

Three-dimensional Phased Array Antenna Analysis and Simulation

Zhao Yan-Qiu¹,

2nd research department of China Shipbuilding
Industry Corporation No.723 Research
CSIC No.723
China, Yangzhou
60311120@163.com

Peng Zong²

College of Aeronautics, Nanjing University of
Astronautics and Aeronautics
NUAA
China, Nanjing,
pengzong@nuaa.edu.cn

Abstract - The beam width of planar phased array antenna becomes large as increasing of the scanning angle, so that the scanning range is limited. Conformal phased array antennas might be applied to improve the scanning range. However, the element distribution of a conformal phased array is not regularization, so the analysis is very complicate. In this article, a three-dimensional (3D) array antenna is investigated and implemented on Matlab. 3D array antenna can form different types of arrays to scan any space through choosing different elements. This paper introduces a general direction function of 3D array antenna and simulations of three typical types of arrays formed: diagonal planar array, trapezoidal array and curved surface array. It also investigates the differences of the beam patterns.

Keywords-Phased array antenna, three dimensional (3D) array antenna, beam pattern, curved surface array, element spacing

I. INTRODUCTION

Usually, phased array antenna means planar array antenna, and scanning range can be up to 120°. The analytical model of planar array antenna is simple, and implementation technology becomes perfect gradually. However it has some shortcomings in [5-7]: such as scanning range is too limited; antenna-gain and angle measurement accuracy decrease with beam-width increasing; and sidelobe level increases with increasing the scanning angle. In order to scan the entire space, the plane of phased array is conducted by driving machinery to make up the limitation of scanning angle.

Conformal phased array antenna, a kind of phased array antenna, has incompatible advantages comparing to common phased array antennas. Such as it does not degrade existing performance of aerodynamics of a carrier and its scanning angle is larger than the common planar array's. However, because of various reasons as following in [8]: (1) different position of array element makes directional pattern various; (2) the principle of antenna array multiplication can not be directly used; (3) computation of mutual coupling among the sensors is very complicated. All these reasons make the study on conformal phased array very difficult.

By applying three-dimensional arrays, we do not need mechanical device to rotate 2D phased array, but only choose different elements composing of multiple phased arrays corresponding to the individual maximum direction of their own; it achieves whole space electronic scanning functions. Consequently, it gets ride of the disadvantages of mono-array antenna electronic scanning. The array elements fall into regular distribution and have defined position relationship of each other, thus it can directly use the antenna array multiplication principle.

II. ANTENNA ARRAY MULTIPLICATION PRINCIPLE

For arbitrary N elements array, all elements have the same directional function. Since different elements have different propagation path r , and the phase lag $e^{jk\hat{r}\cdot r}$, thus the exciting current amplitude in is different and so is the exciting phase $e^{j\alpha_n}$. Hereby the electromagnetic field strength of array antenna is given in [9]

$$E(r) = E_0 f(\theta, \varphi) \sum_{n=1}^N I_n e^{j\alpha_n + jk\hat{r}\cdot r} \quad (1)$$

The directional function $D(\theta, \varphi)$ is given by

$$D(\theta, \varphi) = |f(\theta, \varphi) F(\theta, \varphi)| \quad (2)$$

Where $f(\theta, \varphi)$ denotes the directional function of each antenna element. $F(\theta, \varphi)$ denotes the array factor and is given by

$$F(\theta, \varphi) = \sum_{n=1}^N I_n e^{j\alpha_n + jk\hat{r}\cdot r} \quad (3)$$

Equation (2) obeys to the antenna array multiplication principle: the directional function can be considered to be the product of the directional functions of antenna element and the array. The directional function of the array depends on the position of each

element and amplitude and phase of the exciting current.

For convenience of calculations, we assume that the radiated pattern of each array antenna element is isotropic, thus $f(\theta, \varphi) = 1$. The direction of array is determined by the array factor. In following discussion we only concern the array factor $F(\theta, \varphi)$.

III. PHASED ARRAY ANTENNA

Generally, phased array antenna means one-dimensional (1D) linear array or two-dimensional (2D) planar array. When the elements are arranged properly in a space, they can form three-dimensional (3D) array or conformal array.

A. 2D Rectangular-shaped Array (RsA)

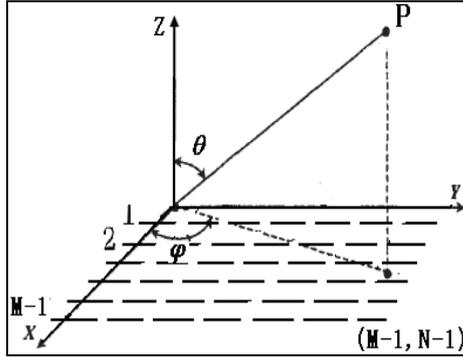


Figure 1. Planar array antenna

2D Rectangular-shaped Array (2DRsA) is shown in Fig. 1. The entire array is a matrix of $N \times M$ elements. Each element has the same amplitude of current and the phases increasing or descending based on equal series along x axis or along y axis. The phase of exciting current is $\alpha = kd_x \sin \theta_s \cos \varphi_s$ ($k = 2\pi/\lambda$) shifting along x axis and $\beta = kd_y \sin \theta_s \sin \varphi_s$ shifting along y axis. And the directional of main lobe is at (θ_s, φ_s) .

The directional function is given by

$$F(\theta, \varphi) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} e^{jn(kd_y \sin \theta \sin \varphi - \beta)} e^{jm(kd_x \sin \theta \cos \varphi - \alpha)} \quad (4)$$

B. 3D Rectangular-shaped Array (RsA)

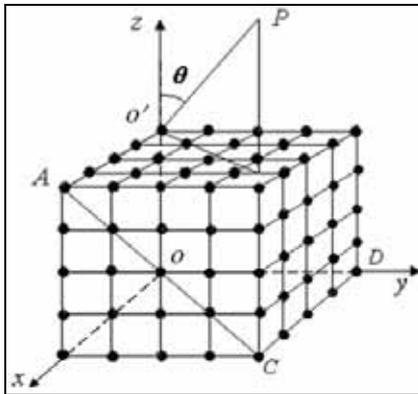


Figure 2. 3D Rectangular-shaped Array

The 3D Rectangular-shaped Array (3DRsA) is shown in Fig. 2, and we establish the presented coordinates, where o is the coordinates origin. With N elements in each x -axis, M elements in each y -axis and H elements in each z -axis, so the entire array has $N \times M \times H$ elements. The array scans above the x - y plane, and the scanning angle can be expressed by elevation angle θ and azimuth angle φ . The phase shift of the exciting current along x, y, z -axis are α, β and γ , and the element spacing is d_x, d_y , and d_z along x, y, z -axis respectively. The element spacing d_{mx}, d_{ny} and d_{hz} can be uniform or disunion.

So the lag phase of each element increases (or decreases) $\alpha_m + \beta_n + \gamma_h$ in turn, where $\alpha_m = kd_{mx} \sin \theta_s \cos \varphi_s$, $\beta_n = kd_{ny} \sin \theta_s \sin \varphi_s$, and $\gamma_h = kd_{hz} \cos \theta_s$.

The directional function of single element is

$$f_{hnm}(\theta, \varphi) = (e^{j(kd_{hz} \cos \theta - \gamma_h)} e^{j(kd_{ny} \sin \theta \sin \varphi - \beta_n)} e^{j(kd_{mx} \sin \theta \cos \varphi - \alpha_m)}) \quad (5)$$

The directional function of 3DRsA antenna is

$$F(\theta, \varphi) = \sum_{h=0}^{H-1} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} (e^{j(kd_{hz} \cos \theta - \gamma_h)} e^{j(kd_{ny} \sin \theta \sin \varphi - \beta_n)} e^{j(kd_{mx} \sin \theta \cos \varphi - \alpha_m)}) \quad (6)$$

Now, let's discuss the 3DRsA in the established coordinates as shown in Fig. 2, scanning above x - y plane, and the uniform spacing along x, y and z -axis.

The directional function is given by

$$F(\theta, \varphi) = \sum_{h=0}^{H-1} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} (e^{jh(kd_z \cos \theta - \gamma)} e^{jn(kd_y \sin \theta \sin \varphi - \beta)} e^{jm(kd_x \sin \theta \cos \varphi - \alpha)}) \quad (7)$$

As $d_x = d_y = d_z = 0.5\lambda$, we choose multi-layers and 100 elements each layer to simulate.

The simulation result of the beam pattern of 3 layers 3DRsA is shown in Fig. 3. Fig. 4 is the beampattern of a mono-layer, also named 2DRsA.

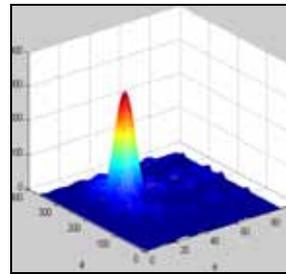


Figure 3. 3DRsA (3 layers)

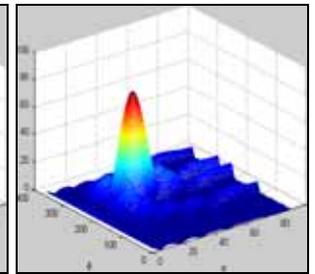


Figure 4. 2DRsA

For ordinary planar array, the mainlobe width decreases and the number of sidelobe increases when the number of element increases. There is little change

for the mainlobe width of multi-layers 3DRsA, only the gain increases and sidelobe level decreases when the number of element increases. But comparing to the planer array having same elements, its mainlobe is wider.

Supposed the mainlobe of 3DRsA is pointed to this direction (θ_s, φ_s) . The phase of the h^{th} layer shifts $e^{jh(kd_z \cos\theta - \gamma)}$ ahead than that of the lowest layer. So the directional function can be written as

$$|F(\theta, \varphi)| = \left| \frac{(1 - e^{jH(kd_z \cos\theta - \gamma)})(1 - e^{jN(kd_y \sin\theta \sin\varphi - \beta)})}{(1 - e^{j(kd_z \cos\theta - \gamma)})(1 - e^{j(kd_y \sin\theta \sin\varphi - \beta)})} \right| \cdot \left| \frac{(1 - e^{jM(kd_x \sin\theta \cos\varphi - \alpha)})}{(1 - e^{j(kd_x \sin\theta \cos\varphi - \alpha)})} \right| \quad (8)$$

The extrema are at

$$\begin{cases} N(kd_x \sin\theta \cos\varphi - \alpha) = \pm 2n\pi \\ M(kd_y \sin\theta \sin\varphi - \beta) = \pm 2n\pi \\ H(kd_z \cos\theta - \gamma) = \pm 2n\pi \end{cases} \quad n=0,1,2,\dots \quad (9)$$

where

$$\begin{cases} \alpha = kd_x \sin\theta_s \cos\varphi_s \\ \beta = kd_y \sin\theta_s \sin\varphi_s \\ \gamma = kd_z \cos\theta_s \end{cases} \quad (10)$$

From (9), we can find the new extrema appear only related to θ . Because phase changing with d_z is only related to pitch direction or elevation angle θ , thus it only affects the mainlobe width against elevation angle θ , does not affect mainlobe width against azimuth angle φ . If d_z increases, mainlobe width decreases at θ direction.

The following Fig. 5 and Fig. 6 are simulation for $d_x=d_y=0.5\lambda$, $d_z=0.5\lambda$ and $d_z=2.5\lambda$. When $d_z=2.5\lambda$, although the mainlobe width decreases at pitch angle θ direction, almost does not change at azimuth angle φ direction, but there are more sidelobes appear.

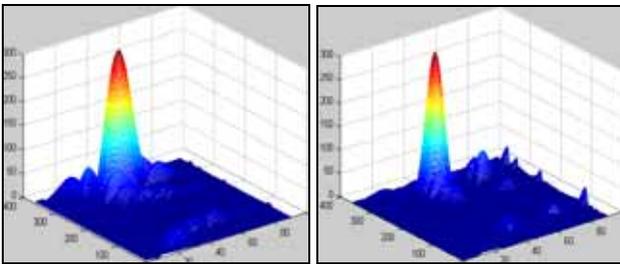


Figure5. $d_z=0.5\lambda$

Figure6. $d_z=2.5\lambda$

For a given height of z axis, the lower the element spacing of d_z is, the more the element is and the higher the gain is. Besides, when scanning θ increases, there may be appear high lobe. If all elements of 3DRsA contribute to scanning at the same time, its computing load is very large. So considering select partial array to

scan, if d_z is too great, there may be appear grating lobe or high lobe. Therefore, d_z can not be too large.

1) Diagonal planar array (DpA)

If only choosing the elements in x - y plane of the 3D array, they form a normal planer array. We can also choose other elements, such as in the $ACDO'$ plane (the cubical diagonal plane). It can be regarded as y - z plane rotating an angle ζ on the x -axis.

For studying the characteristics and scanning range of selected elements, we can reestablish the coordinates and construct the elements into an array on x' - y' plane. In Fig. 2 the elements are selected in $ACDO'$ plane, x' -axis is coincident with x -axis. Cartesian Coordinates transformation for z' -axis, and y' -axis adheres to the right-hand rule.

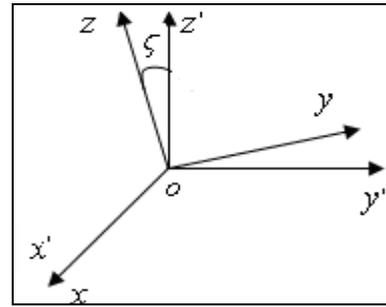


Figure7. Cartesian Coordinates

As shown in Fig. 7, x - y - z is the original coordinates and x' - y' - z' is the new coordinates. If $d_x=d_y=d_z$, ζ is 45° . The relationship between them can be expressed by the transformation equation

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = L_x(\zeta) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\zeta & -\sin\zeta \\ 0 & \sin\zeta & \cos\zeta \end{bmatrix} \begin{bmatrix} \sin\theta \cos\varphi \\ \sin\theta \sin\varphi \\ \cos\theta \end{bmatrix} \quad (11)$$

The new (θ', φ') in new coordinates is calculated and the scanning range can be obtained under the new coordinates easily.

We can also choose other diagonal planar array, which can be considered as rotating different angle ζ on the different axis. If the array composed by selected elements is not a right diagonal planar array, it can be reestablished a coordinates based on a plane of selected elements as x' - y' plane even if it is an approximate plane. The new coordinates plane can be realized through several elementary rotations from the original coordinates one.

2) Trapezoidal Array

Fig. 8 and Fig. 9 show the side face (y - z plane) of 3DRsA, in which the elements are selected along the diagonal line and kept consistence along x -axis, therefore it forms a diagonal planar array.

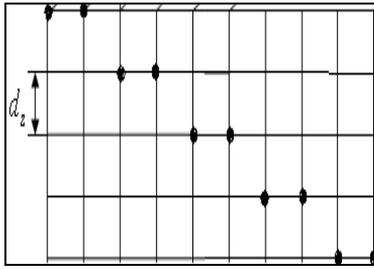


Figure 8. Trapezoid 1

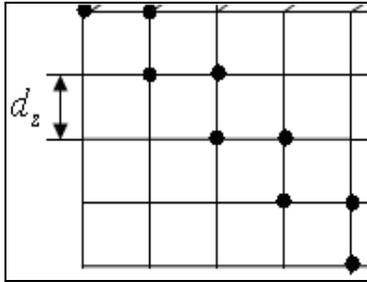


Figure 9. Trapezoid 2

The selected elements of the side face compose a trapezoid array. Trapezoid 1 can be regarded as a combination of two-layer slope array. Trapezoid 2 can be regarded as a combination of two-layer diagonal planar array.

For a 2DRsA, the element spacing only is d_x or d_y , whereas for 3DRsA, d_z is in addition to space the elements along z axis. The different of d_z also affects the beam width.

When $d_x=d_y=d_z=0.5\lambda$, the coordinates is still the same in Fig. 2, the mainlobe direction $(\theta_s, \phi_s) = (45^\circ, 80^\circ)$ and the number of elements is 70, The simulations are taken for Trapezoid 1 and Trapezoid 2. The beam pattern or the scanning characteristics of Trapezoid 1 and Trapezoid 2 are similar to the planar array.

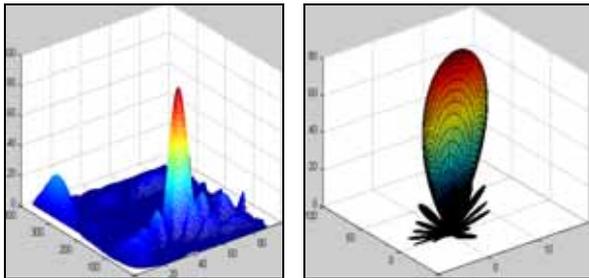


Figure10. Beampattern of Trapezoid 1

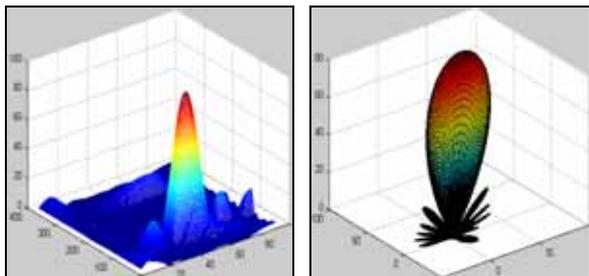


Figure 11. Beampattern of Trapezoid 2

Fig. 10 is the simulation result of Trapezoid 1, indicates that the effect of changing d_x and d_y just likes that of the planar array. The mainbeam becomes wide and grating lobe or high-level sidelobe appears when d_x or d_y increases. The mainbeam width decreases but sidelobe level rise when d_z increases.

Fig. 11 shows the effect of changing d_x , d_y and d_z of Trapezoid 2, which is similar, but not significant as that of Trapezoid 1, shown in Fig. 10.

We can reestablish the coordinates by rotating the diagonal or slope plane as $x'-y'$. The element spacing $d_{y'1}$ is greater than element spacing $d_{y'2}$ in the new coordinates and area of Trapezoid 1 is larger than that of Trapezoid-2. Therefore, mainbeam width of Trapezoid 1 is smaller than that of Trapezoid 2, but the grating lobe is easy to appear.

3) Curved surface Array (CsA)

The elements in 3DRsA can also be selected around the curved surface to approximately construct a Cured Surface Array (CsA). The rule of constructing the cured surface is to find the grid node which is the most close to the surface.

Fig. 12 is the side image of Cured Surface Array (CsA) whose elements are 70, the equivalent element number of projection on $x-y$ plane is about 42.

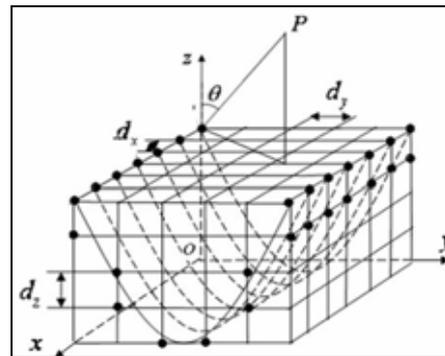


Figure12. Curved surface Array

Fig. 13 is the pattern of CsA in spherical coordinates system. Fig. 14 is the pattern of a 42-element 2DRsA. Both mainbeam directions are $(\theta_s, \phi_s) = (40^\circ, 60^\circ)$. Two mainlobe patterns are similar, but the sidelobes are quite different. The sidelobe level of the CsA is higher than that of 2DRsA.

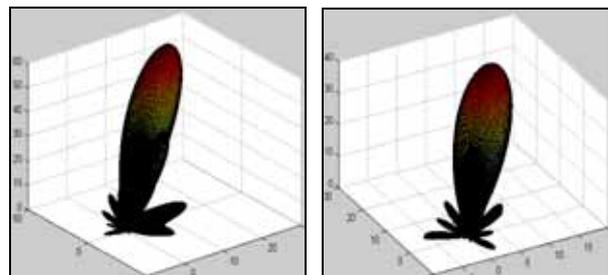


Figure 13. CsA beampattern

Figure14. 2DRsA beampattern

Fig. 15 and Fig. 16 are beam patterns of the CsA and 2DRsA respectively, $d_x=d_y=d_z=0.5\lambda$, and the mainlobe direction $(\theta_s, \phi_s)=(70^\circ, 140^\circ)$. The mainlobe of 2DRsA becomes wider when the scanning angle increasing, whereas the mainlobe of the CsA changes very little.

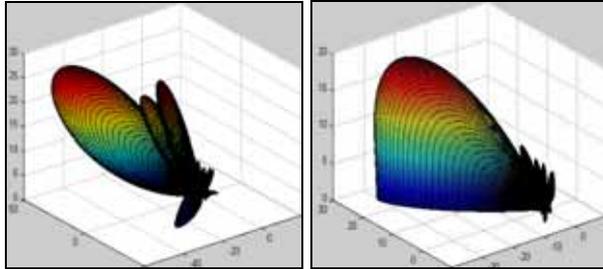


Figure 15. CsA beam pattern Figure 16. 2DRsA beam pattern

In the above discussions, we do not concern the symmetrical pattern, which is common for the plane arrays. For a multi-layer diagonal planar array or an approximate planar array, which has a certain included angle with x - y plane, like DpA, the range of θ may be in the range of $0\sim\pi/2$ or beyond $\pi/2$. There may be symmetrical lobes at the scanning angle.

IV. THE SELECTION OF ELEMENT SPACING

The greater the element spacing is, the narrower mainbeam width is, however the higher possibility of grating lobe appearing is. Let's determine the maximum element spacing. The scanning range of 2DRsA shown in Fig. 1, is cone-shaped whose symmetrical axis is z -axis (scanning above x - y plane) and the angle is θ . The maximum element spacing of d_x and d_y can be determined in [10].

$$\begin{cases} d_{x\max} = \frac{\lambda}{1 + \sin \theta_{z\max}} \\ d_{y\max} = \frac{\lambda}{1 + \sin \theta_{z\max}} \end{cases} \quad (12)$$

About 3DRsA, there adds a element d_z , we can divide the whole space into several ranges. For each range, we may choose different elements to scan, and the scanning ranges might be a part of cone-shaped. The lower θ is, the narrower main lobe is. Last, according to the maximum element spacing of d_x , d_y and d_z of every part, we determine the maximum element spacing.

The principle for classification of the scanning ranges is to ensure that adjacent scanning range should not have mesh hole or dark area and the whole space can be scanned, then determine the maximum element spacing.

V. CONCLUSIONS

Since mono-layer or conformal array is not enough to meet the demand of antenna gain and pattern, Multi-layer element or 3D array antenna can be adopted to increase the gain and scanning range. We can select

elements to construct arbitrary arrays for scanning. They beehive as mixed multiple planar arrays. Thus, using single antenna can realize scanning the whole space. Surely, the element combination is not constrained by three types of arrays introduced in this article.

REFERENCES

- [1] Chen MengRao, Xu FuYong and Zhao KeYu, *Electromagnetic and Microwave Technique*, High Education Publication House, 1989.
- [2] Wang Wei, Wang JiLi, *Microwave Technique and Antenna*, Northern-West University Publishing House, 1986.
- [3] Jin RongHong, Geng JunPing and Fan Yu, *Smart Antenna in Radio Communication*, Beijing Post University Publishing House, 2006.
- [4] Joseph C. Liberti, JR. and Theodore S. Rappaport, *Smart Antenna for Wireless Communications: IS-95 and Third Generation CDMA Applications*, Prentice Hall, 1999.
- [5] Yu Yan-min, Zhao Jian-zhong and Wu Wen, "Research on Conformal Wraparound Millimeter Wave Microstrip Antenna," *Guidance & Fuze*, 26(3): 43~46, 2005.
- [6] Lv Shan-Wei, *Antenna Array Synthesis*, Beijing: Beijing Aeronautical College Publication, pp.91~97, 1988.
- [7] Ling Chang-Lu and Nie Zai-ping, *Engineering Handbook for Antenna*, Beijing: Electronic Industry Publishing House, pp. 656~ 660, 2002.
- [8] Xiao Lei, "Analysis of Microstrip Conformal Antenna Based on Cylindrical Carriers," *University of Electronic Science and Technology of China*, 2004.
- [9] Lu Wan-Zheng, *The Theory and Technique of Antenna*, Xi-an University of Electronic Science and Technology of China Press, pp.259~260, 2004.
- [10] Guo Yan-Chang, Qian Ji-ceng, Huang Fu-xiong and Feng Zu-wei, *The principle of Phased Array Antenna and Frequency Scanning*, Beijing: Defense Industry Publishing House , pp.48~56, 1978.
- [11] Zhao Yan-Qiu, Peng Zong and Lu Yu-Ping, A Novel Multi-beam Array Antenna for Tracking Formation Satellites, Eleventh International Space Conference of Pacific-basin Societies, 2007.05