Electromagnetic Wave Propagation
Lecture 7: Pulse propagation in dispersive media

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Outline

1. **Propagation of narrow-band pulses**
2. **Group velocity as velocity of peak: linear approximation**
3. **Pulse broadening: quadratic approximation**
4. **Slow, fast, and negative group velocities**
5. **Application: chirp radar and pulse compression**
6. **Conclusions**

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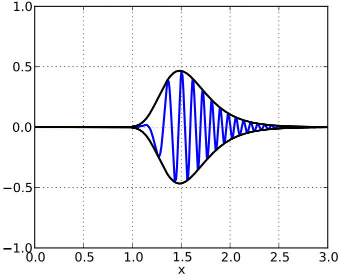
Key questions



- How does a narrow-band pulse propagate?
- How can pulse distortion be characterized?
- How can pulse distortion be compensated?

Lots of text in Orfanidis, especially when discussing applications.
We concentrate on the fundamental ideas rather than the details.

Basic approach

- The time domain pulse $E(z,t)$ is decomposed in Fourier components, $\hat{E}(\omega)\mathrm{e}^{\mathrm{j}(\omega t - k(\omega)z)}$.
- Each component is propagated according to its wavenumber $k(\omega)$.
- The resulting pulse is synthesized as

$$E(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{E}(\omega)\mathrm{e}^{\mathrm{j}(\omega t - k(\omega)z)}\,\mathrm{d}\omega$$

- To simplify the computation of the Fourier integral, various approximations of $k(\omega)$ and $\hat{E}(\omega)$ are introduced.

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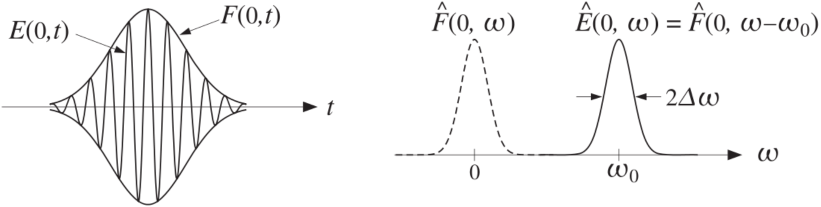
Narrow-band pulse with carrier frequency and envelope

A pulse with carrier frequency ω_0 and envelope $F(0,t)$ can be written $E(0,t) = \mathrm{e}^{\mathrm{j}\omega_0 t} F(0,t)$, corresponding to the frequency shift $\hat{E}(0,\omega) = \hat{F}(0,\omega-\omega_0)$



(Fig. 3.5.1 in Orfanidis)

This can be seen as the translation of a low-frequency spectrum $\hat{F}(0,\omega)$ to be centered around ω_0, $\hat{F}(0,\omega-\omega_0)$. F (or $\hat{F}$) is called the *envelope* of the carrier signal ω_0.

Propagation of narrow-band pulse

Propagation in the frequency domain corresponds to $\hat{E}(z,\omega) = \mathrm{e}^{-\mathrm{j}k(\omega)z}\hat{E}(0,\omega)$, which implies (where $k_0 = k(\omega_0)$)

$$\begin{aligned} E(z,t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}(\omega t - k(\omega)z)}\hat{F}(0,\omega-\omega_0)\,\mathrm{d}\omega \\ &= \mathrm{e}^{\mathrm{j}(\omega_0 t - k_0 z)}\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}(\omega-\omega_0)t - \mathrm{j}(k(\omega)-k_0)z}\hat{F}(0,\omega-\omega_0)\,\mathrm{d}\omega \\ &= \mathrm{e}^{\mathrm{j}(\omega_0 t - k_0 z)}F(z,t) \end{aligned}$$

with

$$F(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}(\omega-\omega_0)t - \mathrm{j}(k(\omega)-k_0)z}\hat{F}(0,\omega-\omega_0)\,\mathrm{d}\omega$$

The factor $\mathrm{e}^{\mathrm{j}(\omega_0 t - k_0 z)}$ is the carrier wave, propagating with the *phase velocity* $\boxed{v_\mathrm{p} = \omega_0/k_0}$.

Taylor expansion of the wavenumber

Making the Taylor expansion

$$k(\omega) = k(\omega_0) + k'(\omega_0)(\omega - \omega_0) + \cdots$$

implies (with $\omega' = \omega - \omega_0$, $k_0 = k(\omega_0)$, $k_0' = k'(\omega_0)$)

$$\begin{aligned} F(z,t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}((\omega-\omega_0)t-(k(\omega)-k_0)z)} \hat{F}(0, \omega - \omega_0) \, \mathrm{d}\omega \\ &\approx \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\omega'(t-k_0'z)} \hat{F}(0, \omega') \, \mathrm{d}\omega' = F(0, t - k_0'z) \end{aligned}$$

Thus, the envelope F is propagating with the *group velocity*

$$\boxed{v_\mathrm{g} = \frac{1}{k'(\omega_0)}}$$

If $k(\omega) = \beta(\omega) - \mathrm{j}\alpha(\omega)$ is complex, replace by $v_\mathrm{g} = 1/\beta'(\omega_0)$.

Envelope impulse response

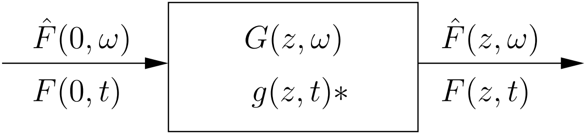
The impulse response can be written

$$h(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}(\omega t - k(\omega)z)}\,\mathrm{d}\omega = \mathrm{e}^{\mathrm{j}(\omega_0 t - k_0 z)} g(z,t)$$

where

$$g(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}(\omega-\omega_0)t - \mathrm{j}(k-k_0)z)}\,\mathrm{d}\omega$$

We can define transfer functions for the envelope F as



$\hat{F}(0,\omega)$

$F(0,t)$

$G(z,\omega)$

$g(z,t)*$

$\hat{F}(z,\omega)$

$F(z,t)$

with $G(z,\omega') = \mathrm{e}^{-\mathrm{j}(k(\omega'+\omega_0)-k(\omega_0))z}$.

Group velocity and envelope impulse response

Making the approximation

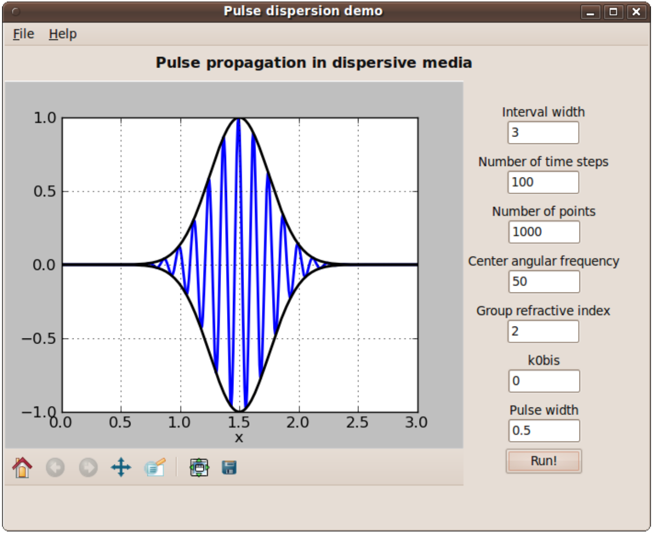
$$k(\omega) = k_0 + k_0' \omega'$$

implies the envelope impulse response is

$$\begin{aligned} g(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}(\omega-\omega_0)t - \mathrm{j}(k-k_0)z} \, \mathrm{d}\omega &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\omega'(t-k_0' z)} \, \mathrm{d}\omega' \\ &= \delta(t - k_0' z) \end{aligned}$$

Thus, in the linear approximation the envelope propagates undistorted with the group velocity.

Simulation illustration



Pulse dispersion demo

Pulse propagation in dispersive media

Interval width
3

Number of time steps
100

Number of points
1000

Center angular frequency
50

Group refractive index
2

k0bis
0

Pulse width
0.5

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Quadratic expansion

Keeping one more term in the Taylor expansion

$$k(\omega) = k(\omega_0) + k'(\omega_0)(\omega - \omega_0) + \underbrace{k''(\omega_0)(\omega - \omega_0)^2/2}_{\text{quadratic term}} + \cdots$$

implies (with $\omega' = \omega - \omega_0$, $k_0 = k(\omega_0)$, $k'_0 = k'(\omega_0)$, $k''_0 = k''(\omega_0)$)

$$\begin{aligned} F(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}[(\omega-\omega_0)t-(k(\omega)-k_0)z]} \hat{F}(0, \omega - \omega_0)\, \mathrm{d}\omega \\ \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\omega'[t-(k'_0+k''_0\omega'/2)z]} \hat{F}(0, \omega')\, \mathrm{d}\omega' \end{aligned}$$

The quadratic term leads to pulse spreading. Frequency dependent velocity:

$$v(\omega') = \frac{1}{k'_0 + k''_0\omega'/2} \approx \frac{1}{k'_0} - \frac{k''_0\omega'}{2k'_0}$$

Spreading of Gaussian envelope

Assume a Gaussian envelope,

$$F(0,t) = \mathrm{e}^{-t^2/(2\tau_0^2)} \quad \Leftrightarrow \quad \hat{F}(0,\omega) = \sqrt{2\pi\tau_0^2}\,\mathrm{e}^{-\tau_0^2\omega^2/2}$$

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The propagated envelope is then

$$\begin{aligned} F(z,t) &= \frac{\sqrt{2\pi\tau_0^2}}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\omega'(t-(k_0'+k_0''\omega'/2)z)}\mathrm{e}^{-\tau_0^2(\omega')^2/2}\,\mathrm{d}\omega' \\ &= \cdots = \sqrt{\frac{\tau_0^2}{\tau_0^2+\mathrm{j}k_0''z}}\exp\left[-\frac{(t-k_0'z)^2}{2(\tau_0^2+\mathrm{j}k_0''z)}\right] \end{aligned}$$

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Since $\exp[-\frac{1}{a+\mathrm{j}b}] = \exp[-\frac{a}{a^2+b^2}+\mathrm{j}\frac{b}{a^2+b^2}]$, the width of $|F(z,t)|$ in time is proportional to

$$\tau(z) = \left(\frac{\tau_0^4+(k_0''z)^2}{\tau_0^2}\right)^{1/2} = \left[\tau_0^2+\left(\frac{k_0''z}{\tau_0}\right)^2\right]^{1/2}$$

Spreading of envelope impulse response

Making the approximation

$$k(\omega) = k_0 + k_0'\omega' + k_0''(\omega')^2/2$$

implies the envelope impulse response is

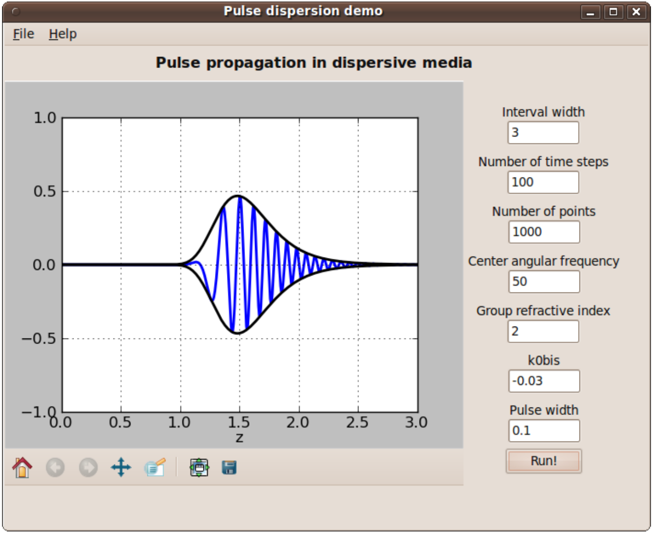
$$\begin{aligned} g(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}(\omega-\omega_0)t-\mathrm{j}(k-k_0)z}\,\mathrm{d}\omega \\ = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\omega'(t-k_0'z)-\mathrm{j}k_0''z(\omega')^2/2}\,\mathrm{d}\omega' \end{aligned}$$

This has the exact solution

$$g(z,t) = \frac{1}{\sqrt{2\pi \mathrm{j} k_0'' z}} \exp\left[-\frac{(t-k_0'z)^2}{2\mathrm{j}k_0''z}\right] \quad \rightarrow \quad \delta(t - k_0'z), \quad k_0'' \rightarrow 0$$

Thus, the typical time scale of the envelope impulse response is $\sqrt{k_0''z}$.

Simulation illustration



Pulse dispersion demo

Pulse propagation in dispersive media

Interval width	3
Number of time steps	100
Number of points	1000
Center angular frequency	50
Group refractive index	2
k0bis	-0.03
Pulse width	0.1

Example: pulse broadening in optical fibers

A Gaussian pulse of initial width τ_0 has extra width after propagation from 0 to z:

$$\Delta t \approx \frac{k_0'' z}{\tau_0}$$

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A typical value of k_0'' for a standard single mode optical fiber at $\lambda = 1.55\,\mu\mathrm{m}$ is

$$k_0'' = -21.67 \frac{(\mathrm{ps})^2}{\mathrm{km}}$$

and attenuation about $0.2\,\mathrm{dB/km}$. With a data rate of 40 $\mathrm{Gbit/s}$, the pulses are spaced by $25\,\mathrm{ps}$. How long can a $10\,\mathrm{ps}$ pulse propagate before it blends with its neighbors?

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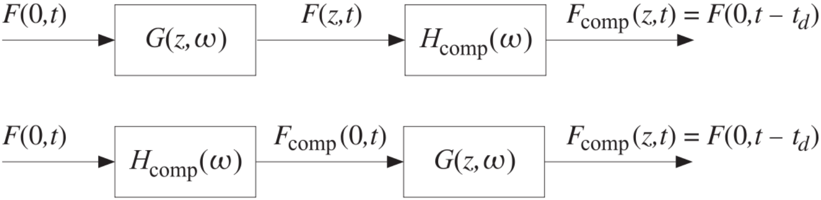
$$z \leq \frac{\Delta t \tau_0}{k_0''} = \frac{25\,\mathrm{ps} \cdot 10\,\mathrm{ps}}{21.67\,(\mathrm{ps})^2/\mathrm{km}} = 11.5\,\mathrm{km}$$

This limits the length of individual fibers, and compensations must be inserted at regular intervals.

Dispersion compensation

The dispersion due to propagation in a dispersive medium can be compensated by a suitable filter at the receiver or transmitter:



(Fig. 3.8.1 in Orfanidis)

With *a priori* knowledge of the dispersion, the pulse can be predistorted to compensate for the propagation effects.

Naively we would expect $H_{\text{comp}} = 1/G$, but this violates causality. With additional delay, $H_{\text{comp}} = \mathrm{e}^{-\mathrm{j}\omega t_{\mathrm{d}}}/G$, causality is preserved.

Dispersion compensation

In the frequency domain, the compensation filter is given by

$$H_{\mathrm{comp}}(z,\omega) = \frac{\mathrm{e}^{-\mathrm{j}\omega t_{\mathrm{d}}}}{G(z,\omega)}$$

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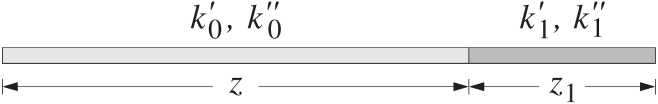
Realize the filter via an additional propagation medium:



(Fig. top p. 105 in Orfanidis)

Dispersion compensation

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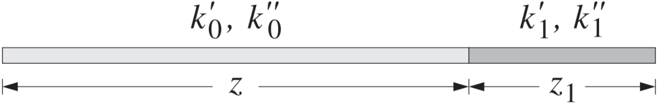


(Fig. top p. 105 in Orfanidis)

The total envelope transfer function will be

$$\begin{aligned} G_0(z,\omega)G_1(z_1,\omega) &= \mathrm{e}^{-\mathrm{j}k_0' z\omega}\mathrm{e}^{-\mathrm{j}k_0'' z\omega^2/2}\mathrm{e}^{-\mathrm{j}k_1' z_1\omega}\mathrm{e}^{-\mathrm{j}k_1'' z_1\omega^2/2} \\ &= \underbrace{\mathrm{e}^{-\mathrm{j}(k_0' z + k_1' z_1)\omega}}_{=\text{delay}}\underbrace{\mathrm{e}^{-\mathrm{j}(k_0'' z + k_1'' z_1)\omega^2/2}}_{=\text{dispersion}} \end{aligned}$$

Dispersionless (only delay) if

$$k_0'' z + k_1'' z_1 = 0$$

Observations on pulse propagation

For a narrow-band pulse with carrier frequency ω_0 and envelope $F(z,t)$, we have $E(z,t) = \mathrm{e}^{\mathrm{j}(\omega_0 t - k_0 z)} F(z,t)$.

- The envelope $F(z,t)$ propagates with the group velocity $v_\mathrm{g} = 1/k_0'$.
- The width of the pulse increases with propagation length as $\tau(z) = \sqrt{\tau_0^2 + (k_0'' z/\tau_0)^2}$. This limits the propagation length in communication applications.
- The local frequency varies with z (or t), which is called *chirping*.
- The propagation dispersion can be compensated for by the use of filters.

Outline

When can we trust the group velocity?

The concept of group velocity relies on **narrow-band pulses**. With τ_0 being the duration of the pulse we have implicitly assumed:

$$|k_0'' z| \ll \tau_0^2 \quad \text{and} \quad |\operatorname{Im}(k_0) z| \ll 1$$

Typical risks:

- Fast frequency variations (typical at resonance).
- Active/gain medium.

Typical situation: resonance

A resonant medium is typically described by the susceptibility

$$\chi(\omega) = \frac{f\omega_{\mathrm{p}}^2}{\omega_{\mathrm{r}}^2 - \omega^2 + \mathrm{j}\omega\gamma}$$

where $f = +1$ for an absorbing medium, and $f = -1$ for a gain medium. The refractive index is

$$n = \sqrt{1+\chi} \approx 1 + \frac{1}{2}\chi = 1 + \frac{f\omega_{\mathrm{p}}^2/2}{\omega_{\mathrm{r}}^2 - \omega^2 + \mathrm{j}\omega\gamma}$$

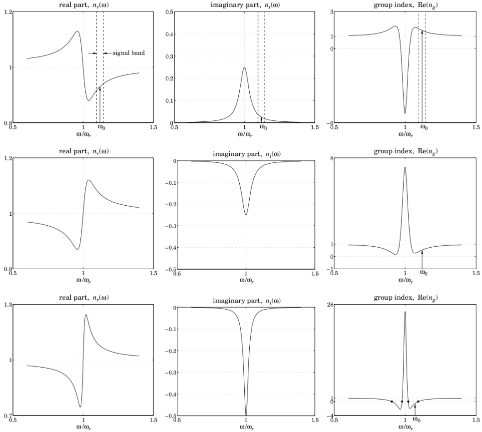
At resonance $\omega = \omega_{\mathrm{r}}$:

$$n = 1 - \mathrm{j}\frac{f\omega_{\mathrm{p}}^2}{2\gamma\omega_{\mathrm{r}}}, \quad n_{\mathrm{g}} = 1 - \frac{f\omega_{\mathrm{p}}^2}{\gamma^2}$$

The group refractive index can be just about anything, depending on the choice of parameters!

Slow, fast, and negative group velocities

Default values: $\omega_{\mathrm{p}} = 1$, $\omega_{\mathrm{r}} = 5$, $\gamma = 0.4$. Top row: $f = +1$. Middle row: $f = -1$. Bottom row: $f = -1$, $\gamma = 0.2$. $n = n_{\mathrm{r}} - \mathrm{j}n_{\mathrm{i}}$.



Slow, $n_{\mathrm{g}} = 1.48 + 0.39\mathrm{j}$

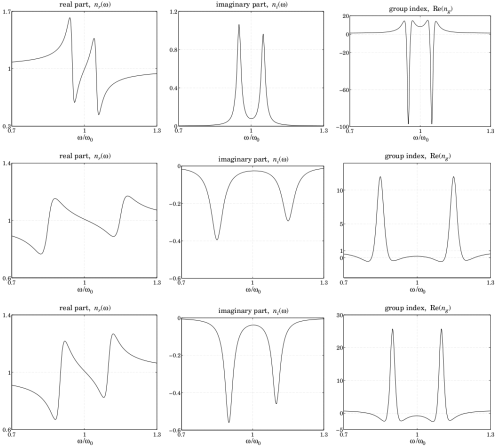
Fast, $n_{\mathrm{g}} = 0.52 - 0.39\mathrm{j}$

Negative, $n_{\mathrm{g}} = -0.58 - 1.02\mathrm{j}$

Fig. 3.9.1 in Orfanidis. Observe active media are used!

Slow, fast, and negative group velocities

Using two nearby resonances:



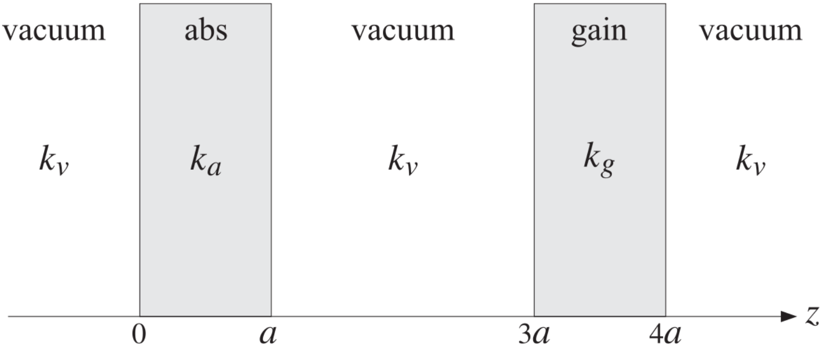
$n_\mathrm{g} = 8.104 + 0.063\mathrm{j}$

$n_\mathrm{g} = 0.208 - 0.021\mathrm{j}$

$n_\mathrm{g} = -0.778 - 0.032\mathrm{j}$

Fig. 3.9.2 in Orfanidis. Observe active media are used!

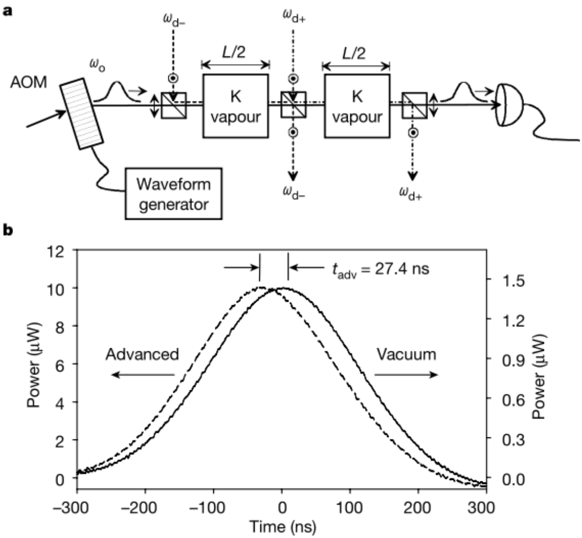
Propagation through slow and fast media



(Fig. 3.9.3 in Orfanidis)

Matlab functions `grvmovie1.m` and `grvmovie2.m`. Movies plot only envelope propagation.

Stenner, Gauthier, Neifeld, Nature 2003



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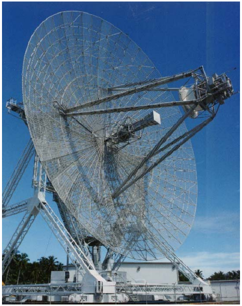
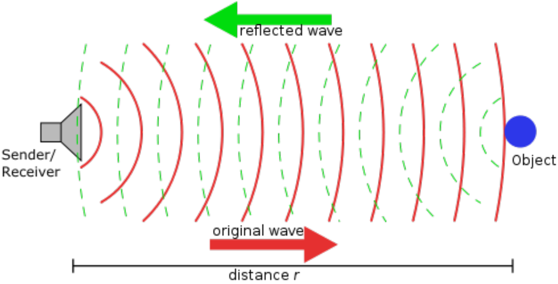
Radar system

RADAR = RAdio Detection And Ranging, was developed mainly in the 1940s. The principles have been transferred to many other areas, for instance LIDAR = Light Detection and Ranging, atmospheric measurements, and speed measurements.

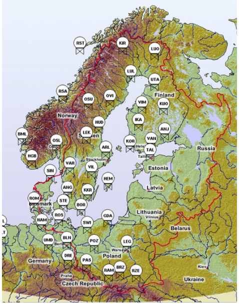




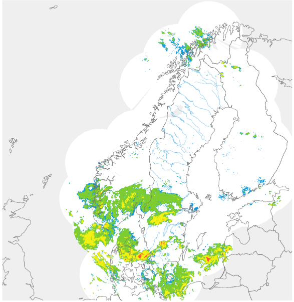
Weather radar (www.smhi.se)

Frequency 5.6 GHz, wavelength 5 cm, range 240–250 km.

Radar in Vilebo



BALTEX stations



Resulting image

Design tradeoffs

Simple estimates for signal-to-noise ratio SNR, range resolution $\mathit{\Delta} R$, and Doppler resolution (velocity resolution) $\mathit{\Delta} v$:

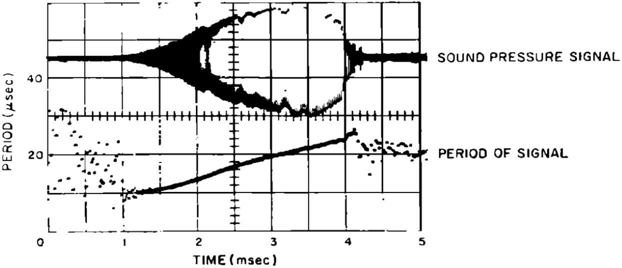
$$\mathrm{SNR} = \frac{\mathcal{E}_{\mathrm{rec}}}{N_0} \approx \frac{P_{\mathrm{rec}} T}{N_0}, \quad \mathit{\Delta} R \approx \frac{c}{2B}, \quad \mathit{\Delta} v \approx \frac{c}{2 f_0 T}$$

For good range *and* Doppler resolution, the product BT must be large.

So how to design a pulse with large bandwidth and long duration?

The bats have a suggestion!

Wavelength $\approx (340\,\mathrm{m/s}) \cdot (20\,\mu\mathrm{s}) = 7\,\mathrm{mm}.$

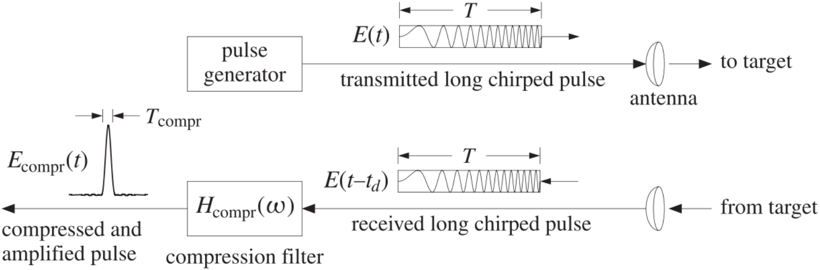


(Altes & Titlebaum, 1970)

Change the frequency linearly with time (chirp).

Chirp radar system

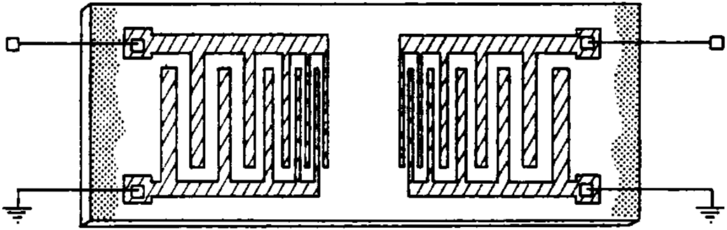
Stretch the pulse before sending it out, compress it when coming back.



(Fig. 3.10.1 in Orfanidis)

Realization of chirp filter: SAW

A chirp filter can be realized by a *surface acoustic wave* device.



The device couples electromagnetic energy to acoustic waves, where the coupling is strongest when the distance between the metal fingers correspond to $\lambda/2$ for the acoustic wave. Chirping is obtained by different acoustic propagation lengths. Works up to about $3\,\mathrm{GHz}$.

Pulse compression

Chirped pulse (input)

$$E(t) = F(t)\mathrm{e}^{\mathrm{j}\omega_0 t + \mathrm{j}\dot{\omega}_0 t^2/2}$$

Compression filter

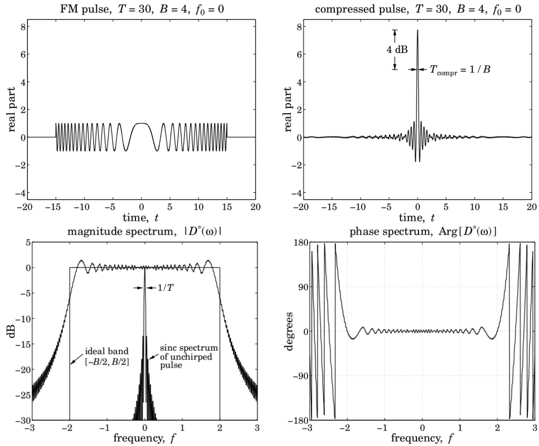
$$h_{\mathrm{compr}}(t) = \sqrt{\frac{\mathrm{j}\dot{\omega}_0}{2\pi}}\mathrm{e}^{\mathrm{j}\omega_0 t - \mathrm{j}\dot{\omega}_0 t^2/2} \quad \Leftrightarrow \quad H_{\mathrm{compr}}(\omega) = \mathrm{e}^{\mathrm{j}(\omega-\omega_0)^2/(2\dot{\omega}_0)}$$

Compressed pulse

$$E_{\mathrm{compr}}(t) = [h_{\mathrm{compr}} * E](t) = \cdots = \sqrt{\frac{\mathrm{j}\dot{\omega}_0}{2\pi}}\mathrm{e}^{\mathrm{j}\omega_0 t - \mathrm{j}\dot{\omega}_0 t^2/2}\hat{F}(-\dot{\omega}_0 t)$$

Compression since if $F(t)$ is wide, then $\hat{F}(\omega)$ is narrow.

Rectangular pulse

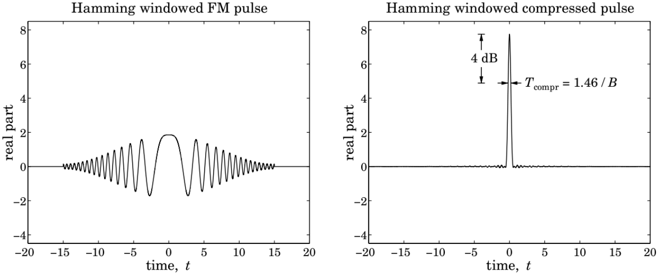


(Figs. 3.10.3–3.10.4 in Orfanidis)

$$\hat{E}(\omega) = \sqrt{\frac{2\pi \mathrm{j}}{\dot{\omega}_0}}\, \mathrm{e}^{-\mathrm{j}(\omega-\omega_0)^2/(2\dot{\omega}_0)} D^*(\omega)$$

Pulse compression

A smoother window function like the Hamming $w(t) = 1 + 2\alpha\cos(2\pi t/T)$ for $-T/2 < t < T/2$ and zero otherwise, can reduce the sidelobes.



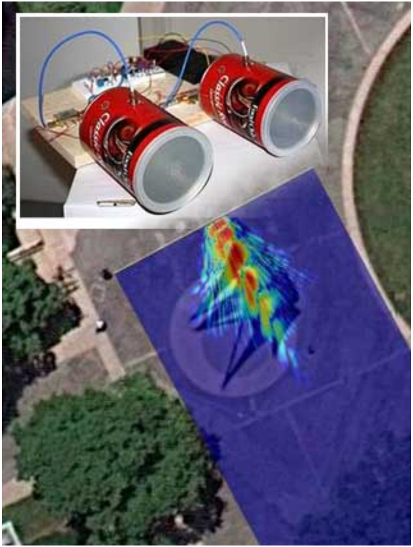
(Fig. 3.10.5 in Orfanidis)

Even better with matched filters, $h(t) = E^*(-t)$ and $H(\omega) = \hat{E}^*(\omega)$. Implemented digitally after recording impulse response (time reversal) or by SAW.

Advertisement: Build your own radar!

MIT inspired course (click on the image to the right or google "MIT build radar").

- Design, build and test your own simple radar system.
- Trial run during fall 2013 for PhD students.
- Hopefully a version will be available for undergraduates in spring 2014.
- ETEN10 Antenna Technology in ht2 is a good preparation!



Outline

1. Propagation of narrow-band pulses
2. Group velocity as velocity of peak: linear approximation
3. Pulse broadening: quadratic approximation
4. Slow, fast, and negative group velocities
5. Application: chirp radar and pulse compression
6. **Conclusions**

Conclusions

- Narrow-band pulses propagate with the group velocity.
- Dispersive media leads to pulse broadening.
- High and low frequencies are spread throughout the pulse.
- Dispersion leads to finite lengths of propagation in a communication application.
- The dispersion can to some extent be counteracted by suitable filters.
- Pulses can be tailored to specific needs.