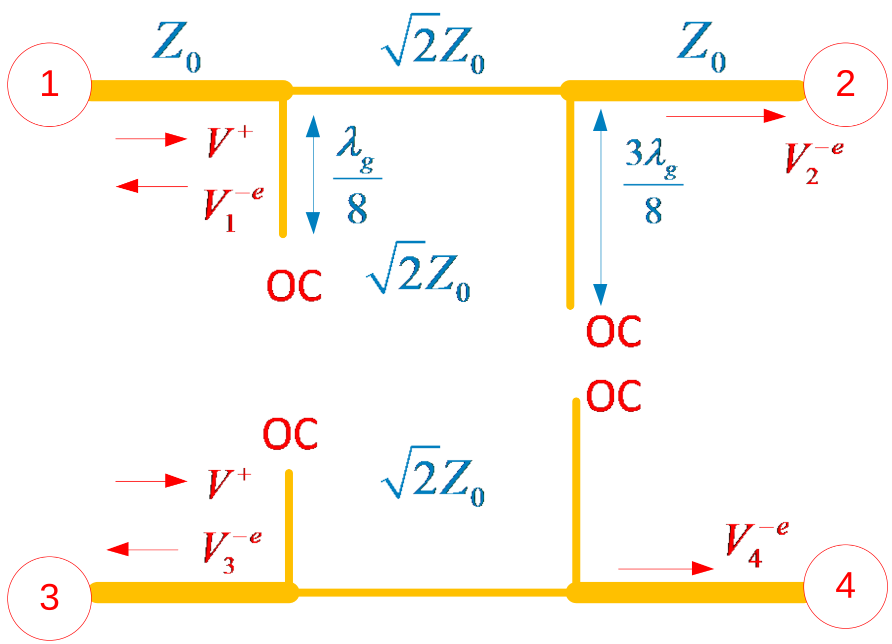

Layout



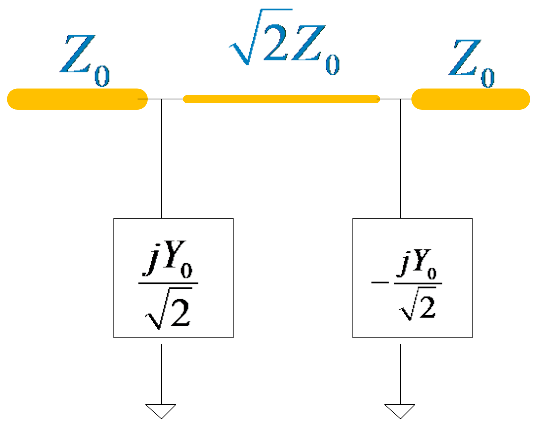

Schematic

# Appendix (cont.)

## Even Analysis




$\Rightarrow$




$$Y_{s1} = jY_{0s1}\tan\left(\beta_s l_{s1}\right) = j\left(Y_0/\sqrt{2}\right)\tan\left(\pi/4\right) = jY_0/\sqrt{2}$$

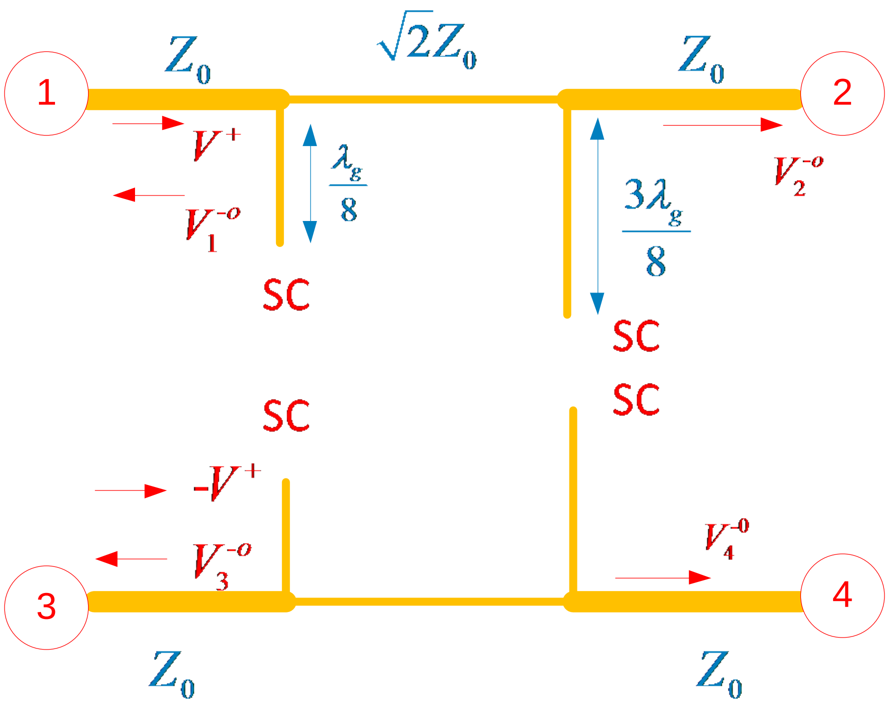
$$Y_0 = 1/Z_0$$

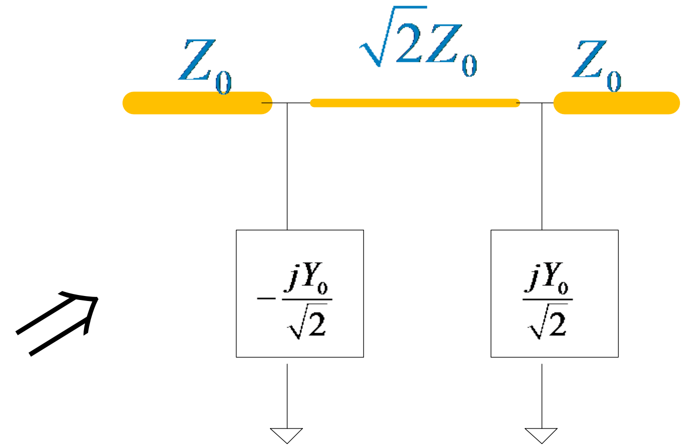
$$Y_{0s} = Y_0/\sqrt{2}$$

$$Y_{s2} = jY_{0s2}\tan\left(\beta_s l_{s2}\right) = j\left(Y_0/\sqrt{2}\right)\tan\left(3\pi/4\right) = -jY_0/\sqrt{2}$$

# Appendix (cont.)

## Odd Analysis




$$Y_0 = 1/Z_0$$

$$Y_{0s} = Y_0/\sqrt{2}$$

$$\begin{aligned} Y_{s1} &= -jY_{0s1}\cot(\beta_s l_s) \\ &= -j\left(Y_0/\sqrt{2}\right)\cot(\pi/4) \\ &= -jY_0/\sqrt{2} \end{aligned}$$

$$\begin{aligned} Y_{s1} &= -jY_{0s2}\cot(\beta_s l_s) \\ &= -j\left(Y_0/\sqrt{2}\right)\cot(3\pi/4) \\ &= jY_0/\sqrt{2} \end{aligned}$$

# Appendix (cont.)

Proceeding as for the 90° coupler, we have:

$$\left[ABCD\right]_{0}^{e} = \begin{bmatrix} 1 & 0 \\ \dfrac{\pm jY_0}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2}Z_0 \\ j\dfrac{1}{\sqrt{2}Z_0} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \mp\dfrac{jY_0}{\sqrt{2}} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \dfrac{\pm jY_0}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} \pm 1 & j\sqrt{2}Z_0 \\ j\dfrac{1}{\sqrt{2}Z_0} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \pm 1 & j\sqrt{2}Z_0 \\ j\dfrac{\sqrt{2}}{Z_0} & \mp 1 \end{bmatrix}$$

# Appendix (cont.)

Converting from the *ABCD* matrix to the *S* matrix, we have

$$\left[ABCD\right]_{0}^{e} = \begin{bmatrix} \pm 1 & j\sqrt{2}Z_0 \\ j\dfrac{\sqrt{2}}{Z_0} & \mp 1 \end{bmatrix}$$

Note: We are describing a two-port device here, in the even and odd mode cases.

↓ Table 4.2 in Pozar

$$\left[S\right]_{0}^{e} = \frac{-j}{\sqrt{2}} \begin{bmatrix} \pm 1 & 1 \\ 1 & \mp 1 \end{bmatrix}$$

# Appendix (cont.)

For the $S$ parameters coming from port 1 excitation, we then have:

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{a_2=a_3=a_4=0}$$

$$S_{11} = \frac{V_1^{-e} + V_1^{-o}}{2V^+} = \frac{1}{2}\left(S_{11}^e + S_{11}^o\right) = \frac{1}{2}\left(\frac{-j}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = 0$$

$S_{11} = S_{33} = 0$

(symmetry)

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{a_2=a_3=a_4=0}$$

$$S_{21} = \frac{V_2^{-e} + V_2^{-o}}{2V^+} = \frac{1}{2}\left(S_{21}^e + S_{21}^o\right) = \frac{1}{2}\left(\frac{-j}{\sqrt{2}} + \frac{-j}{\sqrt{2}}\right) = \frac{-j}{\sqrt{2}}$$

$$S_{21} = S_{12} = S_{34} = S_{43} = \frac{-j}{\sqrt{2}}$$

(symmetry and reciprocity)

# Appendix (cont.)

$$S_{31} = \frac{V_3^-}{V_1^+}\bigg|_{a_2=a_3=a_4=0}$$

$$S_{31} = \frac{V_3^{-e} + V_3^{-o}}{2V^+} = \frac{1}{2}\left(S_{11}^e - S_{11}^o\right)$$

$$= \frac{1}{2}\left(\frac{-j}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$$

$$= \frac{-j}{\sqrt{2}}$$

$$S_{31} = S_{13} = -\frac{j}{\sqrt{2}}$$

(symmetry)

Similarly, exciting port 2, and using symmetry and reciprocity, we have the following results (derivation omitted):

$$S_{22} = S_{44} = 0$$

$$S_{23} = S_{32} = S_{14} = S_{41} = 0$$

$$S_{24} = S_{42} = \frac{j}{\sqrt{2}}$$